國科會成果精簡報告

應用於頻率選擇性通道之分離式多調調變收發器
DMT Transceivers for Frequency Selective Channels

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1 摘要

In this report, we introduce the BTDM (Block-based Time Division Multiplexing) system, which can be obtained from the OFDM system through a remarkably simple modification. For a given signal to noise ratio, it enjoys a smaller transmission power than the OFDM system for all practical range of bit error rate.

Keywords: multicarrier system, OFDM, bit error rate, BTDM,

2 緣由與目的

The OFDM system is receiving growing attention as an important transceiver system for wireless channels [1][2]. It has been adopted in standards for various wireless applications, e.g., wireless LAN [3]. The transmitter and receiver perform respectively M-point IDFT and DFT computation, where M is the number of tones or subchannels.

In this report, we propose a new class of DFT based transceivers. In the new system, both IDFT and DFT computation are done at the receiver end, the transmitter is an identity matrix followed by cyclic insertion. It will be called the BTDM (Block Time Division Multiplexing) transceiver. The overall complexity of the BTDM system is the same as the OFDM system. The BTDM system with a zero-forcing receiver (ZF-BTDM) can be obtained from the OFDM system by moving the IDFT matrix at the transmitting side to the end of the receiver. The BTDM system, though seemingly simple modification of the OFDM system, has important advantages over the OFDM systems. In the BTDM system, the transmitter performs only parallel to serial operation, the PAPR is very low. Furthermore, spectral nulls of the channel do not lead to prominent performance degradation. For all practical range of bit error rate, the BTDM system outperforms the OFDM system significantly.

3 結果與討論:

Zero-forcing Receivers. The block diagram of the BTDM system is as shown in Fig. 1(a). The new transceiver can be obtained from the OFDM system by cascading the DFT matrix $\mathbf{W}$ to the front and the IDFT matrix $\mathbf{W}^{\dagger}$ to the end. At the transmitting side, the DFT and IDFT matrices cancel off to become the i-

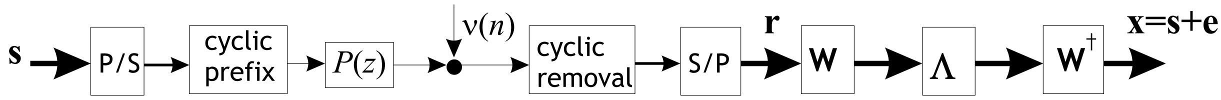

圖 1: The block diagram of the BTDM with a zero-forcing receiver.

dentity matrix. The new transceiver continues to be ISI free and the receiver is a zero-forcing receiver. The transceiver shown in Fig. 1(a) will be referred to as ZF-BTDM. Now that the IDFT matrix at the transmitter of the OFDM system is moved to the receiving end, there is no computation at the transmitter and the receiver performs both DFT and IDFT. The overall complexity of the transceiver is: one $M \times M$ DFT matrix $\mathbf{W}$, one $M \times M$ IDFT matrix $\mathbf{W}^\dagger$, and M multipliers $1/P_i$. It is exactly the same as the OFDM system. The channel dependent part is still the set of M scalars $1/P_i$, for $i = 0, 1, \cdots, M-1$.

As the BTDM transceiver in Fig. 1(a) is ISI free, the output noise vector $\mathbf{e}$, defined as $\mathbf{e} = \mathbf{x} - \mathbf{s}$, comes entirely from the channel noise $\nu(n)$. It can be verified that the autocorrelation matrix of the noise vector $\mathbf{e}$ is given by $\mathbf{R}_e = N_0 \mathbf{W}^\dagger \mathbf{\Lambda}^\dagger \mathbf{\Lambda} \mathbf{W}$. Notice that the matrix $\mathbf{W}^\dagger \mathbf{\Lambda}^\dagger \mathbf{\Lambda} \mathbf{W}$ is circulant, all of its diagonal elements are the same. One can verify that the (0,0)th element of $\mathbf{W}^\dagger \mathbf{\Lambda}^\dagger \mathbf{\Lambda} \mathbf{W}$ is given by $\frac{1}{M}\sum_{i=0}^{M-1} 1/|P_i|^2$. All subchannels have the same noise variance, given by

$$\sigma_{e_k}^2 = N_0/M \sum_{i=0}^{M-1} 1/|P_i|^2, \quad k = 0, 1, \cdots, M-1. \tag{1}$$

Therefore the bit error rates in all subchannels are identical and average bit error rate is also the same. For BPSK case, it is,

$$Pe_{ZF-BTDM} = Q\left(\sqrt{\frac{\gamma}{\frac{1}{M}\sum_{i=0}^{M-1} 1/|P_i|^2}}\right). \tag{2}$$

Thresholding Receivers. Consider the introduction of a thresholding operation to the receiver of the ZF-BTDM. We modify the transceiver in Fig. 1(a) to Fig. 1(b) by inserting a diagonal matrix $\mathbf{\Theta}$ in the receiver. The matrix $\mathbf{\Theta}$ is a diagonal matrix with diagonal elements θ_i equal to zero or one. The transmitter remains the same. The system in Fig. 1(b) will be called TH-BTDM as it has an additional thresholding device. If $\theta_i = 0$, the i-th band is discarded. If $\theta_i = 1$ for all i, it reduces to the ZF-BTDM. As some of the bands may be discarded, the transceiver is no longer ISI free. The following theorem gives the optimal thresholding matrix $\mathbf{\Theta}$ for minimizing the total output noise power.

