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電路中間聯結的模式簡化-基於平衡實現的方法(II)

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計畫主持人： 林清安

計畫參與人員： 吳建賢

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利用平衡實現法之電路中間聯結模式簡化

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利用平衡實現法之電路中間聯結模式簡化

Balanced Realization Based Interconnet Model Reduction

計畫編號：NSC-91-2219-E009-069
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主持人: 林清安 交通大學電機與控制工程系
計畫參與人員: 吳建賢 交通大學電機與控制工程系

中文摘要

利用平衡時實現法來做RLC 電路中間聯結的模式簡化有許多優點, 如穩定性及誤差上限等。這個方法的主要缺點在於解李奧波夫方程式所需的大量運算。目前用來減少計算量的方法, 在穩定性及準確度均有所不足。後續的研究需針對這點進行探討, 使此法更有效率。

Abstract

Balanced realization method as a tool for RLC interconnect circuit model reduction has many advantages over the moment-matching based methods. Computation complexity associated with the solutions of Lyapunov equations seems to be the main disadvantage of the method. Methods currently available for reducing computations are still inadequate and further investigation is needed if the method, as a tool for circuit model reduction, is to be competitive.

1 Introduction

It has been asserted that interconnect delay dominates gate delay in next generation nanometer scale IC's [1]. Rapid and accurate analysis of interconnect delay is crucial for the success of nano-scale IC design. Accurate analysis of interconnect delay taking into account the transmission line effect is computationally unrealistic. Lump approximations usually result in RLC circuits of sufficient high order that are too computation intensive for simulation tools such as SPICE. Circuit model order reduction is thus an important research topic and many techniques have been proposed over the last 10 years.

The techniques can be classified into two groups: one based on moment matching and the other based on balanced realization. Both approaches have their origin in system and control theory. The idea of moment matching is first applied to circuit model order reduction in the name of asymptotic waveform evaluation (AWE) [2]. The method of balanced realization [3] appears more recently [9], [8]. It is well-known that the moment matching method may yield unstable reduced models. Our study shows that for RLC trees, even a second-order reduced model may be unstable [4]. For high-order circuits, direct computation of moments becomes very ill-conditioned. Much effort is devoted to improving numerical robustness [7] with some success, but the stability problem remains. The method of balanced realization on the other hand guarantees stability of reduced model at the expense of higher computation complexity. For high order circuits, computation of the balancing transformation may also be ill-conditioned.

We study application of balanced realization method and its variations to circuit model order reduction. The report is organized as follows. Section 2 introduces balanced realization method, properties of the reduced model, and variations of the method. A method that reduces computation complexity based on low-rank Lyapunov solutions is discussed in Section 3. Simulation results are shown in Section 4 and brief conclusions are given in Section 5.

2 Method of balanced realization

A linear time-invariant passive RLC circuit, possibly obtained from lump approximation of an interconnect, can be described by a standard linear state equation

$$\begin{array}{l} \dot{x}(t) = Ax(t) + bu(t) \\ y(t) = cx(t) \end{array} \quad (1)$$

where $u(t)$ is the source input, $y(t)$ is the output of interest, $x(t)$ is the state, $A \in \mathbf{R}^{n\times n}$, $b \in \mathbf{R}^{n\times 1}$, and $c \in \mathbf{R}^{1\times n}$. The transfer function $H(s) = c(sI-A)^{-1}b$ is nth-order. By model order reduction we mean finding a transfer function $H_r(s)$, of order $r < n$, so that $H_r(s)$ is close to $H(s)$ in some sense. In moment matching approach, the two transfer function are consider close in that their first r moments are identical. In balanced realization approach, the order reduction is obtained through a reduction in state space dimension. The two transfer functions, or equivalently the input-output relations, are close in that only the most important (most controllable and most observable) subspace is kept in the reduction.

2.1 Model reduction procedure

A standard balanced realization procedure to obtain reduced order model is as follows:

Step0: Given $A \in \mathbf{R}^{n\times n}$, $b \in \mathbf{R}^{n\times 1}$, and $c \in \mathbf{R}^{1\times n}$.

