Evaluating the performance improvement of differential phase-shift keying signals by amplitude regeneration and phase-noise suppression

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Theoretical bit error rates of the differential phase-shift keying format with various regeneration schemes is presented. A comparison with ideal regeneration reveals that averaging the phase noise of adjacent bits is found to eliminate most penalties that are induced by phase noise. © 2008 Optical Society of America

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The differential phase-shift keying (DPSK) format has recently attracted considerable attention in long-haul high-speed optical communication systems. A comparison with the traditional on–off keying (OOK) indicates that the DPSK format offers a sensitivity improvement of ~3 dB and higher nonlinear tolerance [1]. Nevertheless, most all-optical regeneration schemes for the OOK format are not suitable for the DPSK format, since the phase information of the DPSK signals will be distorted. Accordingly, several phase-preserving (PP) amplitude regenerators have been proposed to suppress the amplitude noise (AN) of the DPSK signals with negligible extra phase distortion [2,3]. Additionally, both coherent and incoherent methods can be used to suppress the phase noise (PN) of the DPSK signals, such as a phase-sensitive amplifier (PSA) [4] and a phase noise-averaging (PNA) regenerator [5–8]. While the PSA is proposed to simultaneously remove PN and AN the requirement of a phase-locking pump beam is difficult to implement. In the approach of PNA regeneration the PN, $\theta(t)$, is converted into the average of neighboring PNs, $[\theta(t)+\theta(t-T)]/2$, where T is the bit period. However, the improvement by averaging PN has only been shown by reducing the variance of PN. Actually, less than 30% of PN is removed by PNA regeneration when the PNs of adjacent pulses are uncorrelated [6].

In this Letter, various regeneration schemes are considered, and the probability density function (pdf) of the phase distribution and the bit error rate (BER) of a DPSK system with complex Gaussian noise are analytically given for the first time, to the best of our knowledge. Comparing the results with an ideal PN elimination demonstrates that PNA regeneration can eliminate most of the PN-induced penalty. Since only Gaussian noise is considered, these results show the upper limits of various regeneration schemes.

Figure 1 presents the configuration of a linear DPSK system with a regenerator in front of the receiver, and $n_1(t)$ and $n_2(t)$ represent complex Gaussian noises that are accumulated in transmission and loaded before the receiver to enable BER evaluation. When a polarizer is applied to filter out the noise, which is orthogonal to DPSK signals, the signals at point **1** can be represented as $s(t)=A(t)+n_1(t)$, where $A(t)=\pm A_0$ indicates phase-modulated signals and the variance of n_1 is $2\sigma_1^2$. To better understand the noise suppression capability of a regenerator, all regenerators are assumed to be noiseless and lossless (or gain=1). With PP regeneration the signals become $\mathrm{AR}\{s(t)\}=\pm\sqrt{A_0^2+2\sigma_1^2}\exp(j\theta(t))$, where $\mathrm{AR}\{\cdot\}$ indicates the PP amplitude regeneration without changing the average power and the pdf of $\theta(t)$ can be expressed as a Fourier series [9],

$$p_\Theta(\theta)=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\left[\frac{\sqrt{\pi\rho_1}}{2}e^{-\rho_1/2}\times\Pi_m(\rho_1)\right]e^{jm\theta}, \quad (1)$$

where $\Pi_m(x)\equiv I_{(m-1)/2}(x/2)+I_{(m+1)/2}(x/2)$, $\rho_1\equiv A_0^2/(2\sigma_1^2)$ is the signal-to-noise ratio (SNR) at point **1**, and $I_n(\cdot)$ is the nth modified Bessel function of the first kind. By assuming the variance of n_2 to be $2\sigma_2^2$, the pdfs of differential phase noise (DPN), $\Delta\theta=\theta(t)-\theta(t-T)$, at points **1** and **2** are

$$p_{\Delta\Theta}(\Delta\theta)=\frac{\rho_1 e^{-\rho_1}}{8}\sum_{m=-\infty}^{\infty}\Pi_m^2(\rho_1)e^{jm\Delta\theta}, \quad (2)$$