Theorem 1 *Consider the transceiver in Fig. 1(b). The input modulation symbols have variance E_s and the noise is AWGN with variance N_0. For minimum mean squared error, the diagonal matrix $\mathbf{\Theta}$ with diagonal $\theta_i = 0$ or 1 should be chosen as,*

$$\theta_i = \begin{cases} 1, & \frac{1}{|P_i|^2} < \gamma \\ 0, & \textit{otherwise}, \end{cases} \quad 0 \le i < M, \quad \gamma = E_s/N_0. \tag{3}$$

In this case the noise variance of each subchannel is the same, given by, $\sigma_{e_k}^2 = \frac{N_0}{M}\sum_{i=0}^{M-1} \min\left(\gamma, 1/|P_i|^2\right)$. *For BPSK modulation, the bit error probability is,*

$$Pe_{TH-BTDM} = Q\left(\sqrt{\frac{\gamma}{\frac{1}{M}\sum_{i=0}^{M-1} \min\left(\gamma, \frac{1}{|P_i|^2}\right)}}\right) \tag{4}$$

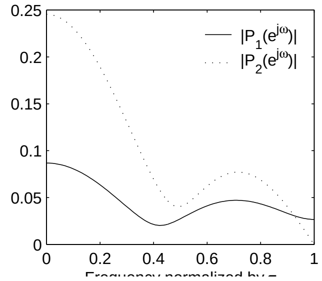

圖 2: The magnitude responses of $P_1(z)$ and $P_2(z)$.

Numerical Examples. The two channels $p_1(n)$ and $p_2(n)$ to be used for the examples have length = 4 ($L = 3$) and coefficients,

$$p_1(n) : [0.0394 \;\; 0.0112 \;\; 0.0664 \;\; 0.0708],$$

$$p2(n) : [0.0503 \;\; 0.0443 \;\; 0.0724 \;\; 0.0784].$$

The coefficients are obtained from iid Gaussian random variables with zero mean and unit variance. The magnitude responses of the two channels are as shown in Fig. 2. The channel noise is AWGN with zero mean. The modulation symbols are BPSK. We choose $M = 64$. We will consider the BTDM system with zero-forcing and thresholding receivers. The bit error rates are respectively $Pe_{ZF-BTDM}$ in (2), and $Pe_{TH-BTDM}$ in (4).

Example 1. *ZF-BTDM and OFDM.* Using channel $p_1(n)$, Fig. 3(a) shows the bit error rate (BER) performance of the OFDM and the ZF-BTDM system. The two curves crosses when BER is equal to 0.03 and $\gamma = E_s/N_0$ is equal to 29 dB. The OFDM system has a lower BER than the BTDM system for $\gamma < 29$dB. However, for the range of SNR beyond the crossing, the ZF-BTDM has a smaller BER for a given SNR. The OFDM system is seriously affected by some of the worst tones and the ZF-BTDM system has a sharper roll-off by comparison. To explain this, we plot for $\gamma = 35$ dB, the BERs of the 64 subchannels in the OFDM and ZF-BTDM systems in Fig. 3(b). The subchannels have been ordered such that $|P_0| \le |P_1| \le \cdots |P_{M-1}|$. In the OFDM system the subchannel noise variances will be in decreasing order and hence the BERs of individual subchannels are also in decreasing order, as shown in Fig. 3(b). For each of the 64 subchannels, the SNR and the corresponding BER is shown as one dot in Fig. 3(c). In the ZF-BTDM system, the noise variances of all the subchannels are identical, resulting in the constant line shown in Fig. 3(b). In the BER v.s. SNR plot of individual subchannels (Fig. 3(c)), there is only one point, shown as a solid square. From Fig. 3(b), we see that only around one quarters of subchannels are worse than the ZF-BTDM and the other 3 quarters are actually better or much better than the ZF-BTDM system. Fig. 3(c) shows that for $\gamma = 35$ dB, although some of the subchannels in the OFDM systems have very low BER, some of the worst subchannels still fall in the relatively flat part of the Q-function and the BER in these subchannel is high, with the worst BER ≈ 0.1. The average BER for $\gamma = 35$ dB is around 0.005.

Example 2. Here we compare the BER performances of BTDM system with zero-forcing receiver (ZF-BTDM), BTDM system with thresholding receivers (TH-BTDM) to that of the OFDM system. Fig. 4(a) shows the bit error rate (BER) performance when the channel $p_1(n)$ is used. The thresholding receiver performs better than the zero-forcing receiver, especially for small SNR. The gap narrows as SNR increases and the two curves converges as one for SNR over 32.5 dB. This can also be observed from the thresholding rule in (3); the bands are retained if $\gamma > 1/|P_i|^2$ and dropped otherwise. When $\gamma > 1/|P_i|^2$ for all i, all the bands are kept, in which case the receiver becomes a zero-forcing receiver. We also see that the TH-BTDM system outperforms the OFDM system for all SNR.

