行政院國家科學委員會專題研究計畫　期中進度報告

關於 conditional inference 不一致性的研究(1/2)

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計畫主持人： 洪慧念

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一、中文摘要

我們的重點是關於樞紐量的推論，與條件推論一樣，他也是一個用來解決多餘參數模型的重要工具。但因爲樞紐量一定會與感興趣的參數有關，也不是唯一（甚至也不等同），也因此經由樞紐量來造出的概似函數也自然的會受到樞紐量選取的影響。這與條件推估的不一致性有很密切的關係。我們發現，這兩問題是屬於同一類的現象。我們能從微分子的觀點仔細分析概似函數，找出一個一致的答案，也就是說我們期待能找到一個標準的程序。只要尋此程序，條件推論的結果不會受到 conditional 的統計量的影響，且樞紐量的推論也不會受到樞紐量選取的影響。同時，我們也找到一個合理的解釋爲何運用樞紐量來造出的概似函數是一個正確的概似函數。

關鍵詞：樞紐量，多餘參數模型，一致，概似函數

二、英文摘要

The most important thing we have done in this year is about the pivotal inference. This contribution is same as the conditional inference that is a powerful tool for solving the nuisance parameter problem. Since the pivotal quantity is not, in general, unique (or even not equivalent) and involve parameter of interest. Therefore the likelihood function about the parameter of interest derived from the pivotal quantities will have different results. The main reason is related to the inconsistence problem of the conditional inference. We try to solve this problem from a differential point of view. And, we find a good approximation for this problem. From out point of view. We can find a consistence result from the pivotal quantity. This contribution may very important in the field of statistical inference.

Keywords: pivotal inference, nuisance parameter problem, likelihood function, consistence, conditional inference

三、緣由與目的

The most important thing we have done in this year is about the inconsistence of the pivotal inference. This contribution is same as the conditional inference inconsistence problem that is a powerful tool for solving the nuisance parameter problem but having some inconsistence results. Since the pivotal quantity is not, in general, unique (or even not in equivalent form) and involve parameters of interest. Therefore the likelihood function about the parameter of interest derived from the pivotal quantities will have different results. This behavior confused many people for a long time. The main reason for this problem is related to the inconsistence problem of the conditional inference. We try to solve this problem from a differential calculation point of view. And, we find a good approximation for this problem. From out point of view. We can easily find a consistence result from the pivotal quantity. We feel that this contribution may very important in the field of statistical inference.

四、結論

In many statistical models, a pivotal quantity exists and they have the same structure as follows. Suppose that we have i.i.d. samples from a statistical distribution with parameter (theta,g), and (X,V) is a minimum sufficient statistic, where theta is the parameter of interest and g in G is a nuisance parameter. Let T (X,V,theta) denote a pivotal quantity in the model. Then we can regard the sampling scheme as sampling v from the marginal distribution of V first. Secondly, we sample t from the conditional distribution of t given v. Since, for any fixed v and theta, T(X,V,theta) is not always a one to one transformation of x, the final step we need to sample x conditioning on v and t. If (1) in the first step, the marginal distribution of v depends on g only, (2) in the last step, the conditional distribution of x given t and v depends on theta only, and (3) the distribution of t given v depends on theta only, then we think that the right inference about theta should be based on the distribution of t given v and the distribution of x given v and t. In general, the conditions (1) and (2) are satisfied in many models. But, unfortunately, the distribution of t given v will involve g in most situations. For this difficulty, we notice that in many models, the distribution of v given t is a group transformation model satisfies (N). Therefore, we claim that v does not provide any information about t. In other words, we claim that there should be no difference on the information about t before observed v and after observed v. In this sense, the ``right'' inference about theta should be based on the marginal distribution of t and the conditional distribution of x given t. From the following subsections, we will see that our average likelihood function can lead to the right likelihood in this model, Hencefore, we assume the statistical models are as described above, i.e., pivot T exists, (1) and (2) are satisfied, and the distribution of v given t is a group transformation model satisfying (N).

In many statistical models, we can find the pivotal quantities easily, and we use it to set a confidence interval. On the other hand, when there exists nuisance parameters, we also use the density of the pivot to construct a likelihood function about the parameter of interest. In this case, if we choose different pivots, then we will have different likelihood functions. It confuses statisticians for a long time about which likelihood we should use to construct the likelihood function.

Form our results in last section, it gives an answer about this question. We can choose any equivalent pivot T, and then use a special function to be the likelihood function of parameter of interest (i.e., different equivalent pivots will provide the same answer).

Example 1.

Normal Mean With Unknown Variance

Let X_1,…,X_n be i.i.d. with distribution N(theta,sigma ^2). We know that theta is orthogonal to sigma and it is not difficult to get the likelihood function of the parameters. If theta is the parameter of interest, by the result of Hung and Wong (1996), we can easily to get the likelihood function of the mean parameter.

In this example (Y,V)=(X, sum(x_i-bar x)^2) is minimal sufficient and T={Y-\theta}/{sqrt{V}} is a pivotal quantity. The distribution of V depends on sigma only and the conditional distribution of V given T is{sigma^2}/{1+nt^2}chi_{n}^2 which satisfies (N). Also, for each fixed v, T is a one-to-one function of T, condition (C_1) is satisfied. For (C_2'), U(t,v,theta)= sum_y {partial t}/{partial theta} P(y|t,v) = {1}/{sqrt{v}},therefore, U(t,sigma^{-1} (z),theta)= {1}/{sqrt z}sqrt{sigma}. For finding the orthogonal nuisance parameter, it is not difficult to see that (4) and (5) have a common solution. Hence all the conditions in above are satisfied.

In many statistical models, we can find the pivotal quantities easily, and we use it to set a confidence interval. On the other hand, when there exists nuisance parameters, we also use the density of the pivot to construct a likelihood function about the parameter of interest. In this case, if we choose different pivots, then we will have different likelihood functions. It confuses statisticians for a long time about which likelihood we should use to construct the likelihood function. Form our results in last the section, it gives an answer about this question. We can choose any equivalent pivot T, and then use f_T(t)={partial t}/{partial x}|P(x|t,v) to be the likelihood function of parameter of interest (i.e., different equivalent pivots will provide the same answer).

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