$$p_{\Delta\Theta,\mathrm{pp}}(\Delta\theta)=\frac{\pi\rho_1\rho_{\mathrm{pp}}e^{-(\rho_1+\rho_{\mathrm{pp}})}}{32}\sum_{m=-\infty}^{\infty}\Pi_m^2(\rho_1)\Pi_m^2(\rho_{\mathrm{pp}})e^{jm\Delta\theta}, \quad (3)$$

where $\rho_{\mathrm{pp}}\equiv(A_0^2+2\sigma_1^2)/(2\sigma_2^2)=\rho_2(1+\rho_1^{-1})$ and $\rho_2\equiv A_0^2/(2\sigma_2^2)$. Equations (2) and (3) are derived based on the fact that the characteristic function of the sum of independent random variables is the product of individual ones. Then, the BER of signals with a PP am-

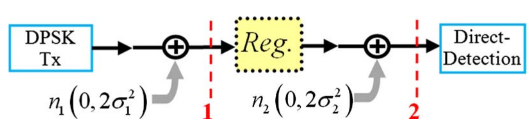

Fig. 1. (Color online) Schematic of regeneration configuration.

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plitude regeneration is $1-\int_{-\pi/2}^{\pi/2} p_{\Delta\Theta,\mathrm{pp}}(\Delta\theta)\mathrm{d}\Delta\theta$:

$$BER_{\mathrm{pp}} = \frac{1}{2} - \frac{\pi\rho_1\rho_{\mathrm{pp}}e^{-(\rho_1+\rho_{\mathrm{pp}})}}{8}\sum_{m=0}^{\infty}\frac{(-1)^m}{2m+1} \times \Pi_{2m+1}^2(\rho_1)\Pi_{2m+1}^2(\rho_{\mathrm{pp}}). \quad (4)$$

For comparison, the BER of direct-detection DPSK signals without regeneration is $\mathrm{BER}_0 = \exp(-\rho_0)/2$ [9,10], where $\rho_0^{-1} = \rho_1^{-1} + \rho_2^{-1}$ is the final SNR at the receiver. Furthermore, since phase is a relative parameter, a coherent reference beam is required to regenerate phase information, and a PSA has been proposed to realize phase regeneration [4]. In this Letter, an ideal coherent regenerator is assumed to be able to completely eliminate both AN and PN. The BER determined by a coherent beam at the regenerator is identical to the case of coherent detection, $\mathrm{erfc}(\sqrt{\rho_1})/2$, where $\mathrm{erfc}(\cdot)$ is the complementary error function [9,10]. Because the regenerator is lossless, the BER at the direct-detection receiver is $\exp(-\rho_{\mathrm{pp}})/2$. With negligible differences the cross term can be omitted, and the final BER with coherent regeneration is $\mathrm{BER}_{\mathrm{co}} = \mathrm{erfc}(\sqrt{\rho_1})/2 + \exp(-\rho_{\mathrm{pp}})/2$.

Moreover, it has been proposed that PP amplitude regenerators can simultaneously realize AN elimination and PN averaging [6]. As shown in Fig. 2, a delay interferometer (DI) converts the DPSK signals into two phase-modulated OOK signals: duobinary, $[A(t)+A(t-T)+n_1(t)+n_1(t-T)]/2$ and alternate-mark inversion (AMI), $[A(t)-A(t-T)+n_1(t)-n_1(t-T)]/2$. Ideally, the PP amplitude regenerators can eliminate AN of both marks and spaces, i.e., the output power of both OOK signals is either 0 or $A_0^2+\sigma_1^2$ and the signals become $\bar{s}(t) = \mathrm{AR}\{A(t)+[n_1(t)\pm n_1(t-T)]/2\}$, where a 3 dB loss is neglected and $\pm$ depends on $A(t)$ and $A(t-T)$ being in-phase or out-of-phase. Since $\theta(t) \approx \Im\{n_1(t)\}/A(t)$, where $\Im\{\cdot\}$ is the imaginary part, the PN of $\bar{s}(t)$ can be approximated as $[\theta(t)+\theta(t-T)]/2$, which turns out to be the averaged PN. Although cascading another DI after the PNA regenerator can further increase the correlation between PNs of adjacent pulses, it also induces additional AN [6], and this is beyond the scope of this Letter. Owing to $\bar{s}(t)$ and $\bar{s}(t-T)$ influenced by identical noise $n_1(t-T)/2$ as well as independent noise $n_1(t)/2$ and $n_1(t-2T)/2$, the pdfs of DPN at points **1** and **2** are (Appendix A)

