行政院國家科學委員會專題研究計畫　成果報告

修正具有違約風險之選擇權評價模式

計畫類別：個別型計畫
計畫編號：NSC91-2416-H-009-021-
執行期間：91 年 08 月 01 日至 92 年 07 月 31 日
執行單位：國立交通大學財務金融研究所

計畫主持人：陳勝源

報告類型：精簡報告

處理方式：本計畫可公開查詢

中　華　民　國 92 年 9 月 22 日

行政院國家科學委員會補助專題研究計畫成果報告

※※※※※※※※※※※※※※※※※※※※※※※※※※※
※　　　　　　　　　　　　　　　　　　　　　　　　　※
※　　　　修正具有違約風險之選擇權評價模式　　　　※
※　　　　　　　　　　　　　　　　　　　　　　　　　※
※※※※※※※※※※※※※※※※※※※※※※※※※※※

計畫類別： 個別型計畫
計畫編號：NSC 91-2416-H-009-021
執行期間： 91 年 8 月 1 日至 92 年 7 月 31 日

計畫主持人：陳勝源
共同主持人：
計畫參與人員：

成果報告類型(依經費核定清單規定繳交)：精簡報告

處理方式：得立即公開查詢

執行單位：國立交通大學財務金融研究所

中　華　民　國　九十二　年　七　月　三十一　日

行政院國家科學委員會專題研究計畫成果報告

修正具有違約風險之選擇權評價模式

The Valuation of Options Subject to Default Risk－A Correction 計畫編號：NSC 91-2416-H-009-021

執行期限：91 年 8 月 1 日至 92 年 7 月 31 日

主持人：陳勝源　e-mail: chen051688@hotmail.com

執行機構及單位名稱：國立交通大學財務金融研究所

中文摘要

由於金融市場不斷發展與創新，企業界對於規避金融風險之需求亦日益增加，因而許多由金融機構所發行、但未上市交易之選擇權乃不斷因應而生。與一般上市交易之選擇權不同的是，這些店頭選擇權並沒有足夠而完善的保證金結算制度，藉以保障選擇權持有者之權益，所以購買此類選擇權的投資者就必須面對賣方可能潛存的違約風險。因此，當投資者在評估這種具有違約風險之選擇權價值時，Black & Scholes (1973)的一般選擇權評價模式就不能適用。

本計畫之主要目的即在修正 Klein（1996）之脆弱選擇權評價模式，期望能推導出路徑相依（path-dependent）的具有違約風險選擇權之評價模式；同時，亦擬以數值模擬方法比較分析本計畫之修正評價模式與 Black & Scholes（1973）選擇權評價模式、Klein（1996）評價模式之差異。

關鍵詞：選擇權、選擇權評價模式、信用風險、違約風險、路徑相依

Abstract

As the needs to hedge financial risks increase, a variety of non-exchange options or over-the-counter options are widely traded between financial institutions and their institutional costumers. However, there is no sufficient margin settlement mechanism for these unlisted options, thus the credit risk of the option writer must be considered when investors evaluate the price of unlisted options. As a consequence, the Black & Scholes (1973) option pricing model is not adequate to assess the options which are subject to counterparty credit risk.

The purpose of this project is to extend the vulnerable option pricing model in Klein (1996) and derive a path-dependent valuation model for option subject to counterparty's default risk. At the same time, this project also employs numerical simulation to compare the pricing results of our derived model with both of the Black & Scholes (1973) default-free option model and the Klein (1996) path-independent vulnerable option model.

Keywords: option, option pricing, credit risk, default risk, path-dependent

I. Introduction

In contrast to exchange-listed options markets, there is no clearing organization, such as an Options Clearing Corporation, in the over-the-counter markets that requires that the option writer pay an initial margin and be daily resettled. Thus it is unreasonable to assume that options investors are not exposed to possible default risk from their writers. As a consequence, the Black & Scholes (1973) option pricing model is not adequate for evaluating the options which are subject to counterparty credit risk. Johnson & Stulz (1987) refer to this type of option as a “vulnerable option.”

Existing papers consider the credit risk in their pricing models only on the option expiration date. Under this assumption, the default risk is only examined at the date of maturity and the path-dependent process of the asset value is ignored. So even if the asset value of the option writer is ever below its debt prior to the expiration date but it ends with an asset value above its debt, then the in-the-money option holders can still claim a full payoff from the writer. In fact, this is seldom the real situation.