Step1: Solve the Lyapunov equations

$$AW_c + W_cA^T + bb^T = 0 \quad (2)$$

$$A^TW_o + W_oA + c^Tc = 0 \quad (3)$$

for the controllability gramian W_c and observability gramin W_o.

Step2: Compute the similarity transformation T by

(1) Cholesky factorization of W_c

$$W_c = RR^T$$

(2) forming $W = R^TW_0R$

(3) singular value decomposition

$$W = V\Sigma^2V^T$$

where $\Sigma = \mathrm{diag}(\sigma_1, \cdots, \sigma_n)$, $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n > 0$

(4) setting $T = RV\Sigma^{-1/2}$.

Step3: Obtain the balanced system

$$\begin{array}{l} \dot{\overline{x}} = \overline{A}\overline{x} + \overline{b}u \\ y = \overline{c}\overline{x} \end{array}$$

where $\overline{A} = T^{-1}AT$, $\overline{b} = T^{-1}b$, and $\overline{c} = cT$.

Step4: To obtain a rth order reduced model, partition compatibly $\overline{A}$, $\overline{b}$, and $\overline{c}$ as

$$\overline{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad \overline{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad \overline{c} = \begin{bmatrix} c_1 & c_2 \end{bmatrix}$$

where $A_{11} \in \mathbf{R}^{r\times r}$, and the reduced order model is

$$\begin{array}{l} \dot{x} = A_{11}x + b_1u \\ y = c_1x \end{array}$$

with transfer function

$$H_r(s) = c_1(sI - A_{11})^{-1}b_1$$

2.2 Properties of reduced models

Two most important properties of reduced model are (i) stability and (ii) exact error bound. More precisely, the first property means that if the original model is stable, then the reduced model is also stable. We note that this property is very desirable and the lack of it is the main disadvantage of the moment matching method.

The error bound is expressed in term of frequency response and it is tight. More precisely, for the original nth order circuit model and the rth order reduced model, the error frequency response $E(j\omega) = H(j\omega) - H_r(j\omega)$ satisfies

$$\max_{0\leq\omega<\infty} |E(j\omega)| \leq 2 \sum_{k=r+1}^{n} \sigma_k$$

The main disadvantage of balanced realization method is its computation complexity. To find a reduced model, we need to find the balancing transformation T which requires the solutions of two Lyapunov equations in addition to the singular value decompositions that follow. For high order circuits, the amount of computation is huge. Furthermore, if the circuit contains modes that are either nearly uncontrollable or nearly unobservable, the computation leading to a balanced system is ill-conditioned and serious numerical difficulties may occur.

The error frequency response spreads (in general, quite evenly) over the entire frequency range. In particular, the dc-gains of the models do not match. This is undesirable, since many interconnect (especially gate-to-gate interconnect) do have unit dc-gain and it is important to maintain this property in the reduced model. (The moment matching method, in contrast, always maintain this property by matching the 0th moment.)

2.3 Modification of balanced realization

To remove some of the disadvantages of the method, a number of modifications have been proposed. We mention three of them below.

(i) DC-gain matching

Consider the balanced system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u$$
$$y = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Instead of removing the state component x_2 completely as is done in the original balanced realization method, the modification keep the steady-state value of x_2 in order to make the dc-gain of the reduced model the same as that of the original model. The idea is borrowed from the so called singular perturbation method: the state x_1 contains the 'slow' dynamics and the state x_2 contains the 'fast' dynamics. In considering the slow dynamics, the fast dynamics can be assumed to be at steady state. More precisely, for a given $x_1(t)$, the steady-state value of $x_2(t)$ satisfies

$$0 = A_{21}x_1(t) + A_{22}x_2(t) + b_2u(t)$$

that is

$$x_2(t) = -A_{22}^{-1}(A_{21}x_1(t) + b_2u(t))$$

Putting $x_2(t)$ into the first equation, we get the reduced model as

$$\dot{x}_1 = (A_{11} - A_{12}A_{22}^{-1}A_{21})x_1 + (b_1 - A_{12}A_{22}^{-1}b_2)u$$
$$y = (c_1 - c_2A_{22}^{-1}A_{21})x_1 + (-c_2A_{22}^{-1}b_2)u$$

The reduced model now has the same dc-gain as the original model.

