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A study of developing an input-oriented ratio-based comparative efficiency model

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ABSTRACT

Data envelopment analysis (DEA) is a representative method to estimate efficient frontiers and derive efficiency. However, in a situation with weight restrictions on individual input–output pairs, its suitability has been questioned. Therefore, the main purpose of this paper is to develop a mathematical method, which we call the input-oriented ratio-based comparative efficiency model, DEA-R-I, to derive the input-target improvement strategy in situations with weight restrictions. Also, we prove that the efficiency score of DEA-R-I is greater than that of CCR-I, which is the first and most popular model of DEA, in input-oriented situations without weight restrictions to claim the DEA-R-I can replace the CCR model in these situations. We also show an example to illustrate the necessity of developing the new model. In a nutshell, we developed DEA-R-I to replace CCR-I in all input-oriented situations because it sets a more accurate weight restriction and yields a more achievable strategy.

1. Introduction

Data envelopment analysis (DEA) is one popular method for identifying efficient frontiers and evaluating efficiency. An efficient frontier is based on the concept of a non-dominated condition, which was first expressed by the Italian economist Pareto in 1927. This concept was adapted to production by Koopmans in 1951 and to evaluate efficiency by Farrell in 1957 (Cooper et al., 2002). Charnes, Cooper, and Rhodes (1978) applied linear programming (LP) to identify efficient frontiers and measure productivity. This method, which measures productivity by LP, is called "data envelopment analysis". They derived both an output-oriented (CCR-O) model and an input-oriented (CCR-I) model, which are not only the first but also most popular models of DEA. Many scholars have used DEA as the representative method to estimate an efficient frontier and measure productivity (Amirteimoori, 2007; Jahanshahloo, Hosseinzadeh Lotfi, & Zohrehbandian, 2005a). Over the past two decades, DEA has been established as a robust and valuable methodology (Chen & Ali, 2002; Liu & Chuang, 2009).

One advantage of DEA is objective weight selection, and there are many studies that focus on weight (Bernroider & Stix, 2007; Jahanshahloo, Soleimani-damaneh, & Nasrabadi, 2004; Jahanshahloo, Memariani, Hosseinzadeh, & Shoja, 2005b; Lotfi, Jahanshahloo, & Esmaeili, 2007; Wang, Parkan, & Luo, 2008). However, when applying the typical DEA model, which is based on $(\sum vx)/(\sum uy)$ or $(\sum uy)/(\sum vx)$, to a situation with weight restrictions on individual input–output pairs, its suitability is questionable. We take a case in hospitals as the example of the necessity of weight restrictions. Sickbeds, physicians, outpatients, inpatients, and surgery are important variables for hospital performance evaluations, where the sickbed variable contributes only to the inpatient and surgery variables but not the outpatient variable. In this situation, it is hard to assign a suitable weight restriction to an outpatient-sickbed pair. Golany and Roll (1989) argue that input–output pairs must correspond to an isotonicity assumption to avoid this problem. However, an isotonicity assumption represents a statistical rather than a causal relationship. For example, the statistical relationship between outpatient services and sickbeds is high, but the causal relationship between them is low. Therefore, conformance to the isotonicity assumption does not always avoid this problem. Dyson et al. (2001) argue that handling weight restrictions is still a pitfall in DEA applications from a theoretical perspective. Despic, Despic and Paradi (2007) claim that this kind of problem is difficult to solve with a typical DEA model and therefore developed DEA-R, a model to solve the problem of weight restriction. DEA-R is expressed as follows:

$$\hat{\hat{e}}_k = \max_{c_{(j,i)}\geqslant 0}\left\{\sum_{j=1}^{s}\sum_{i=1}^{m}c_{(j,i)}r_{(j,i)k}\,\middle|\,\sum_{j=1}^{s}\sum_{i=1}^{m}c_{(j,i)}r_{(j,i)p}\leqslant 1,\ \text{for } p=1,2,\ldots,n\right\}. \quad (1)$$

However, the DEA-R model developed by Despic et al. (2007) is an output-oriented model (we call it DEA-R-O). In some situations, we

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need an input-oriented model to provide an input-target improvement strategy with weight restrictions. Using Taiwan's private hospitals as an example again, the output was bounded by National Health Insurance; they have to adopt an input-targeted improvement strategy (reduce inputs), rather than an output-targeted strategy, to improve their efficiency. Therefore, a new mathematical method of deriving the input strategy (we call it input-oriented DEA-R, or DEA-R-I) has been developed.

