行政院國家科學委員會專題研究計畫　成果報告

連結運算在影像切割上之研究

計畫類別：個別型計畫
計畫編號：NSC91-2213-E-009-098-
執行期間：91 年 08 月 01 日至 92 年 07 月 31 日
執行單位：國立交通大學資訊科學學系

計畫主持人：薛元澤

計畫參與人員：張尹彬、石永靖

報告類型：精簡報告

處理方式：本計畫可公開查詢

中　華　民　國 93 年 3 月 1 日

行政院國家科學委員會專題研究計畫成果報告
連結運算在影像切割上之研究
A Study on Connected Operators on Image Segmentation

計畫編號：NSC 91-2213-E-009-098
執行期限：2002 年 8 月 1 日至 2003 年 7 月 31 日
主持人：薛元澤　國立交通大學資訊科學系
計畫參與人員：張尹彬、石永靖　國立交通大學資訊科學系

中文摘要

在分析一連串影像時，影像切割是非常重要的影像處理方法。分水嶺切割法在數學形態學中是相當有效的影像切割法。它觀念簡單、實作容易，但卻易造成過度切割。對過度切割問題，很多學者提出各種解決之道，其中以連結運算之含意最廣，幾乎涵蓋所有解決過度切割問題的方法。本計劃探討連結運算之定義、性質與應用，並將之用於實際的影像切割上。我們獲致相當滿意的實驗結果。

關鍵詞：影像切割；分水嶺切割法；連結運算

Abstract

Image segmentation is essential in sequential image analysis. In mathematical morphology, segmentation by watersheds is a well-known morphological segmentation. However, segmentation by watersheds will yield the so-called over-segmentation problem. Many operations have been proposed to solve the over-segmentation problem. Among them, the connected operators have become more and more popular. In this report, we investigate the theoretical background of connected operators and study their properties. When applying to motion segmentation, satisfactory results are obtained.

Keywords: Image Segmentation,
Segmentation by Watersheds,
Connected Operators

1. INTRODUCTION

In mathematical morphology, watershed algorithm is a well-known approach to image segmentation. An image is segmented by applying the watershed algorithm to its gradient image. Due to the presence of noise, the gradient image may contain many small regional minima. Which will result in the problem of watershed oversegmentation.

Flooding from markers is a very effective way to reduce oversegmentation. However, it needs experience to choose suitable markers. Connected operators have a great potential to extract markers automatically. They form a large class of operators including opening by reconstruction, area opening, etc.

In this report, we will investigate the properties and possible applications of connected operators. Experimental results by employing connected operators are exhibited and discussed. Then, some conclusions are made and future works are suggested.

2. Connected Operators

Let E denote the 2D Euclidean space. A **partition** P on E is a collection of sets in E which are exhaustive and disjoint. For each element x in E, denote $[x]_P$ the set in P such that $x \in [x]_P$, called the **class** of x in P. Then partition P' is said to be **finer** than partition P if each class in P' is a subset of some class in P, and P is said to be **coarser** than P'. A partition is called **connected** if all classes in it are connected.

Given a binary image A. Then A induces a unique partition, denoted P_A, consisting of the connected components of A and its complement. An operation Ψ on binary images is called a **connected operator** [18] if the induced partition $P_{\Psi(A)}$ is coarser than P_A for every binary image A.

Example. (Area opening [20])
Let A be a binary image with connected components $C_1, C_2, ..., C_k$. For each $r \geq 0$, the **area opening** α_r of A is the union of those C_i that has area greater than or equal to r.

Example. (Reconstruction by markers [21])
Let A be a binary image. For any two points x and y, the **geodesic distance** between them in A, denoted $d_A(x, y)$, is the length of the shortest path (if any) included in A linking x and y. Then, **geodesic discs** are of the form

$$B_{A,\lambda}(x) = \{y \in E : d_A(x, y) \leq \lambda\}$$

Given a marker M, a subimage of A, we can find the union of all connected components of A that containing M by

$$\begin{aligned} & R_A(M) \\ & = \{x \in A : d_A(x, y) < \infty \quad \text{for some} \quad y \in M\} \\ & = \bigcup_{y \in M, \lambda \geq 0} B_{A,\lambda}(y) \end{aligned}$$

The resulted image $R_A(M)$ is called the **reconstruction** of A by M.