(ii) Frequency weighting

Many interconnect parameters, such as delay time and rise time, depends mainly on the low-frequency characteristics of the transfer function. A good low-order reduced model should have small approximation error in the low-frequency band, while larger error in the high-frequency band can be tolerated. In the balanced realization framework, frequency weighting techniques are introduced to achieve this desirable property [5], [6]. The basic idea is to augment the system with low-pass filter and then compute balancing transformation for an "equivalent " system. This method is shown to be effective [6].

(iii) The Schur method

The computation of balancing transformation is usually ill-conditioned, this is especially so for RLC circuits of high order. Chiang and Safonov [10] proposes a method to obtain the same reduced model as the balanced realization method but without explicitly computing a balanced realization of the original system. Their idea is to compute the Schur decomposition of the product gramian W_cW_o which then provides an orthogonal basis for the eigenspace associated with the selected (large) Hankel singular values. Restrict the dynamics to this subspace gives the reduced model. The method is shown to be numerically stable even the system is nearly uncontrollable or unobservable.

All the three modifications mentioned above still require solving the Lyapunov equations for the controllability and observability gramians, each of which amounts to solving a set of $\frac{n(n+1)}{2}$ linear equations for an nth order system. To obtain an efficient method for model reduction, it is very desirable to avoid solving large Lyapunov equations.

3 Reducing computation complexity

Much of the computation in obtaining a reduced model using balanced realization method is devoted to the solutions of two large Lyapunov equations. To reduce computation, the Lyapunov equations can be solved approximately. The basic approach is to restrict the solutions, respectively, to the Krylov subspace $K_m(A,b)$ and $K_m(A^T,c^T)$, where

$$\mathcal{K}_m(A,b) = \text{span}\{b, Ab, \cdots, A^{m-1}b\}$$

and

$$\mathcal{K}_m(A^T,c^T) = \text{span}\{c^T, A^Tc^T, \cdots, (A^T)^{m-1}c^T\}$$

A procedure to obtain a set of m orthonormal basis of $\mathcal{K}_m(A,b)$, known as the Arnoldi procedure, is as follows.

Arnoldi Procedure

(1) Choose b and compute $q_1 = b/\|b\|_2$

(2) for $j = 1, 2, \cdots, m$

(a) Compute $w = Aq_j$

(b) for $i = 1, 2, \cdots, m$, $\begin{cases} h_{ij} = w^T q_i \\ w = w - h_{i,j} q_i \end{cases}$

(c) $h_{j+1,j} = \|w\|_2$ and $q_{j+1} = w/h_{j+1,j}$.

The procedure gives the set $\{q_1 \cdots q_m\}$ as orthonormal basis of $\mathcal{K}_m(A,b)$. Let $Q_m = [q_1 \cdots q_m]$, we have

$$Q_m^T Q_m = I_m$$

and

$$\mathcal{R}(Q_m) = \mathcal{K}_m(A,b)$$

The Arnoldi procedure produces a Hessenberg matrix H_m satisfying

$$Q_m^T A Q_m = H_m$$

The matrix H_m is the representation of A, restricted to $\mathcal{K}_m(A,b)$, with respect to $\{q_1 \cdots q_m\}$.

A method proposed by Saad [11] to approximately solve the controllability gramian W_c is as follows.

Step1: Using Arnoldi procedure to find Q_m so that

$$Q_m^T A Q_m = H_m$$

where $H_m \in \mathbf{R}^{m\times m}$ is Hessenberg matrix.

Step2: Solving the low rank Lyapunov equation

$$H_m G_m + G_m H_m^T + \beta^2 e_1 e_1^T = 0$$

where $\beta = \|b\|_2$ and $e_1 = [1\ 0 \cdots 0]^T$

Step3: The approximate controllability gramian is

$$\hat{W}_c = Q_m G_m Q_m^T$$

Now the $\hat{W}_c \in \mathbf{R}^{n\times n}$ has rank at most m is a low rank approximation of W_c.

