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互耦合系統渾沌的再同步與廣義同步(I)

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（一） 計畫中英文摘要

中文摘要

關鍵詞：渾沌再同步化、互耦合、廣義同步化

渾沌同步化是近年來發現的重要研究主題。對於此主題，大多數的研究僅止於單向耦合系統，本計劃欲對更複雜的互耦合自治系統作深入研究。對於互耦合系統而言，一般來說我們並不知道同步後的吸引子型態及個數，即吾人無法預測同步化發生後的行為是規則或是渾沌的。因此，本計劃提出再同步化的方法，使得系統的行為能隨需求而表現出特定的吸引子型態，以俾利用。在研究中，橫截李亞普諾夫指數、條件熵及 SRB 測度等渾沌特徵值將被用來找出發生再同步化的觸發臨界值，不變流形法可判別再同步發生後的穩定性，穩定性支配函數則用以找出影響同步化與否的參數範圍，用符號動力法研究互偶合系統再同步化後 K-S 熵與同步化誤差的情況，再用改良內插胞映射法對耦合系統作全域分析，以求得再同步化發生的初值及參數區域，並藉以判斷此區域是否為碎形。接著，再運用各種非線性控制方法來達成不同種類非線性耦合系統的再同步化。

最後，將研究方向指向耦合自治系統的廣義渾沌同步特性。用互錯近傍法及相關函數研究兩渾沌訊號之廣義同步的程度，利用橫截李亞普諾夫指數及條件熵找出發生廣義同步的觸發臨界值，並用穩定性支配函數找出影響廣義同步化與否的參數範圍，以決定能保持廣義同步化穩定性的參數極值，再用改良內插胞映射法求得廣義同步化發生的初值及參數區域。

英文摘要

key words: Chaos resynchronization, mutually coupled, generalized synchronization

Chaos synchronization is an important research topic discovered in recent years. Since the focus of most research is on the unidirectionally coupled chaotic systems, mutually coupled chaotic systems will be explored in this project. In general, we do not know the types of attractors and the number of attractors when synchronization occurs. This means that we could not predict whether the behaviors are regular or chaotic once synchronization occurs. Thus, new method of resynchronization is proposed in this project to force the systems to have a specific attractor which we want. So, synchronized chaotic behaviors can be applied easily. In our work, the characteristic quantities of chaos such as transversal Lyapunov exponents, conditional entropy and SRB measure are used to find out the threshold of resynchronization. The stability after resynchronization can be verified by invariant manifold method. Master stability functions can be used to calculate the range of parameters affecting the resynchronization. The relations between K-S entropy and errors are researched by

symbolic dynamics. After resynchronization, MICM method is used to locate the basins of initial values and parameters for coupled systems so as to clarify whether these regions are fractals. Moreover, we use various nonlinear control methods to achieve chaos resynchronization for coupled chaotic systems.

Finally, we will focus on generalized synchronization of chaos for coupled systems. Degrees of generalized synchronization of chaos are explored via mutual false nearest neighbor method and correlation function. Transversal Lyapunov exponents and conditional entropy are used to find out the threshold of generalized synchronization of chaos. The ranges of parameters affecting the generalized synchronization of chaos are calculated by master stability functions. These can determine the limit of parameters affecting the stability of generalized synchronization of chaos. MICM method is used to locate the basins of initial values and parameters for coupled systems while generalized synchronization of chaos occurs.

（二） 報告內容

前言：

對於不同耦合型態的同步化得到不少穩定性判準，也利用控制方法達成同步化。發展出穩定性支配函數以判別耦合陣列系統的同步穩定性[13]。再者，發現了互耦合系統可能隨著耦合強度增加而破壞同步穩定性。最後，鑒於同步化有許多種類，且未有一統之定義，嘗試統一同步化之研究出現[15]，雖未盡完善，但也使研究進展跨出一步。渾沌同步的理論與應用正迅速發展中。目前可謂初露鋒芒，其前景未可限量，國內目前研究尚不多。職是之故，這方面的研究是極為重要的。在單向耦合系統中，我們明確知道系統同步化後行為模式，即其被偶合子系統會表現如同主動子系統一般；但對於互耦合的情況，一般來說，我們並不能預測同步化後的行為是規則或是渾沌的，而且系統吸引子的型態及個數皆屬未知。本計劃對於互耦合系統提出再同步化的方法，俾使系統行為能隨需求而表現出特定的吸引子型態，方能使系統能被有效的加以利用。