The following is an interest property of connected operators.

Proposition 1. [18]
An operator Ψ on binary images is a connected operator if and only if the symmetric difference of a binary image A and $\Psi(A)$ consists of connected components of A and the complement of A.

The notion of connected operators can be easily extended to grayscale images by using the concept of flat zones. Given a grayscale image f. We consider the level set of f at level t:

$$L_t(f) = \{x \in E : f(x) = t\}$$

Then a **flat zone** [16] of f is a maximal connected component of a level set. Note that flat zones of f constitute a partition of E, which will be written as P_f. Then an operation Ψ on grayscale images is called a **connected operator** [16] if partition $P_{\Psi(f)}$ is coarser than P_f for every grayscale image f.

Area openings and reconstruction by markers can also be extended to grayscale images. We denote $X_t(f)$ the set of all points x with $f(x) \geq t$ and $\hat{X}_t(f)$ the set of all points x with $f(x) \leq t$.

Example. (Area opening)
Let f be a grayscale image. For each $r \geq 0$, the **area opening** α_r of f is given by

$$\alpha_r(f) = \sup\{t : x \in \alpha_r(X_t(f))\}$$

Example. (Reconstruction of f by g)
Let f and g be two grayscale images. If $f \geq g$, the **reconstruction** of f by g is given by

$$R_f(g) = \sup\{t : x \in R_{X_t(f)}(X_t(g))\}$$

If $f \leq g$, the **dual reconstruction** of f by g is given by

$$R_f^*(g) = \sup\{t : x \in R_{\hat{X}_t(f)}\left(\hat{X}_t(g)\right)\}$$

Proposition 2. [7]
An operator Ψ on grayscale images is a connected operator if for any grayscale image f and for any two neighbors x and y,

$$\Psi(f)(x) \neq \Psi(f)(y) \quad \text{implies} \quad f(x) \neq f(y).$$

3. APPLICATIONS

Connected operators can be used to reduce the oversegmentation problem when images are segmented by watershed algorithm [1, 19, 20]. For instance, the area opening α_r can be used to eliminate those regions with areas less than *r*. The reconstruction of *f* by $f-h$ will remove any local maximum with relative height less than *h*, while the dual reconstruction of *f* by $f+h$ will remove any local minimum with relative depth less than *h*.

In our experiment, we apply reconstruction operators to motion segmentation. Consider extracting a frame from the sequence Hall-Monitor using the method proposed by Chang [3] with two different threshold values. The followings are the extracted results after smoothing.

Figure 1

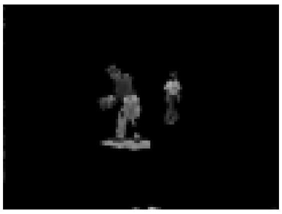

Figure 2

Figure 1 is the extracted image with smaller threshold value. Now, we let *f* to be the image in Figure 1 and *g* the image in Figure 2. Then the dual reconstruction of f by g will produce the image shown in Figure 3.

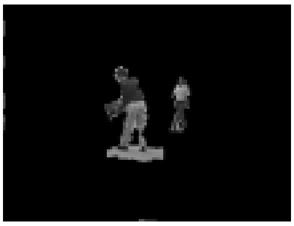

Figure 3

Observe that the image in Figure 3 preserves more object information than that in Figure 2.

4. CONCLUSIONS

In this study we investigate the theoretical background and properties of connected operators. These operators do not process images by pixels, instead, they process images by zones. This property enables them to become a very powerful approach to reduce oversegmentation.

The property listed in proposition 2 is very useful to derive the notion of levelings [7, 8]. They are the fundamental operations to build segmentation pyramids. We will work on this subject in the future.

