行政院國家科學委員會補助專題研究計畫成果報告

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※　　用於通道等化之高效率卡門演算法　　※
※　Adaptive Channel Equalization Using Efficient　※
※　　　　Kalman Algorithms　　　　※
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計畫類別：¦**個別型計畫**　　□**整合型計畫**
計畫編號：NSC 89－2213－E－009－161－
執行期間：89 **年**　8 **月**　1 **日至** 90 **年**　7 **月**　31 **日**

計畫主持人：吳文榕
共同主持人：

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執行單位：國立交通大學電信工程學系

中　華　民　國 90　**年**　8　**月**　17　**日**

行政院國家科學委員會專題研究計畫成果報告
用於通道等化之高效率卡門演算法
Adaptive Channel Equalization Using Efficient Kalman Algorithms

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一、中文摘要

眾所周知，卡門等化器比適應性有限脈衝濾波器來得有效率，卡門等化器卻沒有廣泛的被使用，這主要的原因是其運算量太高的緣故。本計劃即是針對此問題提出解決的方法，我們推導了一個穩態之卡門增益的數學式，利用這個式子，穩態之卡門增益可以快速的算出，一旦如此，濾波的動作就可以有效的進行而無需任何矩陣的運算，因此計算量可以大大的降低，若 N 是濾波器的長度的話，傳統的卡門等化器之運算量是正比於 N^2，而我們的方法只正比於 N。

關鍵詞：通道等化，卡門等化器，卡門濾波器

Abstract

It is known that the Kalman equalizer (KE) has superior efficiency than the finite impulse response (FIR) equalizer. Because of the high computational complexity problem, the KE has not widespreadly used. This project proposes an efficient algorithm overcoming this problem. Using a novel method, we derive a closed-form expression for the steady-state Kalman gain. This steady-state Kalman gain is then used in the filtering process. Using this approach, no matrix operations are required and a tremendous reduction in computation is obtained. It can be shown that the computational complexity for the conventional KE is on the order of $O(N^2)$ where N is the filter length, while that for the proposed KE is on the order of $O(N)$.

Keywords: Channel Equalization, Kalman Filtering, Kalman Equalizer

二、緣由與目的

Adaptive channel equalization has been an important research issue in signal processing. General approaches to this problem involve the use of an finite-impulse-response (FIR) or infinite-impulse-response (IIR) filter. The advantage of the FIR equalizer is that many adaptive algorithms are available and the behavior of the equalizer is well-understood. Probably, the most popular adaptive algorithm is the least mean square (LMS) algorithm. This algorithm has a simple structure and the computational requirement is low.

It is know that the IIR filter is more efficient than the FIR filter. It can equalize a channel with less tap-weights. However, the adaptive IIR filter have stability problem. Another solution to the IIR equalization problem is the use of the Kalman filter. The Kalman filter is known to be stable and fast converging. However, its computational complexity is still high. Lawrence and Kaufman [1] first proposed to apply the Kalman filter in equalization. Luvison and G. Pirani proposed to adaptively adjust the Kalman gain [2] using the LMS algorithm. Using this approach, no matrix operations are required. However, the convergence is slow and the MSE is large. Mulgrew and Cowan [3] proposed another structure that a channel identification algorithm is run in parallel with the Kalman filter. Although this algorithm can yield good results, the high computational complexity problem still remain.

三、結果與討論

Let H=[$h(0)\ h(1) \dots h(M-1)$]^T be the impulse response vector of the channel, $x(n)$ be the channel output, $S(n)$=[$s(n)$ $s(n-1) \dots s(n-M+1)$]^T is the input vector, and $v(n)$ is the channel noise . Then, the output can be written as

$$x(n)=H^T S(n)+v(n). \qquad (1)$$

Let $S(n)$ be the state vector. We then have a state space representation as follows:

$$S(n)=G\,S(n-1)+s(n)b \qquad (2)$$

where G is a MxM shift matrix and b=[$1\ 0 \dots 0$]^T . Thus, we have obtained the state and measure equations and the Kalman filter can be applied.

The idea behind our method is the equivalence of the Wiener and Kalman filters. Using this equivalence, we can explore the relation between the steady-state Kalman gain and the Wiener solution. Due to the special structure of the state transition matrix in (2), the relation can be identified. Once the relation is defined, the Wiener solution can be exactly solved to obtain the steady-state Kalman gain. Let $x(n)$ be the filter input, $d(n)$ be the desired signal, and $W(z)$ be the z-transform of the Wiener filter. Then,

$$W(z)=\frac{1}{rR_{xx}^{+}(z)}\left[\frac{R_{dx}(z)}{R_{xx}^{+}(z)}\right]_{+}$$