In addition, the DEA-R-I seems to substitute for CCR-I in input-oriented situations without weight restrictions because the efficiency score of DEA-R-I is greater than or equal to than CCR-I when the relationship between DEA-R-O and CCR-O is unclear. According to Despic et al. (2007), the efficiency score of DEA-R-O with no weight restrictions is sometimes higher and sometimes lower than the efficiency score of CCR-O. This drawback prevents DEA-R-O from replacing CCR-O in a situation without weight restrictions. But, based on our study, we found two factors that cause this efficiency score discrepancy. The first is a more flexible selection of optimum weight, which affects the efficiency score of DEA-R-O higher than the efficiency score of CCR-O, while the second is the sum of the output-oriented ratio $\sum\left(w \times \frac{y}{x}\right)$, which affects the efficiency score of DEA-R-O less than the efficiency score of CCR-O. Since we will use the sum of the input-oriented ratio $\sum\left(w \times \frac{x}{y}\right)$ to replace the sum of the output-oriented ratio $\sum\left(w \times \frac{y}{x}\right)$ in computing the efficiency score in the DEA-R-I mathematical method, we suggest that the efficiency score of DEA-R-I will always be greater than or equal to the efficiency score of CCR-I (CCR input-oriented). This also means that the strategies of DEA-R-I are easier to achieve than the strategies of CCR-I because the strategy derived from the higher efficiency score needs fewer changes. If we can prove this hypothesis, the CCR-I model can be replaced by DEA-R-I because DEA-R-I provides a more accurate efficiency score in situations with weight restrictions and a better strategy in situations without weight restrictions.

Because of the above reasons, the first goal of this paper is to develop a mathematical method (we call it DEA-R-I) to derive the input-target improvement strategy in a situation with weight restrictions. The second goal is to prove that the input-target improvement strategy developed by DEA-R-I is always better than the CCR-I model in a situation without weight restrictions. Therefore, we can claim that the DEA-R-I model can replace the CCR-I model in all input-oriented situations.

2. Mathematical method to evaluate efficiency scores and derive input-target improvement strategies

Because there are no suitable input-oriented models for situations with weight restrictions on single I/O pairs, we developed a new model to evaluate the efficiency score and derive the input-oriented strategy. We applied a new model to calculate the efficiency score and then derive the input-target strategy from the efficiency score:

Step 1: Compute the efficiency score

DEA-R-I, the mathematical model for computing the efficiency score of a DMU object $\hat{\theta}_o$, is expressed as follows:

$$\text{Max } \hat{\theta}_o \tag{2}$$

$$\text{st.} \quad \sum_{i=1}^{m}\sum_{r=1}^{s} W_{ir}\frac{(X_{ij}/Y_{rj})}{(X_{io}/Y_{ro})} \geqslant \hat{\theta}_o \; j = 1,\ldots,n \tag{3}$$

$$\sum_{i=1}^{m}\sum_{r=1}^{s} W_{ir} = 1 \tag{4}$$

$$W_{ir} \geqslant 0, \quad \hat{\theta}_o \geqslant 0 \tag{5}$$

$X_{i\,j}$: ith input variable of the jth DMU.
$Y_{r\,j}$: rth output variable of the jth DMU.
$X_{i\,o}$: ith input variable of the object.
$Y_{r\,o}$: rth output variable of the object.
W_{ir}: The weight of the ratio of $\frac{\text{the } i\text{th input variable } X_i}{\text{the } r\text{th output variable } Y_r}$.

$\sum_{i=1}^{m}\sum_{r=1}^{s} W_{ir}(X_{ij}/Y_{rj})/(X_{io}/Y_{ro})$: the relative efficient score with jth DMU's.

For each DMU object, the model first computes its relative efficiency score for each specified weight, and the smallest is selected as the efficiency score of this set of weights. Second, by adjusting the weighting set, a maximum efficiency score will be selected as the efficiency score $\hat{\theta}_o$ of the object. Since each DMU can get its optimal weight, we can say objectively that the DMU is inefficient if the efficiency score of this DMU is less than one. It is necessary to provide the improved strategy for this DMU, which we will discuss in next part. Using the data in Table 1 as an example, if we want to calculate the efficiency score of the DMU1 of Table 1, we must first find four relative efficiency scores for the DMU 1 in each weight.