研究目的：

由於渾沌系統表現出對於初始條件極度敏感的特性，使得渾沌現象被認為難以同步化。自從少數研究成果發表以來[1-3]，科學家了解到同步化是可能的，因而吸引了大量的研究興趣 [4-16]，在秘密通訊方面、圖形成型(pattern formation)以及資訊處理(information processing)也有重要的成果[18-36,44]。在這些研究工作中，大多數在於探討單向耦合(unidirectional coupling)系統，本計劃則進一步研究更複雜的互耦合(mutual coupling)系統。除了研究其暫態及穩定特性外，並提出再同步(resynchronization)的方法，使系統的動態行為能受吾人所掌控，接著利用各種控制方法來達到再同步化的效果。以利於同步化系統能夠隨心所欲的被利用。另外，對於耦合系統的廣義渾沌同步(generalized synchronization of chaos)，截至目

前為止尚無令人滿意可應用之成果，本計劃則欲針對諸如充分性判準、穩定性與初值及參數範圍等作研究。

文獻探討：

渾沌現象是二十世紀學術界的重大進展，其基本行為及理論在各領域已有許多專書 [37-42]。在渾沌同步與控制也有專書 [43-44]。渾沌同步除了一些基礎研究之外 [1-17]，其應用於秘密通訊上亦有長足進展 [18-36]，利用控制方法達到同步化同樣有不錯的成果 [45-58]，對於廣義同步也有部分成果[59−63]，在國內也有林文偉、莊重及莊正等人有成果[65-72]；但由於互耦合系統複雜度較高，故其相關的研究仍屬少數 [1,2,64]。再同步的方法則由本計劃率先提出，以解決互耦合系統未知系統吸引子型態此一極為重要的問題，使得此類系統能如同單向耦合系統一樣被應用於秘密通訊、圖形成型及資訊處理.

（三） 研究方法

考慮兩互耦合自治系統與第三個渾沌系統，求得各種再同步之充分條件

1. 利用橫截李亞普諾夫指數(transversal Lyapunov exponents)、條件熵(conditional entropy)及 SRB 測度(Sinai, Ruelle and Bowen measure)等渾沌特徵值找出發生再同步的觸發臨界值。
2. 採用不變流形(invariant manifolds)法判別再同步發生後是否穩定。若同步化流形(synchronization manifolds)是一不變流形，且保有典型 k 雙曲性(normal k-hyperbolicity)，則同步化流形會在攝動情況下維持穩定性。
3. 將穩定性支配函數(master stability functions)應用於再同步化上，找出影響同步化與否的參數範圍，以決定一系統能保持再同步化穩定性的參數極值。
4. 用符號動力(symbolic dynamics)法研究互偶合系統再同步化後 K-S 熵(Kolmogorov-Sinai entropy)與同步化誤差的情況。
5. 利用改良內插胞映射(MICM)法對耦合系統作全域分析(global analysis)，以求得再同步化發生的初值及參數區域，並藉以判斷其是否為碎形(fractal)。

（四） 結果與討論

對於兩個互耦合的系統，若其產生同步化，一般而言，其同步化後的吸引子是未知的，但是此吸引子並不一定符合我們所需求。本研究計劃及針對此類互耦合系統做探討，欲以第三個已知系統對已同步化的耦合系統做再同步，期望能使之同步到第三個系統，即我們所要求且已知的吸引子。

以離心調速器為例，當前兩個互耦合系統分別同步化到平衡點、週期解及渾

沌吸引子時，都可用線性回饋的控制讓已經同步化的系統再同步或實用同步(practical synchronization)到渾沌吸引子。因為偶合系統過於複雜，李亞普諾夫指數(transversal Lyapunov exponents)可能無法作爲同步化發生的指標。雖然有成功的例子，但並未找到使系統再同步化的一般性判據。

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