行政院國家科學委員會專題研究計畫　成果報告

線性結構模式下調節效果檢定之 MRA 修訂法

計畫類別：個別型計畫
計畫編號：NSC91-2416-H-009-004-
執行期間：91 年 08 月 01 日至 92 年 07 月 31 日
執行單位：國立交通大學經營管理研究所

計畫主持人：丁承

計畫參與人員：陳美芳

報告類型：精簡報告

處理方式：本計畫可公開查詢

中　華　民　國 92 年 11 月 12 日

行政院國家科學委員會補助專題研究計畫成果報告

※※※※※※※※※※※※※※※※※※※※※※※※※
※　　　　　　　　　　　　　　　　　　　　　　　　※
※　**線性結構模式下調節效果檢定之 MRA 修訂法**　※
※　　　　　　　　　　　　　　　　　　　　　　　　※
※※※※※※※※※※※※※※※※※※※※※※※※※

計畫類別：☑個別型計畫　　□整合型計畫
計畫編號：NSC 91-2416-H-009-004
執行期間：91 年 8 月 1 日至 92 年 7 月 31 日

計畫主持人：丁　承

本成果報告包括以下應繳交之附件：
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

執行單位：國立交通大學經營管理研究所

中　華　民　國　92　年　10　月　28　日

行政院國家科學委員會專題研究計畫成果報告

線性結構模式下調節效果檢定之 MRA 修訂法

Testing for Moderating Effects in SEM: A Modified MRA Approach

計畫編號：NSC 91-2416-H-009-004
執行期限：91 年 8 月 1 日至 92 年 7 月 31 日

主持人：丁 承　國立交通大學經營管理研究所

一、 中文摘要

在線性結構模式下，本計畫提出一個檢測連續型外顯調節變數調節效果的 MRA 修訂法。針對 MRA 傳統法及修訂法，配以因素基礎分數(factor-based score)或基於驗證型因素分析因素分數(factor scores derived from CFA)之採用，本研究以蒙地卡羅模擬法進行調節效果檢定力及估計偏誤大小比較。將連續型外顯調節變數以中位數作分類處理所導致之檢定力及估計偏誤資訊流失亦一并檢視。研究結果顯示，當構念指標之因素負荷量呈較大之差異時，採用因素分數之 MRA 修訂法可獲得最大的檢定力；然而不同方法在估計偏誤上之差異則未見明確。此外，對連續型調節變數作分類處理會降低調節效果檢定力並導致估計偏誤之資訊大量流失。

關鍵詞：偏誤、驗證型因素分析、調節迴歸分析(MRA)、調節效果、蒙地卡羅模擬、檢定力、線性結構模式

Abstract

A modified moderated regression analysis (MRA) is proposed to detect moderating effects for continuous manifest moderators in structural equation modeling (SEM). Monte Carlo simulation is used to compare the test power and the bias associated with moderating effects by using both traditional and modified MRA with factor-based scores and factor scores derived from confirmatory factor analysis (CFA). Loss of power and bias information due to categorizing a continuous manifest moderator by the median are also examined. The results indicated that if the factor loadings for indicators are substantially different, the modified MRA with factor scores led to the greatest power. However, how the biases resulting from different approaches differ was inconclusive. In addition, categorizing a continuous manifest moderator by the median would reduce the power and lead to much loss of the bias information.

Keywords: Bias, Confirmatory Factor Analysis, Moderated Regression Analysis (MRA), Moderating Effect, Monte Carlo Simulation, Power, Structural Equation Modeling

二、緣由與目的

Moderators often appear in behavioral science research. It affects the form or strength of a relationship between an independent variable and a dependent variable (Baron & Kenny, 1986; Sharma, Durand, & Gur-Arie, 1981). In other words, the independent variable and the moderator interact to reflect that the effect of the independent variable on the dependent variable depends on the level of the moderator (McClelland & Judd, 1993). Moderated regression analysis (MRA), developed by Saunders (1956), has long been applied for examining moderating effects. In traditional MRA, standard multiple regression procedures are used to test for the product terms to determine whether moderating effects are statistically significant.

In social science studies involving psychological constructs, the structural equation modeling (SEM) technique has become popular. The MRA approach can be further applied to examine moderating effects on the construct relationships. However, the measurement adequacy for the constructs must be achieved before moderating effects are examined.

Confirmatory factor analysis (CFA) can be used for a priori defined constructs specified in a SEM model to assess the degree to which the measurement items are valid indicators of the constructs. CFA can be used to select the appropriate indicators of constructs (Gerbing & Anderson, 1988; Grapentine, 2000). There exists indeterminacy in obtaining estimates of latent factors since, for the same observed data, different scoring approaches would lead to different factor scores (e.g., Acito and Anderson, 1986; Lawley & Maxwell, 1971). Factor-based scores are the most commonly used scores for latent constructs. With the factor-based scoring approach, two or more indicators are summed up and divided by the number of the indicators. Alternatively, the factor scores derived from CFA (based on the regression approach) could be used, and their use can reduce the distorting effects resulting from the measurement errors on the coefficient estimates of structural models (Bollen, 1989).