When the weight set is $W_{11} = 1$ and $W_{12} = 0$, the relative efficiency scores of DMU 1 with DMUs 1–4 are $1 \times \left(\frac{2}{4}\right)/\left(\frac{2}{4}\right) + 0 \times \left(\frac{2}{3}\right)/\left(\frac{2}{3}\right) = 1.00$, $1 \times \left(\frac{2}{3}\right)/\left(\frac{2}{4}\right) + 0 \times \left(\frac{2}{5}\right)/\left(\frac{2}{3}\right) = 1.33$, $1 \times \left(\frac{2}{4.2}\right)/\left(\frac{2}{4}\right) + 0 \times \left(\frac{2}{4.2}\right)/\left(\frac{2}{3}\right) = 0.95$, and $1 \times \left(\frac{2}{5}\right)/\left(\frac{2}{4}\right) + 0 \times \left(\frac{2}{3}\right)/\left(\frac{2}{3}\right) = 0.80$, respectively (the right-most points of lines 1, 2, 3, and 4 shown in Fig. 1a). The relative efficiency score of DMU 1 with DMU 4 in weight $W_{11} = 1$ and $W_{12} = 0$ is 0.8, which means that if we need one unit of X_1 from DMU 1 to produce one unit of Y_1, only 0.8 units of X_1 from DMU 4 are needed to produce one unit of Y_1. Repeating the computation, the relative efficiency scores of DMU1 with each DMU in different weight sets are shown in Fig. 1a. In Fig. 1a, when W_{11} is between 0.000 and 0.231, the lowest value of the four relative efficiency scores is the relative efficiency score with DMU 2.

Table 1
One input and two-outputs.

DMU	Input	Output	
	X_1	Y_1	Y_2
1(A)	2.0	4.0	3.0
2(B)	2.0	3.0	5.0
3(C)	2.0	4.2	4.2
4(D)	2.0	5.0	3.0

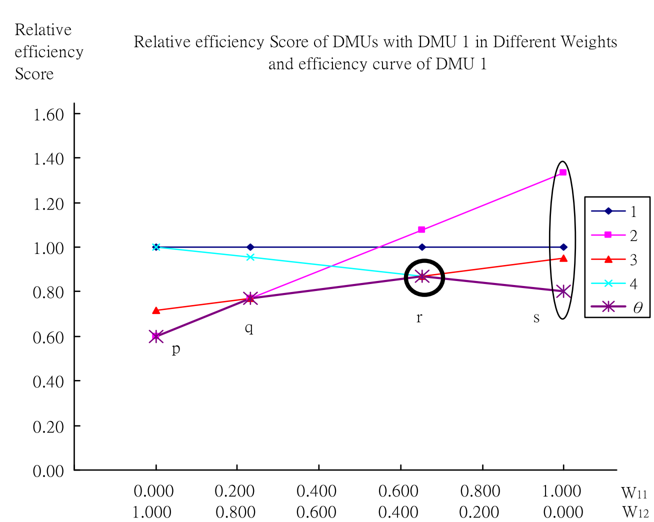

Fig. 1a. Relative Efficiency Score of DMU 1 with DMUs in Different Weights and the Efficiency curve of DMU 1.

When W_{11} is between 0.231 and 0.652, the lowest value of the four relative efficiency scores is the relative efficiency score with DMU 3. Finally, when W_{11} is between 0.652 and 1.000, the lowest value of the four relative efficiency scores is the relative efficiency score with DMU 4. The graph of the lowest values of the four relative efficiency scores in different weight sets is the efficiency curve of DMU 1 (it is not the efficiency frontier of all DMUs but the efficiency of DMU 1 in different weight sets). Since the efficiency score of point r (the least relative efficiency score is 20/23 (about 0.870) when W_{11} = 0.652 and W_{12} = 0.348) on the efficiency curve is the highest relative efficiency score on the curve, we select it as the efficiency score of DMU 1. Repeating the model, the efficiency scores of DMUs 2, 3, and 4 can be found and are shown in Figs. 1b, 1c and 1d, respectively. The efficiency scores of all DMUs are shown in Table 2.