II. Purpose

The purpose of this paper is to extend the vulnerable option pricing model in Klein (1996) and present a path-dependent valuation model for options subject to counterparty default risk. We also compare the pricing results of our model with the Black & Scholes (1973) default-free option model and the Klein (1996) path-independent vulnerable option model.

III. Literature Review

Johnson & Stulz (1987) assume that the option is the sole debt claim of the writer and if the option writer is unable to make a promised payment, the vulnerable option holders will receive all the assets of the option writer. They derive closed-form solutions for European options prices in some different cases by assuming stochastic processes both for the asset value of the option writer and the value of the underlying asset.

Hull & White (1995) extend the Johnson & Stulz (1987) model and allow the counterparty to have other equally ranking liabilities. When the counterparty defaults, option holders are assumed to obtain only a proportion of its no-default claims. Hull & White (1995) not only propose a model for vulnerable options but also numerically simulate and compare the differences between European call, American call and ordinary call options under normal default risk.

By allowing both a stochastic process for the default-free term structure and the term structure for risky debt, Jarrow & Turnbull (1995) provide a new technique for valuing and hedging derivative securities subject to credit risk. This methodology can also be applied to over-the-counter options, corporate bonds and other securities.

Klein (1996) indicates that the default risk in relation to the vulnerable option arises when the asset value of the counterparty declines as the value of the option itself increases as well. Thus Klein (1996) assumes that the credit risk of the option writer and the asset underlying the option is correlated and derives a pricing formula which allows for other debt claims on the part of the counterparty and the existence of capital forbearance.

The Klein (1996) model is more practical than the models of Hull & White (1995) and Jarrow & Turnbull (1995); however, the author still ignores the possibility that the counterparty may default prior to the option's expiration date. Obviously, Klein (1996) assumes the vulnerable option price is path-independent, and thus the default risk can be examined only on the expiration date.

IV. The Pricing Model for Vulnerable Options

Different from Klein (1996), we consider the default risk of the option's writer throughout the option's lifetime instead of only on the maturity date. Furthermore, our model discusses two alternatives to incorporate the situation that the option writer's asset value V_t falls below some amount D^* at any time before the option's maturity date. First, we assume that the option holders receive nothing even though the option is in-the-money, and second, we assume the in-the-money option holders can obtain a proportion of their claims when the option's writer defaults before the maturity date.

Under first alternative, the value of a path-dependent vulnerable option (C_1) can be stated as:

$$\begin{aligned} C_1 = e^{-rT} E\{\max(S_T - K, 0)[(1 \mid \forall V_t > D^*,\ 0 \le t \le \mathrm{T}) \\ + ((1-\alpha)V_T / D \mid \forall V_t > D^*,\ 0 \le t < \mathrm{T}, \text{and } V_T \le D^*)]\} \end{aligned} \tag{1}$$

where r denotes the risk-free rate, S_T, V_T be the value of the underlying stock and option writer's asset at the maturity date, respectively, E denotes risk-neutral expectation over S_T and V_T, K is the strike price of the option; and α is the bankruptcy cost expressed as a percentage of the asset value of the option writer.

Under second alternative, the value of a path-dependent vulnerable option (C_2) can be written as:

$$\begin{aligned} C_2 = e^{-rT} E\{\max(S_T - K, 0)(1 \mid \forall V_t > D^*,\ 0 \le t \le \mathrm{T})\} \\ + e^{-rt} E\{\max(S_t - K, 0)[(1-\alpha)V_t / D \mid\ V_t \le D^*,\ 0 \le t \le \mathrm{T})]\} \end{aligned} \tag{2}$$

Equation (1) can be solved as follows:

$$
\begin{aligned}
C_1 &= S_0 N_2(a_1, a_2, \rho) - Ke^{-rT} N_2(b_1, b_2, \rho) \\
&\quad - S_0 \left(\frac{D}{V_0}\right)^{\frac{2\mu_v}{\sigma_V^2}} N_2(x_1, x_2, \rho) + Ke^{-rT} \left(\frac{D}{V_0}\right)^{\frac{2\mu_v}{\sigma_V^2}} N_2(y_1, y_2, \rho) \qquad (3) \\
&\quad + \frac{V_0(1-\sim)}{D} S_0 e^{(r+\rho\sigma_s\sigma_v)T} N_2(c_1, c_2, -\rho) h(T) - \frac{V_0(1-\sim)}{D} K N_2(d_1, d_2, -\rho) h(T)
\end{aligned}
$$