The test for detecting moderating effects on the construct relationships can be assessed by power. Since how the factor scoring approaches influence the power of the test is rarely seen in the literature, the powers resulting from the two types of scores (factor-based scores and factor scores derived from CFA) will be compared in this study, for the case of continuous moderators.

Particularly, a modified MRA approach for detecting moderating effects will be proposed. The approach combines the features of the measurement model, with acceptable reliability, and traditional MRA to perform statistical inference simultaneously under the SEM frame. The main difference between the modified MRA and the traditional MRA is that simultaneous inference is made for the former, but not for the latter. We contend that the power of the test by using the modified MRA will be relatively higher. The power comparisons between the modified and traditional MRA, together with different types of scores, will be conducted. In addition, the loss of power and bias information due to dichotomizing a continuous moderator by its median will be assessed.

三、結果與討論

3.1 Methods

In SEM, the two-step approach recommended by Anderson and Gerbing (1988) is commonly used. The first step of this approach is to use CFA to develop an acceptable measurement model. The second step deals with the structural model in the theoretical framework. When the theoretical framework includes a third manifest variable having potential moderating effect on the construct relationship, attention will be drawn to test for its significance. In contrast to the traditional MRA, a modified MRA is proposed in this article. The proposed modified MRA treats moderators and the corresponding cross-product terms with the exogenous constructs as exogenous manifest variables and keeps the multiple indicators of latent constructs. By doing so, estimates for exogenous constructs are needed for obtaining cross-product terms before simultaneous statistical inference can be conducted in SEM. Factor-based scores and the factor scores derived from CFA will be used and the resulting effects will be compared. The modified approach applies for categorical moderator through the use of dummy variables. For a dichotomous variable, introduce a dummy variable and its cross-product term with the exogenous construct into the model. To detect moderating effects, simply test if the coefficient associated with the cross-product term is significantly different from zero. If significant, there exist moderating effects.

3.2 Monte Carlo Simulation

Monte Carlo simulation will be used to compare the power and bias for assessing moderating effects of continuous manifest moderators between the traditional and modified MRA with factor-based scores and factor scores. Information loss due to dichotomous categorization by the moderator median will also be examined.

Without loss of generality, we proposed a simple simulation-based model, where the effect of exogenous construct F_X on the endogenous construct F_Y is moderated by the continuous manifest variable M. Each construct is measured by the three indicators separately, with X_1 ~ X_3 measuring F_X, and Y_1 ~ Y_3 measuring F_Y.

The structural model is given by $F_Y = ß_1 F_X + ß_2 M + ß_3 F_X * M + \zeta$, where the error term ζ is independently normally distributed. We test H_0: $ß_3 = 0$ versus H_1: $ß_3 \neq 0$ to detect moderating effects. The t-test can be used by first computing the t-value (= estimate / standard error of the estimate) and then comparing $|t|$ with $z_{\alpha/2}$, the upper $100(\alpha/2)$th percentile of the standard normal distribution.

SAS will be used to carry out simulation.

3.2.1 Simulation Design

Jaccard and Wan (1995) gave the following power-influencing factors: (a) sample size, (b) effect size of the interaction term, (c) size of the latent variable squared multiple correlations, (d) predictor variable intercorrelation, (e) reliability of the indicators, and (f) type of estimation method. Based on Jaccard and Wan (1995), the factors considered and the factor levels are stated below:

1. Factor loadings: The factor loadings for the three indicators on their corresponding constructs were set at (0.6, 0.7, and 0.8) as well as (0.9, 0.6 and 0.5), both leading to alpha values close to 0.7.

2. Total sample size: The sample sizes of 175 and 400 were used.

3. Correlation between the exogenous construct F_X and the moderator M: The correlations of 0.2 and 0.4 were set.

4. Size of the squared multiple correlations for the structural model: The squared multiple correlations for the structural model (reflecting the explanatory power) were set as 0.3 and 0.5. Thus, the corresponding standard deviations of the error term for the structural model were 0.84 ($=\sqrt{1-0.3}$) and 0.71 ($=\sqrt{1-0.5}$).