$$p_{\Delta\Theta}(\Delta\theta) = \frac{e^{-4\rho_1}}{8}\sum_{m=-\infty}^{\infty} H_m(1,4\rho_1)e^{jm\Delta\theta}, \quad (5)$$

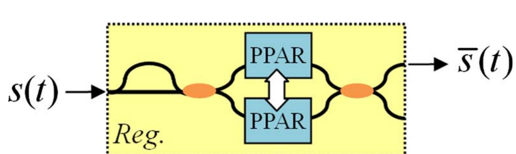

Fig. 2. (Color online) Setup of the PNA regenerator. PPAR, phase-preserving amplitude regenerator.

$$p_{\Delta\Theta,\mathrm{pna}}(\Delta\theta) = \frac{\pi\rho_{\mathrm{pna}}e^{-(4\rho_1+\rho_{\mathrm{pna}})}}{32}\sum_{m=-\infty}^{\infty} H_m(1,4\rho_1) \times \Pi_m^2(\rho_{\mathrm{pna}})e^{jm\Delta\theta}, \quad (6)$$

where

$$H_m(\xi,\rho) \equiv \int_0^{\infty} y e^{-(1+\xi)y} I_0(2\sqrt{\xi\rho y})\Pi_m^2(y)\mathrm{d}y, \quad (7)$$

and $\rho_{\mathrm{pna}} \equiv (A_0^2+\sigma_1^2)/(2\sigma_2^2) = \rho_2[1+(2\rho_1)^{-1}]$. Moreover, since the ideal amplitude regeneration of OOK signals must make a binary decision on signals, the BER determined at the PNA regenerator, BER_1, depends on the criterion of the binary decision. While the amplitude regenerators in Fig. 2 are steplike and mutually independent, the BER can be approximated as double that of the OOK signals: $\mathrm{BER}_1 = \exp(-\rho_1/2)$ [10]. Otherwise, when the decision is made by comparing the power of two OOK signals [7,8], it is similar to direct-detection DPSK: $\mathrm{BER}_1 = \exp(-\rho_1)/2$. Therefore, by integrating Eq. (6) the BER at the receiver is

$$\mathrm{BER}_{\mathrm{pna}} = \frac{e^{-\rho_1}}{2} + \frac{1}{2} - \frac{\pi\rho_{\mathrm{pna}}e^{-(4\rho_1+\rho_{\mathrm{pna}})}}{8}\sum_{m=0}^{\infty}\frac{(-1)^m}{2m+1} \times H_{2m+1}(1,4\rho_1)\Pi_{2m+1}^2(\rho_{\mathrm{pna}}). \quad (8)$$

The first term in Eq. (8) is BER_1 with the best decision criterion. Even though the amplitude regenerators in Fig. 2 only remove the AN of spaces with marks left unchanged, the PN of adjacent bits have been simultaneously averaged. The regenerated signals become $\bar{s}'(t) = A(t)+[n_1(t)\pm n_1(t-T)]/2$ of which the phase is identical to that of $\bar{s}(t)$ but residual AN still exists. Hence, neighboring pulses at point **2** contain identical and mutually independent noises with variances of $\sigma_1^2/2$ and $\sigma_1^2/2+2\sigma_2^2$, respectively, and the pdf of DPN and the BER becomes