Similar procedure is used to compute a low rank approximation $\hat{W}_o \in \mathbf{R}^{n\times n}$ of W_o, the observability gramian. With the gramian $\hat{W}_c$ and $\hat{W}_o$ obtained, the Chiang-Safonov procedure can then be used to obtain a rth order reduced model.

In general, the stability and accuracy of the reduced model is not guaranteed especially when m, the Krylov space dimension, is chosen small compared with n.

4 Simulation results

We consider a 19th order RLC circuit shown in Figure 1, the input is a voltage source and the output of interest is the voltage at node 9. The original transfer function is 19th order with unit dc-gain.

By direct balanced realization, the 2nd-order, 4th-order and 6th-order reduced models are constructed. The dc-gains of the reduced models are respectively 0.8526, 0.9984, and 1.0001. Step responses of the original model and the reduced models, shown in Figure 3, show that 4th- and 6th-order models give very good match, while the steady-state error of the 2nd-order model is about 15%. Figure 3 show that both 4th-order and 6th-order models have very small frequency response error, the error of the 2nd-order model concentrates mostly in the low frequency range $\leq$ 0.3GHz and spreads quite evenly beyond.

By dc-gain matching, the frequency response error in low-frequency range is greatly suppressed, for the 2nd-order model the error spreads quite evenly up to 1GHz as shown in Figure 5. Step responses show that with dc-gain matching, in Figure 4, a 2nd-order reduced model is quite accurate as far delay time and rise time are concerned.

Figure 6 shows the step response of reduced models computed through low rank Lyapunov solution. The dimension of Krylov subspace is chosen 16 and the reduced model is 7th-order. The model exhibits good transient response, while substantial steady state error is observed. The dc-gain mismatch is then corrected by scaling , the error in transient response, however, is then increased.

5 Conclusions

We consider the balanced realization method as a tool for model order reduction of RLC interconnect circuits. The method has many advantages over the moment-matching based methods. The need to solving Lyapunov equations of large size seems to be the main disadvantage associated with the method. Our study shows the method now available for low-rank Lyapunov solution does not substantially reduced computations and further investigation is need if the balanced realization method is to become an efficient tool for interconnect model reduction.

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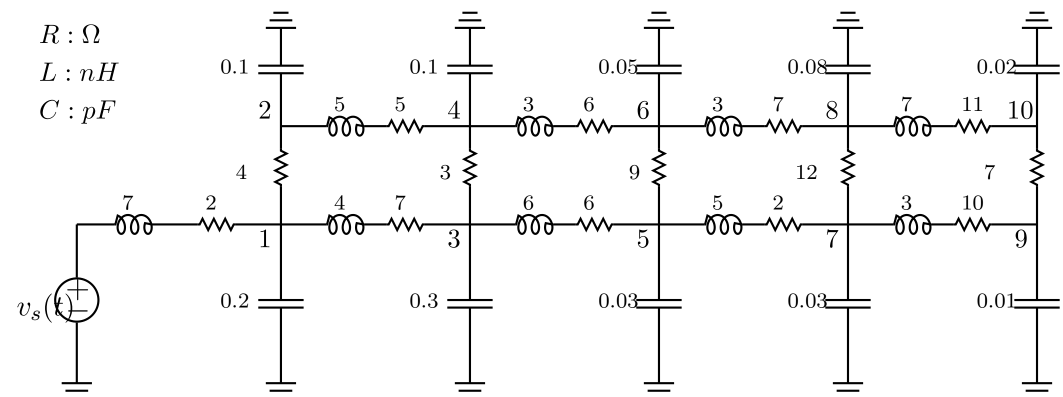

