行政院國家科學委員會專題研究計畫　成果報告

Teichmuller 空間的複幾何

計畫類別：個別型計畫
計畫編號：NSC91-2115-M-009-017-
執行期間：91 年 09 月 01 日至 92 年 07 月 31 日
執行單位：國立交通大學應用數學系

計畫主持人：張麗萍

報告類型：精簡報告

處理方式：本計畫可公開查詢

中　華　民　國 92 年 9 月 8 日

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報告內容

I believe that I have achieved the goal of the proposal for the project “The complex geometry of the universal Teichmueller space.

In collaborating with Leon Takhtajan, we have successfully defined a new topology on the universal Teichmueller space. We proved that each connected component of the manifold is an infinite dimensional complex manifold modeled on the Hilbert space of square-integrable quadratic differentiables on the unit disc. The component that pass through the origin is also a group. The Weil-Petersson metric is a right-invariant Kaehler metric on this space. We compute the Riemann curvature tensor for this metric. As a result, we proved that the sectional curvature, holomorphic sectional curvature and Ricci curvature of the metric is negative. Moreover, we also compute the Ricci curvature explicitly, which gives the constant $13/12\,\pi$, a magic number related to the dimension of space-time in bosonic string theory. On the other hand, we also conclude that the universal Teichmueller space is a Kaehler-Einstein manifold. Our result can be passed to the finite dimensional Teichmueller spaces via suitable regularization, which we have also been able to establish.

In the other direction, we proved that the Kaehler form of the Velling-Kirillov metric I defined in my previous paper [1] coincides with the first Chern form of the canonical line bundle of the of the universal Teichmueller space. This is a very interesting result. In particular, it simplifies the proof of one of my main theorem in [1]—the push forward of the Velling-Kirillov metric on the universal Teichmueller curve to the universal Teichmueller space is the Weil-Petersson metric.

All the above results are contained in our forthcoming paper: “ The curvature properties of the Weil-Petersson metric on the universal Teichmueller space”.

On the other hand, I also have made some progress in understanding the integrable structure of the universal Teichmueller space, which will be helpful for further investigation of the geometric properties of the space. I have proved that the Hirota equations uniquely characterize the tau functions of the dispersionless KP and dispersionless Toda lattice hierarchies. It relies heavily on the concept of Grunsky coefficients and Faber polynomials in classical complex analysis. The result is published in the paper [2]. A well-known mathematician in this area- Takebe, said that he was very impressed by this paper.

References:

[1] L.P. Teo, “Velling-Kirillov metric on the universal Teichmueller curve”, 2003. Accepted by Journal d’ Analyse Mathematique.
[2] L.P. Teo, “Analytic functions and integrable hierarchies—characterization of tau functions”, Letters in Mathematical Physics, Vol 64, No.1, April, 2003.