Fig. 4(b) shows the results when the channel

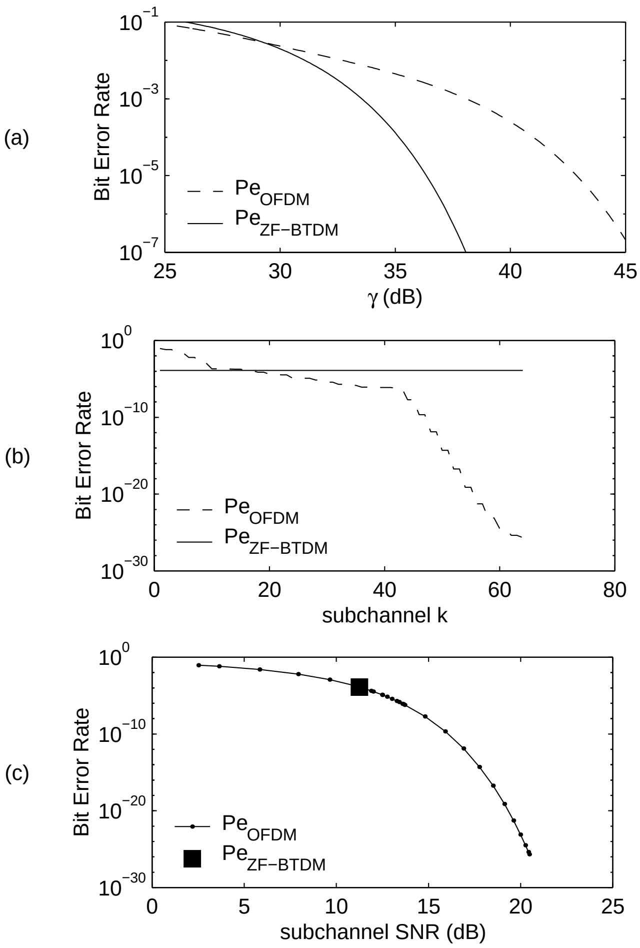

圖 3: Comparison of OFDM and ZF-BTDM systems for the channel $P_1(z)$.

$p_2(n)$ is used, which has a zero at around π. For the ZF-BTDM system, the channel noise goes to infinity in each subchannel and the BER levels at 0.5. However, there is no serious degradation on the performance of the thresholding receiver. For the OFDM system the performance is plagued by the spectral null and the performance is stalling for larger SNR. When SNR is large the bits in all subchannels can be correctly decoded except for the subchannel with the spectral null, whose BER is around 0.5. The overall BER performance floors at around $1/2M$, which is approximately 0.0078 for $M = 64$. By comparison, the TH-BTDM system has a much smaller BER for larger SNR; the spectral null does not lead to prominent degradation.

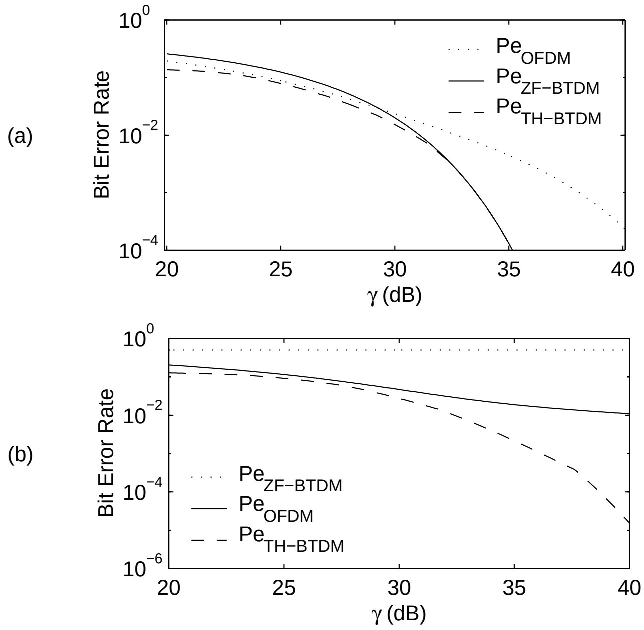

圖 4: Performance of ZF-BTDM, TH-BTDM and OFDM systems for (a) channel $P_1(z)$ and (b) channel $P_2(z)$.

參考書目

[1] J. A. C. Bingham, "Multicarrier Modulation For Data Transmission: An Idea Whose Time Has Come," IEEE Commun. Megazine, vol. 26, no. 6, pp. 5-14, May 1990.

[2] L. J. Cimini, "Analysis and Simulation of a Digital Mobile Channel Using Orthogonal Frequency Division Multiple Access," IEEE Trans. Commun., vol. 30, pp. 665-675, July 1985.

[3] ISO/IEC, IEEE Std. 802.11a, 1999.

[4] J. G. Proakis, *Digital Communications*, McGraw-Hill, 1995.