Step 2: Derive the input-target improvement strategy

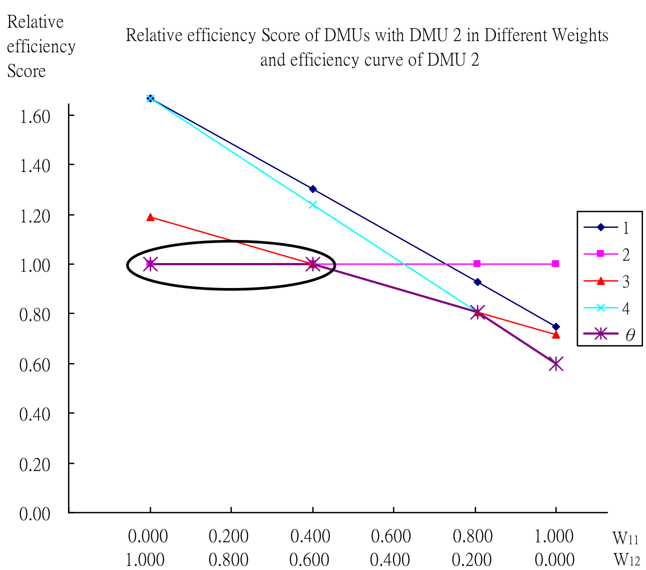

Fig. 1b. Relative Efficiency Score of DMU 2 with DMUs in Different Weights and the Efficiency curve of DMU 2.

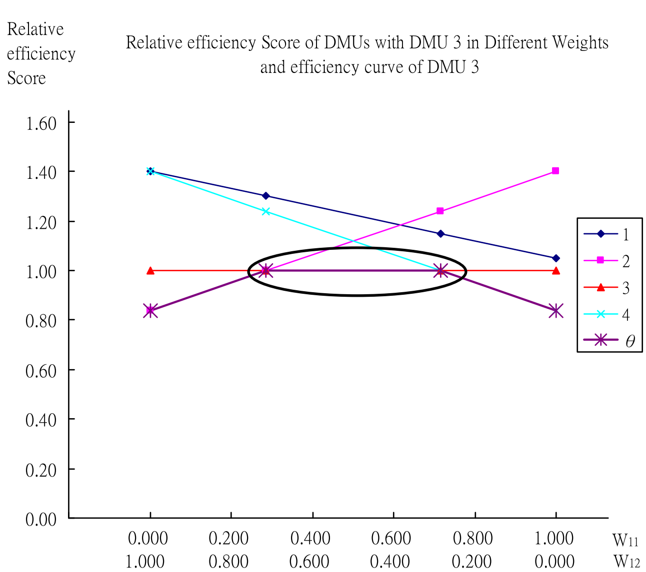

Fig. 1c. Relative Efficiency Score of DMU 3 with DMUs in Different Weights and the Efficiency curve of DMU 3.

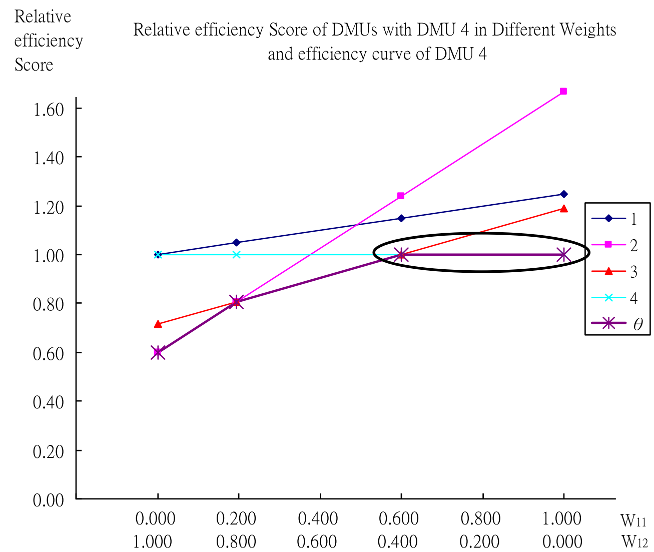

Fig. 1d. Relative Efficiency Score of DMU 4 with DMUs in Different Weights and the Efficiency curve of DMU 4.