Where ¥ is a constant and $R_{xy}^{+}(z)$ denote the causal part of the z-transform of the correlation function between $x(n)$ and $y(n)$. The transfer function in the bracket of (4) can be easily evaluated if the channel response is known. From some derivations, we have found that the steady-state Kalman gain corresponds to a partial response of (4). Let the transfer function of the Kalman gain be $G(z)$. Then

$$G(z)=\frac{\pm_s^{\,2}}{¥}\left[\frac{z^{-²}H(z^{-1})}{R_{xx}^{+}(z^{-1})}\right]_{+}$$

where $\pm_s^{\,2}$ is the variance of the transmitted symbol, $H(z)$ is the z transform of the channel response, and ² is the allowable delay. Note that $R_{xx}^{+}(z)$ corresponds to a causal IIR whitening filter. This filter can be obtained by the spectral factorization method. Of course, this is not desirable. By approximating the whitening filter as an FIR filter, we can apply the Levinson-Durbin algorithm to identify its coefficients. This results in a very efficient algorithm. The conventional and proposed Kalman equalizer is shown in Figs. 1-2. we carry out some computer simulations to demonstrate the efficiency of the proposed algorithm. We consider a wireless multipath channel shown in Fig 3. The equalized results are shown in Fig. 4 and the complexity comparison is shown in Table 1.

四、成果自評

In this project, we have proposed a novel Kalman algorithm. The distinct feature of this algorithm is its efficiency. We have accomplished 90% of the work listed in the original proposal. The result can be published in the international journal.

五、參考文獻

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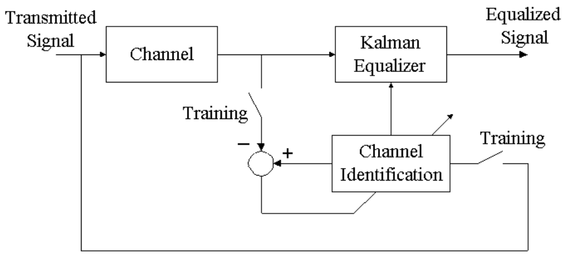


Fig.1 The conventional equalizer

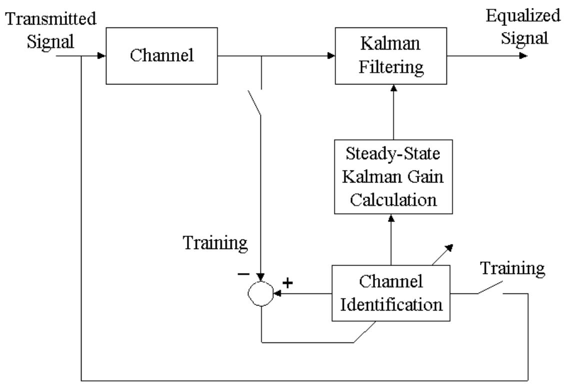


Fig.2 The proposed Kalman equalizer

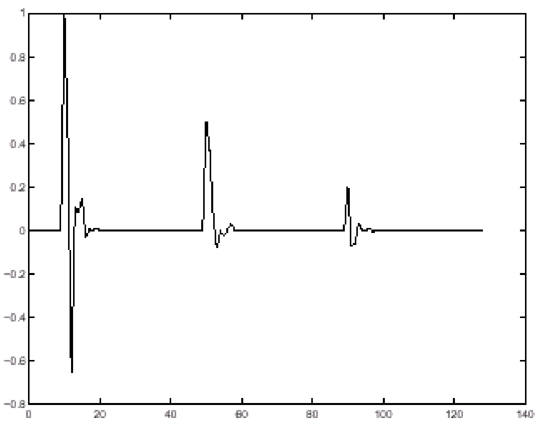
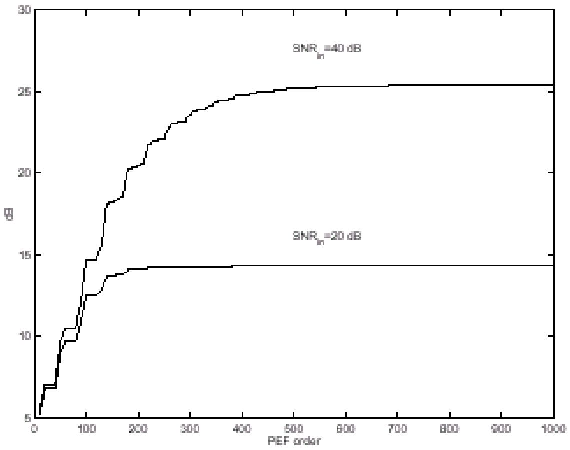


Figure 3: The synthesized wireless TV channel

Fig. 4 The output SNR of the proposed algorithm



PEF : prediction error filter
Efficiency: SNR_proposed/SNR_conventional
MULC: multiplications required for the conventional

PEF Order	Efficiency	MULC	MULP	MULP/MULC
100	57.86%	1,689,800	18,612	1.10%
150	72.30%	2,534,700	31,162	1.23%
200	81.14%	3,379,600	48,712	1.44%
250	90.07%	4,224,500	71,262	1.69%
300	93.95%	5,069,400	98,812	1.95%
350	96.19%	5,914,300	131,362	2.22%
400	97.65%	6,759,200	168,912	2.50%
500	99.30%	8,449,000	259,012	3.07%

MULP: multiplications required for the proposed