To simulate the power functions, the moderating effect size (ES) was started from 0 (reflecting no moderating effect), where the associated power should be close to α, the significance level, through the value with power close to 1. The power functions and the bias will be compared between the traditional and the modified MRA together with factor-based scores and factor scores derived from CFA for each of the sixteen (= $2 \times 2 \times 2 \times 2$) combinations. In addition, the comparisons will be extended, for each combination, to the amount of the power and bias information loss by categorization. Since the continuous moderator Z was set to have a normal distribution, it does not make difference to categorize with the median or the mean.

3.2.2 Simulation Procedure

The simulation procedure, for each combination, is shown by the following steps (The factor loadings of 0.9, 0.6, and 0.5, the sample size of 175, the correlation of 0.2 between F_X and Z, and the error variance of 0.5 are used for illustration):

Step 1. Generate data.

(1) A value of F_X is randomly generated from the standard normal distribution (denoted by N (0,1)), and then use the measurement model to obtain values of X_1 ~ X_3. Under the standardized situation where 1 = Var(X_i) = communality +

specific variance, values of $X_1 \sim X_3$ can be obtained as follows:

$X_1 = 0.9 * F_X + 0.43589 * Z_1$,

$X_2 = 0.6 * F_X + 0.8 * Z_2$,

$X_3 = 0.5 * F_X + 0.866 * Z_3$,

where Z_1, Z_2, Z_3 are independent N (0,1) random variables.

(2) A value of M is generated as follows (Kuan, 2000, Sec.14.3):

$M = 0.2 * F_X + [1 - (0.2)^2]^{1/2} * G$,

where G is an independent N (0,1) random variable.

(3) A value of F_Y is generated through the structural model as follows:

$F_Y = ES*M*F_X + 0.71*H$,

where ES is a specified moderating effect size and H is an independent N (0,1) random variable.

(4) Values of $Y_1 \sim Y_3$ are generated through F_Y as follows:

$Y_1 = 0.9 * F_Y + 0.43589 * Z_4$,

$Y_2 = 0.6 * F_Y + 0.8 * Z_5$,

$Y_3 = 0.5 * F_Y + 0.866 * Z_6$,

where Z_4, Z_5, Z_6 are independent N (0,1) random variables.

Repeat Step 1(1) through Step 1(4) 175 times to obtain 175 observations for $X_1 \sim X_3$, M, and $Y_1 \sim Y_3$. Then, M is categorized by the sample median and dummy variable D is introduced to represent a dichotomous moderator.

Step 2. Estimate scores for latent constructs.

Factor-based scores for F_X and F_Y are $(X_1 + X_2 + X_3) / 3$ and $(Y_1 + Y_2 + Y_3) / 3$, respectively. Factor scores derived from CFA are easy to obtain by SAS (using the PLATCOV command in PROC CALIS).

Step 3. Perform the traditional and the modified MRA we proposed for M as well as for D.

We record the estimates of the ES and the testing conclusions under the significance level $\alpha = 0.05$ for both of the continuous and the categorized moderators.

Step 4. Repeat *Step 1* through *Step 3* 200 repetitions to assess power and bias (The number of repetitions 200 is large enough to obtain stable results).

The power is assessed by computing $\hat{\Pr}$(reject H_0) = the number of rejections / 200. $\hat{\Pr}$(reject H_0 | H_0) should be close to α = 0.05; the higher is the $\hat{\Pr}$(reject H_0 | H_1), the more powerful is the corresponding test. ES were specified at 0, 0.05, 0.10, 0.15, and 0.2 to facilitate power comparison. The bias is assessed by

$$\text{Estimated bias} = \sum_{i=1}^{200} (\hat{ES}_i - ES) / 200,$$

where $\hat{ES}_i$ denotes the estimate of ES for repetition i.

3.3 Results and Discussions

To facilitate presentation, the traditional MRA with factor-based scores, the traditional MRA with factor scores, the modified MRA with factor-based scores, and the modified MRA with factor scores will be referred to as approaches (A), (B), (C), and (D), respectively.

The simulation results for the factor loadings set at 0.9, 0.6, and 0.5 for both F_X and F_Y indicate that the powers using the four approaches are all close to $\alpha = 0.05$ under H_0, and the ranking result of the power performance under H_1 for the four approaches is $D > B > C > A$.

When the moderator is categorized by its median, the power, in spite of keeping the same ranking results, will decrease by 15% ~ 30%. Therefore, categorization should not be considered when detecting moderating effects.

The simulation results for the factor loadings set at 0.6, 0.7, and 0.8 (not substantially different) for both F_X and F_Y

indicate that the power performance for the four approaches is about equally well ($D \approx B \approx C \approx A$). Again, power information loss does occur if the continuous moderator is categorized by its median.