Table 2
The efficiency score of DMUs.

DMU	Efficiency score
1(A)	0.870
2(B)	1.000
3(C)	1.000
4(D)	1.000

Table 3
Original data, efficiency score and strategy of DMU 1 in Table 1.

DMU	Original data			Efficiency score	Input-target strategy		
	Input	Output			Input	Output	
	X_1	Y_1	Y_2		X_1	Y_1	Y_2
1(A)	2.0	4.0	3.0	20/23	40/23	4.0	3.0

As we stated above, if the efficiency score of a DMU is less than one, it is inefficient. The improvement now becomes indispensable. We can simply replace each input variable with a new DMU that equals its original data times its efficiency score, without changing the output variables. We call the new DMU our improved strategy. If we replace the inefficient DMU with this new DMU, then the efficiency score of this new DMU is one, and the efficiency score of the others is the same as before. Table 3 shows the improved strategy of the DMU1. The table indicates that in order to make the DMU efficient, the input variable X_1 should be lowered from 2.0 to 40/23 to get the same outputs Y_1 and Y_2.

3. Mathematical proof that the efficiency score of DEA-R-I is greater than or equal to the efficiency of CCR-I

After studying the relationship between DEA-R-O and CCR-O, we suggest that the efficiency score of DEA-R-I is greater than or equal to the efficiency of CCR-I. If this hypothesis holds, then we can claim that DEA-R-I can replace CCR-I because DEA-R-I is more efficient in situations with weight restrictions and more achievable in situations without weight restrictions.

Inspired by Despic et al. (2007), we define the efficiency of CCR-I

$$\theta_o^* = \max_{\substack{\sum_i v_i=1, v_i \geqslant 0 \\ \sum_r u_r=1, u_r \geqslant 0}} \min_j \frac{\sum_i v_i \frac{X_{ij}}{X_{io}}}{\sum_r u_r \frac{Y_{rj}}{Y_{ro}}}$$

and the efficiency of CCR-I-Harmonic

$$\bar{\theta}_o^* = \max_{\substack{\sum_i v_i=1, v_i \geqslant 0 \\ \sum_r u_r=1, u_r \geqslant 0}} \min_j \sum_i v_i \frac{X_{ij}}{X_{io}} \sum_r u_r \frac{Y_{ro}}{Y_{rj}}$$

to help us prove our claim. The proof requires two steps:

Step 1: Proof that the efficiency of CCR-Harmonic $\bar{\theta}_o^*$ is always greater than or equal to CCR $\theta_o{}^*$.

By replacing X'_{ij}, Y'_{ij} with $\frac{X_{ij}}{X_{io}}, \frac{Y_{rj}}{Y_{ro}}$ and multiplying

$$\frac{\sum_r u_r \frac{Y_{ro}}{Y_{rj}}}{\sum_r u_r \frac{Y_{ro}}{Y_{rj}}} = \frac{\sum_r u_r \frac{1}{Y_{rj'}}}{\sum_r u_r \frac{1}{Y_{rj'}}}$$

in formulation, we can transpose θ_o^* to

$$\max_{\substack{\sum_i v_i=1, v_i \geqslant 0 \\ \sum_r u_r=1, u_r \geqslant 0}} \min_j \frac{\sum_i v_i X'_{ij} \times \sum_r u_r \frac{1}{Y'_{rj}}}{\sum_r u_r Y'_{rj} \times \sum_r u_r \frac{1}{Y'_{rj}}} \quad \text{and}$$

$$\bar{\theta}_o^* = \max_{\substack{\sum_i v_i=1, v_i \geqslant 0 \\ \sum_r u_r=1, u_r \geqslant 0}} \min_j \sum_i v_i X'_{ij} \times \sum_r u_r \frac{1}{Y'_{rj}}.$$

