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Short communication

Thermophoretic particle deposition efficiency in turbulent tube flow

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Abstract

This study investigated the thermophoretic particle deposition efficiency numerically. The critical trajectory was used to calculate thermophoretic particle deposition in turbulent tube flow. The numerical results obtained in turbulent flow regime in this study were validated by particle deposition efficiency measurements with monodisperse particles (particle diameter ranges from 0.038 to 0.498 μm) in a tube (1.18 m long, 0.43 cm i.d., stainless-steel tube). The theoretical predictions are found to fit the experimental data of Tsai *et al.* [Tsai, C. J., J. S. Lin, S. G. Aggarwal, and D. R. Chen, "Thermophoretic Deposition of Particles in Laminar and Turbulent Tube Flows," *Aerosol Sci. Technol.*, **38**, 131 (2004)] very well in turbulent flows. In addition, an empirical expression has been developed to predict the thermophoretic deposition efficiency in turbulent tube flow.

Keywords: Thermophoretic deposition; Aerosol sampling; Turbulent flow

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1. Introduction

Thermophoresis is a physical phenomenon that aerosol particles move toward the direction of decreasing temperature when subjected to a thermal gradient. Knowledge of thermophoresis is of great interest as it has various industrial applications. Extensive experimental and theoretical works have been published on thermophoretic coefficient (Derjaguin *et al.*, 1976; Talbot *et al.*, 1980), thermophoretic particle deposition efficiency in laminar duct and channel flow (Tsai and Lu, 1995; Tsai *et al.*, 2004; Walker *et al.*, 1979) and thermophoretic particle deposition efficiency in turbulent duct and channel flow (He and Ahmadi, 1998; Nishio *et al.*, 1974; Romay *et al.*, 1998). In industrial applications, thermophoretic force has been used to enhance particle deposition efficiency on impactor substrate (Lee and Kim, 2002); to suppress particles deposition on wafer surface or pipe wall (Lin *et al.*, 2004; Stratmann *et al.*, 1988); to design a particle control device for diesel engine exhaust (Messerer *et al.*, 2003).

The objective of this study is to develop a critical trajectory method to evaluate thermophoretic particle deposition in turbulent tube flow. Comparison was made with the experimental thermophoretic deposition efficiency. The results show that theoretical prediction is reasonably well. A non-dimensional model was developed empirically to predict thermophoretic particle deposition in turbulent tube flow.

The derivation follows that of Lin and Tsai (2003), where the critical particle trajectory method is used. A steady, turbulent fluid flow in a circular tube is considered. The thermophoretic velocity $V_{th}(r, z)$ in the radial direction is a function of r and z, and the particle equations of motion can be written as

$$\frac{dr}{dt} = V_{th}(r, z), \tag{1}$$

and

$$\frac{dz}{dt} = u(r) = 2u_m\left(1 - \frac{r}{r_0}\right)^{1/n}\frac{(n+1)(2n+1)}{2n^2}. \tag{2}$$

The critical particle trajectory can be calculated by

$$\int_{r_c}^{r_0}\frac{dr}{V_{th}(r, z)} = \int_0^L \frac{dz}{u(r)}. \tag{3}$$

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Nomenclature

c_p	specific heat capacity at constant pressure (kJ/kg K)
C	slip correction factor
C_m	momentum exchange coefficient
C_s	thermal slip coefficient
C_t	temperature jump coefficient
d_p	diameter of the particle (m)
D_t	tube diameter (m)
f	fanning friction factor
h	convective heat transfer coefficient (W/m K)
k_g	gas thermal conductivity (W/m K)
k_p	particle thermal conductivity (W/m K)
K_{th}	thermophoretic coefficient
L	tube length (m)
Nu_D	Nusselt number
Pe_m	modified Peclet number $(u_m r_0^2)/(\alpha L)$
Pr	gas Prandtl number
Q	inlet gas flow rate (m^3/s)
r	radial coordinate
r_0	tube radius (m)
r_c	critical radial position (m)
R	dimensionless radial coordinate r/r_0
R_c	dimensionless critical radial position r_c/r_0
Re	Reynolds number
$\bar{T}$	average temperature of the fluid (K)
T_e	gas temperature at tube inlet (K)
T_m	mixing-cup temperature (K)
T_w	wall temperature (K)
u_m	average gas velocity (m/s)
$\vec{V}_{th}$	thermophoretic velocity (m/s)
z	axial coordinate
Greek symbols	
α	thermal diffusivity $k_g/(\rho_g c_p)$ (m^2/s)
β_t	thermophoretic parameter $\beta_t = PrK_{th}Nu_D(T_e - T_w)/(T_w Pe_m)$
η_{tur}	thermophoretic deposition efficiency in turbulent tube flow
λ	mean free path of air (m)
υ	air kinematic viscosity (N s/m^2)
ρ_g	gas density (kg/m^3)

It is noted that the thermal entry length in turbulent flow is approximately independent of the flow Reynolds number and can be shown to be (Kays and Crawford, 1993)

$$10 \le \left(\frac{z_{dep}}{D_t}\right)_{tur} \le 60. \tag{4}$$

The thermal entry length in turbulent flow is much shorter than the laminar flow case. As a result, we assume that the temperature is fully developed in any axial position of the tube.