$$p_{\Delta\Theta,\mathrm{pna}'}(\Delta\theta) = \frac{(4\rho_1+\rho_2)e^{-4\rho_1}}{8\rho_2} \times \sum_{m=-\infty}^{\infty} H_m\left(1+\frac{4\rho_1}{\rho_2},4\rho_1\right)e^{jm\Delta\theta}, \quad (9)$$

$$\mathrm{BER}_{\mathrm{pna}'} = \frac{e^{-\rho_1}}{2} + \frac{1}{2} - \frac{(4\rho_1+\rho_2)e^{-4\rho_1}}{2\rho_2} \times \sum_{m=0}^{\infty}\frac{(-1)^m}{2m+1}H_{2m+1}\left(1+\frac{4\rho_1}{\rho_2},4\rho_1\right). \quad (10)$$

Figure 3(a) plots the pdfs of PN, DPN, and averaged DPN without n_2, which are described by Eqs. (1), (2), and (5) with $\rho_1 = 14$ dB. The tail of DPN distribution, which is the main contributor to the BER, is effectively suppressed by PNA regeneration. After loading n_2, the solid curves in Fig. 3(b) show the analytical results of BER_0, $\mathrm{BER}_{\mathrm{co}}$, $\mathrm{BER}_{\mathrm{pp}}$, $\mathrm{BER}_{\mathrm{pna}}$, and

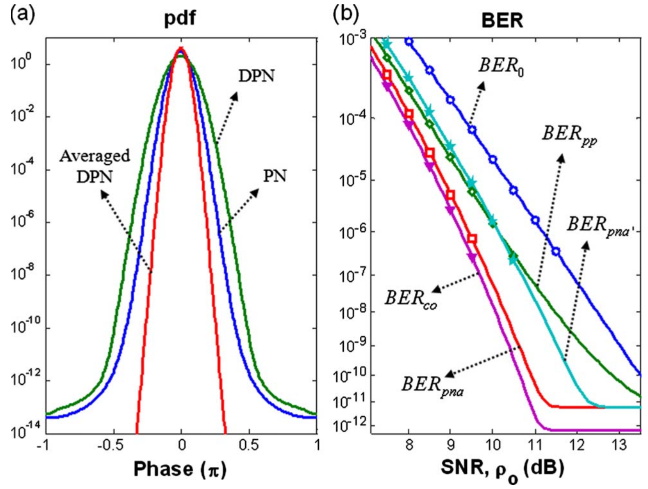

Fig. 3. (Color online) With $\rho_1 = 14$ dB. (a) PDFs of phase distribution. (b) BERs of the DPSK signals with various regeneration schemes, analytical results (curves) and Monte Carlo method (markers).

$\mathrm{BER}_{\mathrm{pna}'}$. The error floors for coherent regeneration and PNA regeneration are determined by n_1, and it indicates that any regenerator can only improve signal performance but cannot correct existing errors. In addition, the BERs indicated by markers in Fig. 3(b) are assessed by the brute-force Monte Carlo method, performed with 10^9 bits, and the results closely agree with each other. The SNR required to achieve the BER of 10^{-9}, shown in Fig. 4, indicates that PNA regeneration can remove most penalty induced by PN, because the difference between its SNR and that of perfect coherent regeneration is less than ~0.3 dB. However, if steplike independent amplitude regenerators are adopted in PNA regeneration (dashed curve in Fig. 4) then regeneration fails when ρ_1 is less than 16.2 dB. Accordingly, the amplitude regenerators shown in Fig. 2 dominate the performance of PNA regeneration if the input SNR is low. Furthermore, Fig. 4 demonstrates that the improvement over PP regeneration vanishes when SNR is lower than 15 dB, because the PN induced by n_1 dominates the whole PN. Similarly, since averaging PN suppresses most PN-induced penalties, the signals with PNA residual AN regeneration can outperform those with PP regeneration, as ρ_1 is low.