To see more about the influence of factor loading setups, we have additionally conducted the power comparison for several different sets of factor loadings such as (0.7, 0.8, 0.9) and (0.7, 0.7, 0.8) as well as their cross setups with (0.9, 0.6, 0.5) and (0.6, 0.7, 0.8) for the constructs, all leading to the Cronbach α values of at least 70%. The following results have been observed:

1. If the factor loadings are substantially different (e.g., (0.9, 0.6, 0.5)) for both exogenous and endogenous constructs, then the ranking result of the power performance is $D > B > C > A$.

2. If the factor loadings are substantially different for the endogenous construct only, then the ranking result of the power performance is $D \approx B \approx C > A$.

3. If the factor loadings are substantially different for the exogenous construct only, then the ranking result of the power performance is $D \approx B > C \approx A$.

4. If none of exogenous and endogenous constructs has substantially different factor loadings, then the ranking result of the power performance is $D \approx B \approx C \approx A$.

The conclusions for the power comparisons among the four different approaches given above apply for different sample sizes (175 or 400), for different correlations (0.2 or 0.4) between the exogenous construct and the moderator, and for different sizes of the squared multiple correlations (0.3 or 0.5) for the structural model, showing the consistency of the comparative results.

When the factor loadings for indicators on their corresponding constructs show substantial difference, as can be frequently seen, the traditional MRA with factor-based scores can hardly detect trivial effect sizes. The modified MRA with factor scores derived from CFA shows much improvement. Although the factor-based scores are meaningful when the measurement model is adequate (Gerbing & Anderson, 1988), the corresponding power is not as high as the power resulting from factor scores. In other words, factor scores could reach the same power level as factor-based scores with a smaller sample size. The influence of different scoring approaches seems stronger than that of simultaneous inference or not. The scoring coefficients for producing factor-based scores are equal while scoring coefficients for producing factor scores are differently weighted. There exists a dominant coefficient, corresponding to the one with the largest loading, for computing factor scores. Different scoring approaches will lead to different scoring results, and in turn, influence the power of the test. On the other hand, when the factor loadings are close, scoring coefficients resulting from CFA will not have dominant one. They are about equally weighted. This may explain why their corresponding simulated power functions do not show remarkable superiority.

In addition, the power loss could be clearly observed for all combinations when we categorize a continuous manifest moderator. Therefore, categorization should be avoided before testing for moderating effects. Categorization may be allowed when the moderating effects are significant and subsequent analysis is needed.

For the bias assessment, how the biases resulting from different approaches differ was inconclusive; nevertheless, categorizing a continuous manifest moderator would seriously enlarge the bias and/or change the direction of bias.

四、計畫成果自評

The findings have made contributions to empirical research by showing that, when substantial differences among factor loadings occur, the traditional MRA with factor-based score (Approach A) is the least preferable while the proposed modified MRA with factor scores (Approach *D*) performs best. Moreover, it has been verified that categorizing a continuous manifest moderator by the median would reduce the power and lose much of the bias information.

Two research papers derived from this study were submitted to *Asian Pacific Management Review* and *British Journal of Mathematical and Statistical Psychology*. The former has been accepted for publication, and the latter is still under review.

五、參考文獻

Acito, F., & Anderson, R. D. (1986). A simulation study of factor score indeterminacy. *Journal of Marketing Research*, 23, 111-118.

Anderson, J. C., & Gerbing, D. W. (1988). Structural equation modeling in practice: A review and recommended two-step approach. *Psychological Bulletin*, 103, 411-423.

Baron, R. M., & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychological research: conceptual, strategic, and statistical consideration. *Journal of Personality and Social Psychology*, 51, 1173-1182.

Bollen, K. (1989). *Structural equations with latent variables*. New York: Wiley.

Gerbing, D.W., & Anderson, J.C. (1988). An updated paradigm for scale development incorporating unidimensionality and its assessment. *Journal of Marketing Research*, 25, 186-192.

Grapentine, T. (2000). Path analysis vs. structural equation modeling. *Marketing Research,* 12, 12-20.

Jaccard, J., & Wan, C. K. (1995). Measurement error in the analysis of interaction effects between continuous predictors using multiple regression: multiple indicator and structural equation approaches. *Psychological Bulletin,* 117, 348-357.

Kuan, C.-M. (2000). *Statistics: concepts and methods*, Taipei, Taiwan: Hwatai.

Lawley, D. N., & Maxwell, A. E. (1971). *Factor analysis as a statistical method*. New York: American Elsevier.

McClelland, G. H., & Judd, C. M. (1993). Statistical difficulties of detecting interactions and moderator effects. *Psychological Bulletin*, 114, 376-390.

Saunders, D. R. (1956). Moderator variables in prediction. *Educational and Psychological Measurement,* 16, 209-222.

Sharma, S., Durand, R. D., & Gur-Arie, O. (1981). Identification and analysis of moderator variables. *Journal of Marketing Research*, 18, 291-300.