The difference between θ_o^* and $\bar{\theta}_o^*$ is

$$\frac{1}{\sum_r u_r Y'_{rj} \times \sum_r u_r \frac{1}{Y'_{rj}}}.$$

Because

$$\begin{aligned}
\sum_r u_r Y'_{rj} \times \sum_r u_r \frac{1}{Y'_{rj}} &= \sum_{\substack{r=1..s \\ t=r+1...s}} u_r^2 + u_r u_t \left(\frac{Y'_{rj}}{Y'_{tj}} + \frac{Y'_{tj}}{Y'_{rj}} \right) \\
&= \sum_{\substack{r=1..s \\ t=r+1...s}} u_r^2 + 2u_r u_t - 2u_r u_t + u_r u_t \left(\frac{Y'_{rj}}{Y'_{tj}} + \frac{Y'_{tj}}{Y'_{rj}} \right) \\
&= \left(\sum_r u_r \right)^2 + \sum_{\substack{r=1..s \\ t=r+1...s}} \frac{u_r u_t}{Y'_{rj} Y'_{tj}} (Y_{rj}\prime - Y_{tj}\prime)^2 \\
&= 1 + \sum_{\substack{r=1..s \\ t=r+1...s}} \frac{u_r u_t}{Y'_{rj} Y'_{tj}} \left(Y'_{rj} - Y'_{tj} \right)^2 \\
&\geqslant 1, \text{ we get } \frac{1}{\sum_r u_r Y'_{rj} \times \sum_r u_r \frac{1}{Y'_{rj}}} \\
&\leqslant 1 \text{ if } \forall\ u_r, Y'_{rj} \geqslant 0
\end{aligned}$$

and infer that $\theta_o^* \quad \leqslant \quad \bar{\theta}_o^*$.

Step 2: Proof that the efficiency of DEA-R-I $\hat{\theta}_o^*$ is always greater than or equal to CCR-Harmonic $\bar{\theta}_o^*$.

We can rewrite the efficiency of DEA-R-I as $\hat{\theta}_o^* = \max_{\substack{\sum_i \sum_r w'_{ir}=1 \\ ,w'_{ir} \geqslant 0}} \min_j \sum_i \sum_r w_{ir} \frac{X'_{ij}}{Y'_{rj}}$ and CCR harmonic $\bar{\theta}_o^* = \max_{\substack{\sum_i v_i=1, v_i \geqslant 0 \\ \sum_r u_r=1, u_r \geqslant 0}} \min_j \sum_i \sum_r v_i u_r \frac{X'_{ij}}{Y'_{rj}}$.. Because $\sum_i v_i = 1$ and $\sum_r u_r = 1$, we get $\sum_i \sum_r v_i u_r = 1$, and we argue that $\bar{\theta}_o^*$ is a special case of $\hat{\theta}_o^*$. The best we obtain is: $\bar{\theta}_o^* \leqslant \hat{\theta}_o^*$. With the two steps, we obtain: $\theta_o^* \leqslant \bar{\theta}_o^* \leqslant \hat{\theta}_o^*$.

4. An example

Like other studies (Ballestero & Maldonado, 2004; Katharaki, 2008), this study takes a hospital as an example to show the necessity of developing DEA-R-I and the advantages of this model. Two-inputs- two-outputs simplified data for four hospitals are shown in Table 4. Because only DMU 5 is inefficient, we show the efficiency scores, improvement strategies, and optimal weight sets that are derived by different models with two oriented situations in Table 5. The upper part of Table 5 shows the results without weight restrictions. Because the sickbed variable has no direct contribution to the outpatient variable, the weight restriction is needed. Although there are not suitable restrictions for CCR, we take the most approximated restriction, $\frac{v_1 x_1}{v_1 x_1 + v_2 x_2} \leqslant \frac{u_2 y_2}{u_1 y_1 + u_2 y_2}$, as the weight restriction of CCR in this case to compare with other models. For the DEA-R models, we set $w_{11} = 0$ as a weight restriction. The results with weight restrictions are shown in the lower part of Table 5.

First, we show the necessity of weight restrictions. Compare the upper part of Table 5 with the lower part. The results show that the efficiency scores of the models without weight restriction are all greater than the same efficiency scores of the models with weight restrictions. Thus, weight restrictions in different models are necessary. Then compare the optimal weight sets of different models with weight restrictions. Although the efficiency score of DEA-R-O with restriction is the same as CCR-O, we show the difference between the optimal weight sets in two output-oriented models. In the input-oriented situation, DEA-R-I derives not only a different efficiency from CCR-I but also a different optimal weight set. In addition, the weight sets derived by DEA-R models are more easily understood than those of CCR because the weight restriction of DEA-R always holds, but the same is not true for CCR. Therefore, we claim that the DEA-R model is more suitable under weight restrictions.