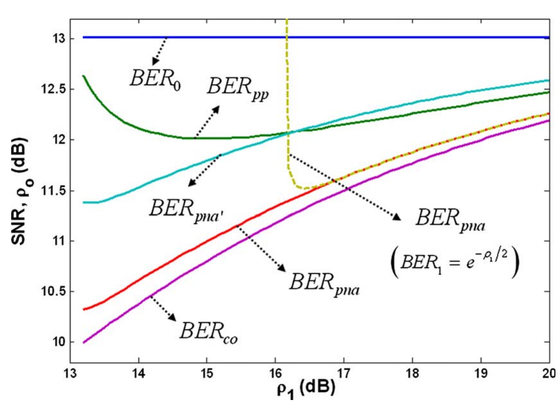

Fig. 4. (Color online) SNR at the BER of 10^{-9}.

This Letter analytically derives the BERs of amplitude-regenerated DPSK signals with no additional phase processing, PN averaging, and ideal phase regeneration. The SNR difference at the BER of 10^{-9} between PNA regeneration and ideal phase regeneration is less than 0.3 dB, indicating that incoherent PNA regeneration eliminates most of the PN-induced penalty, even though PN is statistically reduced by less than 30%.

Appendix A

Assume $u = (A_0 + n_x) + n_y$ and $\tilde{u} = (A_0 + n_x) + \tilde{n}_y$, where n_x, n_y, and $\tilde{n}_y$ are independent complex Gaussian noises whose variances are $2\sigma_x^2$, $2\sigma_y^2$, and $2\sigma_y^2$. Similar to Eq. (1), by defining $A_0 + n_x \equiv \gamma \exp(j\psi)$, $u \equiv r\exp(j\vartheta)$, and $\tilde{u} \equiv \tilde{r}\exp(j\tilde{\vartheta})$, the pdf of $\varphi \equiv \vartheta - \psi$ (or $\tilde{\varphi} \equiv \tilde{\vartheta} - \psi$) with given γ and ψ is

$$p_\Phi(\varphi|\gamma,\psi) = \frac{e^{-(\rho_y/2)}}{4}\sqrt{\frac{\rho_y}{\pi}}\sum_{m=-\infty}^{\infty}\Pi_m(\rho_y)e^{jm\varphi}, \quad \text{(A1)}$$

where $\rho_y \equiv \gamma^2/(2\sigma_y^2)$. Since $\Delta\vartheta \equiv \vartheta - \tilde{\vartheta}$ equals $\varphi - \tilde{\varphi}$, the pdf of $\Delta\vartheta$ without given γ and ψ is

$$p_{\Delta\Theta}(\Delta\vartheta) = \int_0^\infty\int_{-\pi}^{\pi}\left[\frac{\rho_y e^{-\rho_y}}{8}\sum_{m=-\infty}^{\infty}\Pi_m^2(\rho_y)e^{jm\Delta\vartheta}\right] \times p_{\Gamma,\Psi}(\gamma,\psi)\mathrm{d}\psi\mathrm{d}\gamma, \quad \text{(A2)}$$

where $p_{\Delta\Theta}(\Delta\vartheta|\gamma,\psi)$ shown in the brackets is independent of ψ and $p_\Gamma(\gamma)$ is a Rice-distribution,

$$\int_{-\pi}^{\pi} p_{\Gamma,\Psi}(\gamma,\psi)\mathrm{d}\psi = \frac{\gamma}{\sigma_x^2}\exp\left(-\frac{\gamma^2 + A_0^2}{2\sigma_x^2}\right)I_0\left(\frac{\gamma A_0}{\sigma_x^2}\right). \quad \text{(A3)}$$

By inserting Eq. (A3), Eq. (A2) becomes

$$p_{\Delta\Theta}(\Delta\vartheta) = \frac{\xi e^{-\rho}}{8}\sum_{m=-\infty}^{\infty}H_m(\xi,\rho)e^{jm\Delta\vartheta}, \quad \text{(A4)}$$

where $\xi \equiv \sigma_y^2/\sigma_x^2$, $\rho \equiv A_0^2/(2\sigma_x^2)$, and the definition of $H_m(\xi,\rho)$ is Eq. (7).

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