where $a_1 = \frac{1}{\sigma_s\sqrt{T}}\left[\ln\frac{S_0}{K} + \left(r + \frac{1}{2}\sigma_s^2\right)T\right]$, $a_2 = \frac{1}{\sigma_v\sqrt{T}}\left[\ln\frac{V_0}{D^*} + \left(r - \frac{1}{2}\sigma_v^2 + \rho\sigma_v\sigma_s\right)T\right]$

$$
b_1 = \frac{\ln\frac{S_0}{K} + \left(r - \frac{1}{2}\sigma_s^2\right)T}{\sigma_s\sqrt{T}}, \quad b_2 = \frac{\ln\frac{V_0}{D^*} + \left(r - \frac{1}{2}\sigma_v^2\right)T}{\sigma_v\sqrt{T}}
$$

$$
x_1 = \frac{1}{\sigma_s\sqrt{T}}\left[\ln\frac{S_0}{K} + \left(r + \frac{1}{2}\sigma_s^2\right)T\right], \quad x_2 = \frac{1}{\sigma_v\sqrt{T}}\left[\ln\frac{D^*}{V_0} + \left(r - \frac{1}{2}\sigma_v^2 + \rho\sigma_s\sigma_v\right)T\right]
$$

$$
y_1 = \frac{1}{\sigma_s\sqrt{T}}\left[\ln\frac{S_0}{K} + \left(r - \frac{1}{2}\sigma_s^2\right)T\right], \quad y_2 = \frac{1}{\sigma_v\sqrt{T}}\left[\ln\frac{D^*}{V_0} + \left(r - \frac{1}{2}\sigma_v^2\right)T\right]
$$

$$
\mathrm{c}_1 = \frac{\ln\frac{\mathrm{S}_0}{\mathrm{K}} + \left(\mathrm{r} + \frac{1}{2}\sigma_s^2 + \rho\sigma_s\sigma_v\right)\mathrm{T}}{\sigma_s\sqrt{\mathrm{T}}}, \quad \mathrm{c}_2 = -\frac{\ln\frac{\mathrm{V}_0}{\mathrm{D}^*} + \left(\mathrm{r} + \frac{1}{2}\sigma_v^2 + \rho\sigma_v\sigma_s\right)\mathrm{T}}{\sigma_v\sqrt{\mathrm{T}}}
$$

$$
\mathrm{d}_1 = \frac{1}{\sigma_s\sqrt{\mathrm{T}}}\left[\ln\frac{\mathrm{S}_0}{\mathrm{K}} + \left(\mathrm{r} - \frac{1}{2}\sigma_s^2 + \rho\sigma_s\sigma_v\right)\mathrm{T}\right], \quad \mathrm{d}_2 = \frac{-1}{\sigma_v\sqrt{\mathrm{T}}}\left[\ln\frac{\mathrm{V}_0}{\mathrm{D}^*} + \left(\mathrm{r} + \frac{1}{2}\sigma_v^2\right)\mathrm{T}\right]
$$

$$
h(T) = \frac{-\ln\frac{D^*}{V_0}}{\sigma_v T\sqrt{2\pi T}} \exp\{-[\ln(\frac{D^*}{V_0}) - (r - \frac{1}{2}\sigma_v^2)T]^2 / (2\sigma_v^2 T)\}
$$

Similarly, equation (2) can be solved as follows:

$$
\begin{aligned}
C_2 &= S_0 N_2(a_1, a_2, \rho) - Ke^{-rT} N_2(b_1, b_2, \rho) \\
&\quad - S_0 \left(\frac{D}{V_0}\right)^{\frac{2\mu_v}{\sigma_V^2}} N_2(x_1, x_2, \rho) + Ke^{-rT} \left(\frac{D}{V_0}\right)^{\frac{2\mu_v}{\sigma_V^2}} N_2(y_1, y_2, \rho) \qquad (4) \\
&\quad + \frac{V_0(1-\sim)}{D} \int_0^T [S_0 e^{(r+\rho\sigma_s\sigma_v)t} N_2(c_1, c_2, -\rho) - \frac{V_0(1-\sim)}{D} K N_2(d_1, d_2, -\rho)] h(t)\, dt
\end{aligned}
$$

In equations (3) and (4), N_2 is a bivariate standard normal cumulative distribution function, ρ is the correlation coefficient between $\ln S_T$ and $\ln V_T$, and $h(T)$ is the density function for the first time T when the option writer's asset value falls below D^*.