We compare DEA-R-I with DEA-R-O after comparing DEA-R with CCR. Compare the improvement strategies. The left part of Table 5 shows that the strategies of the input-oriented models are different from output-oriented models, both CCR and DEA-R. Then compare the efficiency scores of DEA-R in different orientations. The efficiency scores of DEA-R-I are different from those of DEA-R-O whether or not there are weight restrictions. Unlike the difference between two CCR models, which is only different improvement strategies, the differences between DEA-R-I and DEA-R-O are in both strategies and efficiency. This means that DEA-R-I cannot be replaced with DEA-R-O and that the development of DEA-R-I is necessary.

Finally, compare the DEA-R-I with CCR-I in the situation without weight restrictions. The left part of Table 5 shows that DEA-R-I without weight restrictions has a greater value than CCR-I. This result does not contradict our proof. The middle part of Table 5 shows the improvement strategies, which are translated from the

Table 4
Two-inputs-two-outputs.

DMU	Input		Output	
	Sickbed	Doctor	Outpatient	Inpatient
5(E)	2.0	3.0	4.0	3.0
6(F)	2.0	2.7	3.0	5.0
7(G)	2.0	2.7	4.2	4.2
8(H)	2.0	2.7	5.0	3.0

Table 5
Efficiency scores, strategies and optimal weight sets for DMU 5(E).

Without restriction	θ_o	X'_1	X'_2	Y'_1	Y'_2	v_1x_1	v_2x_2	u_1y_1	u_2y_2
CCR-I	0.857	1.71	2.57	4.00	3.00	1.000	0.000	0.571	0.286
CCR-O	0.857	2.00	3.00	4.67	3.50	1.167	0.000	0.667	0.333
	θ_o	X'_1	X'_2	Y'_1	Y'_2	w_{11}	w_{12}	w_{21}	w_{22}
DEA-R-I	0.870	1.74	2.61	4.00	3.00	0.652	0.348	0.000	0.000
DEA-R-O	0.857	2.00	3.00	4.67	3.50	0.667	0.333	0.000	0.000
With Restriction	θ_o	X'_1	X'_2	Y'_1	Y'_2	v_1x_1	v_2x_2	u_1y_1	u_2y_2
CCR-I	0.800	1.60	2.40	4.00	3.00	0.333	0.667	0.533	0.267
CCR-O	0.800	2.00	3.00	5.00	3.75	0.417	0.833	0.667	0.333
	θ_o	X'_1	X'_2	Y'_1	Y'_2	w_{11}	w_{12}	w_{21}	w_{22}
DEA-R-I	0.811	1.62	2.43	4.00	3.00	0.000	0.324	0.676	0.000
DEA-R-O	0.800	2.00	3.00	5.00	3.75	0.000	0.357	0.643	0.000

efficiency score. DEA-R-I without weight restrictions shows that we need a change of X'_1 from 2 to 1.740 (i.e. 2,000 beds to 1,740 beds) and X'_2 from 3 to 2.610 (that is, 300 doctors to 261 doctors) and keep the same output. CCR-I shows that values of $X'_1 = 1.714$ (1,714 beds) and $X'_2 = 2.571$ (2,571 doctors) are needed. This result shows that the changes of DEA-R-I are less than those of CCR-I in the situation without weight restrictions. Based on these results, we claim that the DEA-R-I strategy is always easier to achieve than the CCR-I strategy. We can conclude that DEA-R-I can replace CCR-I in all input-target situation.

5. Conclusion

In this paper, we developed an input-oriented ratio-based model (DEA-R-I) for calculating efficiency scores and identifying input-target improvement strategies in situations with weight restrictions. We also show further proof of our model in order to claim that this model can replace the CCR-I model in situations without weight restrictions. A numerical example shows the difference between DEA-R-O and DEA-R-I to support our claim that the development of the DEA-R-I model is necessary for input-oriented situations with weight restrictions. This example further supports the claim that DEA-R-I can also provide easier improvement strategies than CCR-I in situations without weight restrictions. Because of its accuracy in situations with weight restrictions and its better strategy, we claim that DEA-R-I can replace CCR-I in all input-oriented situations.

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