Figure 1: An RLC circuit

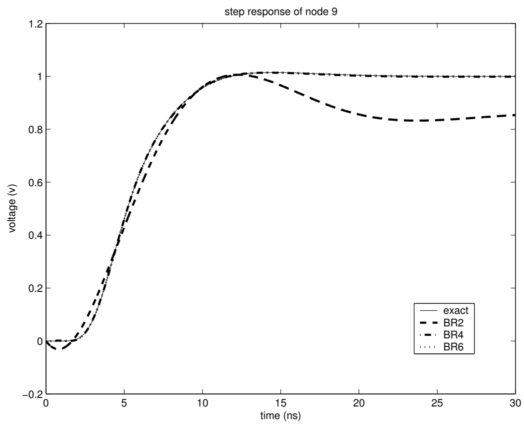

Figure 2: Step responses of original and reduced models

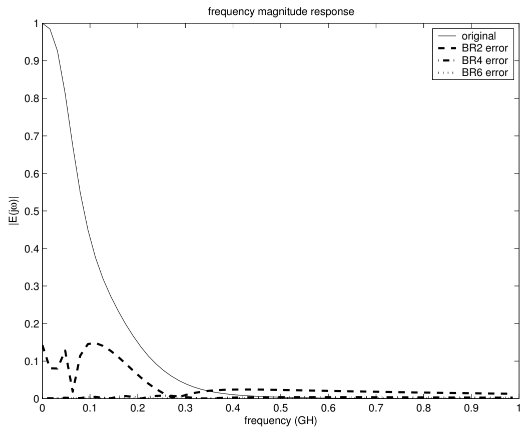

Figure 3: Frequency (magnitude) response of original and error transfer functions

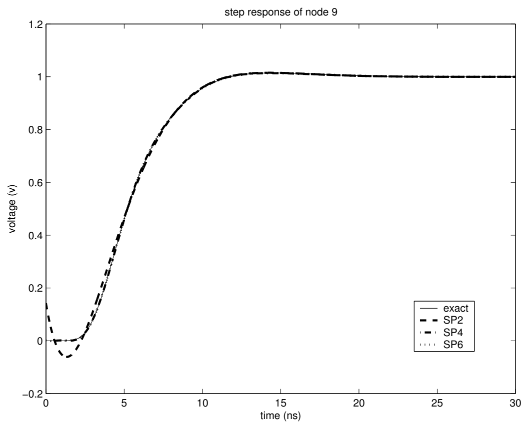

Figure 4: Step responses of original and reduced models after dc-gain matching

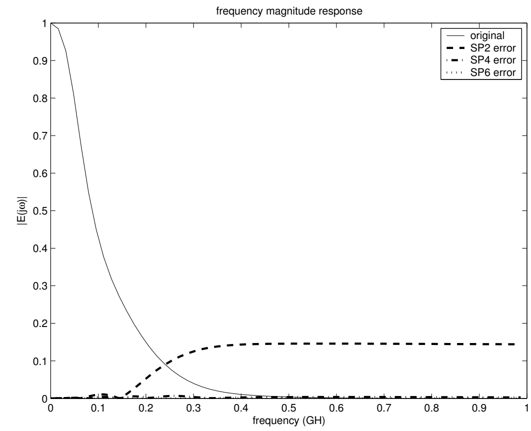

Figure 5: Frequency (magnitude) response of original and error transfer functions after dc-gain matching

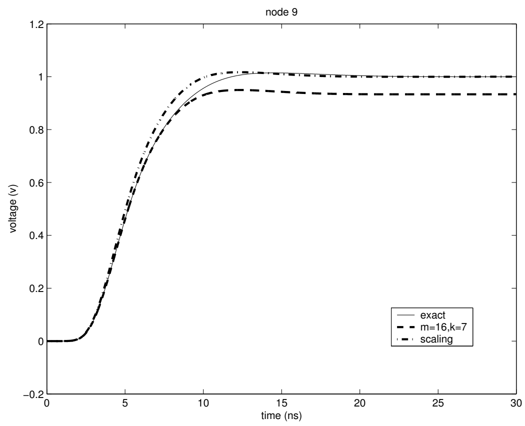

Figure 6: With low rank Lyapunov solution step responses of the original system and a 7th-order reduced model