V. Comparison of Models

In order to compare our models with those of Black & Scholes (1973) and Klein (1996), we construct the same numerical examples as in Klein (1996). The results of the calculation are shown in **Table 1**. All of the calculations of the options values in **Table 1** are based on the following parameter values: $\alpha = 0$, $\pm_s = 0.3$, $\pm_v = 0.3$, $\rho = 0.5$, $r = 0.04833$, $T = 0.3333$, $K = 40$, $S_0 = 40$, $V_0 = 5$, $D = 5$, and $D^* = 5$, unless otherwise noted.

Table 1 Values of vulnerable options and non-vulnerable options

Case	$PDVOC_1$	$PDVOC_2$	B & S	Klein
Base case	0	0	3.0697	3.0049
$\pm_s = 0.2$	0	0	2.1641	2.1160
$\pm_s = 0.4$	0	0	3.9760	3.8954
$\pm_v = 0.2$	0	0	3.0697	3.0312
$\pm_v = 0.4$	0	0	3.0697	2.9772
$\rho = -0.5$	0	0	3.0697	2.7068
$\rho = 0$	0	0	3.0697	2.8801
$T = 0.0833$	0	0	1.4603	1.4439
$T = 0.5833$	0	0	4.1806	4.0681
$K = 30$	0	0	10.5744	10.1628
$K = 50$	0	0	0.4341	0.4305
$S_0 = 30$	0	0	0.1487	0.1478
$S_0 = 50$	0	0	10.9319	10.5427
$V_0 = 10$	3.0691	3.0691	3.0697	3.0697
$r = 0.02833^{\#}$	0	0	2.9394	2.8742
$r = 0.06833$	0	0	3.2031	3.1387

#: The option values in the last two rows are not reported in Table 1 of Klein (1996).

As **Table 1** shows, except in the case where $V_0 = 10$, our models consistently indicate that the vulnerable option values are zero, while Klein's (1996) model still reports that the options are valuable, regardless of the changes in each of the variables. Thus Klein's (1996) path-independent pricing model overestimates the vulnerable option value.

VI. Conclusions

This project derives two path-dependent vulnerable option valuation models which can effectively improve the quality of the Klein (1996) pricing formula and truly reflect the impact of the counterparty's credit risk on the vulnerable option value. We find that the Klein (1996) model overestimates the vulnerable option value. The theoretical value for our path-dependent vulnerable option is lower than both of those estimated by the Black & Scholes (1973) model and the Klein (1996) model.

References

Black F., Scholes, M., 1973. The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 637-654.

Cooper, I. A., Mello, A. S., 1991. The Default Risk of Swaps. *Journal of Finance* 46, 597-620.

Hull, J. C., 1989. Assessing Credit Risk in a Financial Institution's Off-Balance Sheet Commitments. *Journal of Financial and Quantitative Analysis*, 489 -501.

Hull, J. C., White, A., 1995. The Impact of Default Risk on the Prices of Options and other Derivative Securities. *Journal of Banking and Finance*, 299-322.

Jarrow, R. A., Turnbull, S. M., 1995. Pricing Derivatives on Financial Securities Subject to Credit Risk. *Journal of Finance*, 53-85.

Johnson H., Stulz, R., 1987. The Pricing of Options with Default Risk. *Journal of Finance*, 267-280.

Klein, P., 1996. Pricing Black-Scholes Options with Correlated Credit Risk. *Journal of Banking and Finance*, 1211-1229.

計畫成果自評

本計畫順利如期完成各項預定工作目標，特別是改善並修正過去文獻之瑕疵，推導出路徑相依具有違約風險之選擇權評價模式，本計畫為學術理論創新之作，研究結果非常具有貢獻，目前已撰寫成論文，預期將可以發表於國外著名之學術期刊。

本計畫所獲得之研究結果可供我國主管機關於監理金融機構與研擬金融政策之參考，以促進金融市場之健全發展及創新。亦可提供金融機構於經營管理上之參考，特別是在未上市選擇權之定價上，將有重要的貢獻。另一方面，本計畫之研究結果亦同時可供企業界於向金融機構買進未上市選擇權時之參酌，使其持有之選擇權價格更為合理，促進金融市場之公平及效率。

本計畫依預定進度執行，在人力與物力之調度安排與運用上均稱得宜。在研究過程中，利用 MATLAB 撰寫電腦程式，比較各模式差異，使得對於脆弱選擇權有極深入之了解與分析。綜言之，由於本計畫全程進行非常順利，研究成果對理論與實務貢獻均頗大，且較預期豐碩。