5. REFERENCES

[1] A. Bieniek and A. Moga, A Connected Component Approach to the Watershed Segmentation, In H. Heijmans and J. Roerdink, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 215-222, Kluwer Academic, 1998.

[2] U. Braga-Neto and J. Goutsias, Multiresolution Connectivity: An Axiomatic Approach, In J. Goutsias, L. Vincent, and D. Bloomberg, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 159-168, Kluwer Academic, 2000.

[3] Y.P. Chang, Fast and automatic video object segmentation using background information and morphological operations, Master thesis, National Chiao Tung University, Taiwan, 2003.

[4] J. Crespo, Space Connectivity and Translation-Invariance, In P. Maragos, R. Schafer, and M. Butt, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 119-126, Kluwer Academic, 1996.

[5] J. Crespo and R. Schafer, The flat zone approach and color images, In J. Serra and P. Soille, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 85-92, Kluwer Academic, 1994.

[6] J. Crespo, J. Serra, and R. Schafer, Theoretical aspects of morphological filters by reconstruction, *Signal Processing*, 47(2): 201-225, 1995.

[7] F. Meyer, From Connected Operators to Levelings, In H. Heijmans and J. Roerdink, Editors, *Mathematical*

Morphology and It's Application to Image and Signal Processing, pages 191-198, Kluwer Academic, 1998.

[8] F. Meyer, The Levelings, In H. Heijmans and J. Roerdink, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 199-206, Kluwer Academic, 1998.

[9] A. Popov, Approximate Connectivity and Mathematical Morphology, In J. Goutsias, L. Vincent, and D. Bloomberg, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 149-158, Kluwer Academic, 2000.

[10] F. Potjer, Region adjacency graphs and connected morphological operators, In P. Maragos, R. Schafer, and M. Butt, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 111-118, Kluwer Academic, 1996.

[11] C. Ronse, Set-theoretical algebraic approaches to connectivity in continuous or digital spaces, *Journal of Mathematical Imaging & Vision*, 8(1): 41-58, 1998

[12] C. Ronse and J. Serra, Geodesy and connectivity in lattices, Fundamenta Informaticae, 46(4): 349-395, 2001.

[13] P. Salembier and L. Garrido, Connected Operators Based on Region-Tree Pruning, In J. Goutsias, L. Vincent, and D. Bloomberg, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 169-178, Kluwer Academic, 2000.

[14] P. Salembier, L. Garrido, and D. Garcia, Auto-dual Connected Operators Based on Iterative Merging Algorithms, In H. Heijmans and J. Roerdink, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 183-190, Kluwer Academic, 1998.

[15] P. Salembier and A. Oliveras, Pratical Extensions of Connected Operators, In P. Maragos, R. Schafer, and M. Butt, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 97-110, Kluwer Academic, 1996.

[16] P. Salembier and J. Serra, Flat zones filtering, connected operators and filters by reconstruction, *IEEE Transactions on Image Processing*, 3(8): 1153-1160, 1995.

[17] J. Serra, Connectivity on complete lattices, In P. Maragos, R. Schafer, and M. Butt, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 81-96, Kluwer Academic, 1996.

[18] J. Serra and P. Salembier, Connected operators and pyramids, In SPIE, editor, *Image Algebra and Mathematical Morphology*, Vol. 2030, pages 65-76, 1993

[19] V. Vilaplana and F. Marques, Face Segmentation Using Connected Operators, In H. Heijmans and J. Roerdink, Editors, *Mathematical Morphology and It's Application to Image and Signal Processing*, pages 207-214, Kluwer Academic, 1998.

[20] L. Vincent, Grayscale area openings and closings, their efficient implementation and applications, In J. Serra and P. Salembier, editors, *First Workshop on Mathematical Morphology and its Applications to Signal Processing*, pages 22-27, Barcelona, Spain, 1993.

[21] L. Vincent, Morphological gray scale reconstruction in image analysis: Applications and efficient algorithms, *IEEE Transactions on Image Processing*, 2(2): 176-201, 1993.