The fully developed velocity profile follows the power law as (Bhatti and Shah, 1987)

$$u(r) = 2u_m\left(1 - \frac{r}{r_0}\right)^{1/n}\frac{(n+1)(2n+1)}{2n^2}. \tag{5}$$

where n varies slightly with the Reynolds number. According to Nikuradse's experiment data (Bhatti and Shah, 1987), the relation between n and Reynolds number can be fitted as

$$n = -3\times 10^{-10}Re^2 + 4\times 10^{-5}Re + 5.8503, \quad \text{for } 4000 \le Re \le 110,000. \tag{6}$$

For calculating the thermophoretic velocity $V_{th}(r, z)$, the radial temperature gradient $\mathrm{d}T/\mathrm{d}r$ must be found at first. The energy equation is re-written as the following form,

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T}{\mathrm{d}r}\right) = \frac{2u_m}{\alpha}\left[\frac{\mathrm{d}T_m}{\mathrm{d}z}\right]\left[\left(1-\frac{r}{r_0}\right)^{1/n}\frac{(n+1)(2n+1)}{2n^2}\right]\frac{T_w - T}{T_w - T_m}. \tag{7}$$

Thermophoretic velocity, $V_{th}(r, z)$, can be obtained after integrating Eq. (7) with respect to r once. In the turbulent flow, the Nusselt number is much higher than that in the laminar flow. Gnielinski (1976) suggested that the Nusselt number can be expressed as

$$Nu_D = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}. \tag{8}$$

where

$$f = (0.790\ln Re - 1.64)^{-2}, \qquad \text{for } 3000 \le Re \le 5\times 10^6. \tag{9}$$

The fully developed turbulent temperature profile in the turbulent tube flow is

$$\frac{T_w - T}{T_w - T_m} = \left(1 - \frac{r}{r_0}\right)^{1/n}\frac{2(n+2)}{2n+1} \tag{10}$$

From the above equations, Eq. (3) is written as the following dimensionless analytical equation and can be solved to obtain the dimensionless critical radial position, R_c,

$$\int_{R_c}^{1} f(R)\mathrm{d}R = -PrK_{th}\,\ln\left(\frac{T_w}{T_e} + \left(\frac{T_e - T_w}{T_e}\right)\exp\left(-\frac{Nu_D}{Pe_m}\right)\right), \tag{11}$$

where

$$f(R) = \frac{(1-R)^{1/n}(2n+1)}{-2n(1-R)^{(2+n)/n} - \frac{n^2}{(n+1)R}(1-R)^{(2+2n)/n} + \frac{n^2}{(n+1)R}},$$

and $R_c = r_c/r_0$ is the dimensionless critical radial position. The thermophoretic particle deposition efficiency in the turbulent tube flow can then be calculated in the following equation assuming the particle concentration is uniform at the inlet:

$$\eta_{\mathrm{tur}} = R_{\mathrm{c}}(1-R_{\mathrm{c}})^{(1+n)/n}\frac{(2n+1)}{n} + (1-R_{\mathrm{c}})^{(1+2n)/n} \tag{12}$$

2. Results and discussion

2.1. An empirical equation to predict the thermophoretic particle deposition efficiency

It can be seen from Eq. (11), the thermophoretic particle deposition efficiency in the turbulent tube flow is a function of four parameters: the product of the Prandtl number and thermophoretic coefficient, PrK_{th}, the dimensionless temperature $(T_{\mathrm{e}} - T_{\mathrm{w}})/T_{\mathrm{e}}$, the Nusselt number Nu_{D} and the modified Peclet number Pe_{m}. The thermophoretic deposition efficiency depends on the thermophoretic parameter β_{t}, which can be written as

$$\beta_{\mathrm{t}} = \frac{Pr\,K_{\mathrm{th}}Nu_{\mathrm{D}}}{Pe_{\mathrm{m}}}\frac{T_{\mathrm{e}}-T_{\mathrm{w}}}{T_{\mathrm{w}}} \tag{13}$$

The best-fit equation for the thermophoretic particle deposition efficiency in the turbulent tube flow is found to be

$$\eta_{\mathrm{tur}}(\%) = 100 \times (1-\exp(-0.2\beta_{\mathrm{t}})), \qquad \text{for } 0.017 < \beta_{\mathrm{t}} < 34 \tag{14}$$

The above expression is useful for predicting total thermophoretic particle deposition efficiency in a turbulent tube flow. For example, for particles of 0.05 μm in diameter suspended in the tube flow with the flow rate of 45 slpm, inlet gas temperature of 450 K and tube wall temperature of 296 K, the calculated β_{t} value is 0.96 for the present tube geometry and length (i.d. = 0.0043 m, L = 1.18 m). The value for thermophoretic parameter β_{t} is 0.96, which corresponds to particle deposition efficiency of 17.5%.

The present results shows that the predicted results of this equation fit the numerical solutions of Eqs. (11) and (12) very well as illustrated in Fig. 1. Theoretical expressions of the thermophoretic deposition efficiency in the turbulent tube flow of previous studies of Romay *et al.* (1998), Nishio *et al.* (1974) and Housiadas and Drossinos (2005) are given in Table 1. The comparison of thermophoretic deposition efficiencies for these expressions at a flow Reynolds number of 10,200 and 0.5 μm NaCl condition for the tube geometry used in this study is shown in Fig. 2, which illustrates that the predicted efficiencies for all the theories are close to each other.

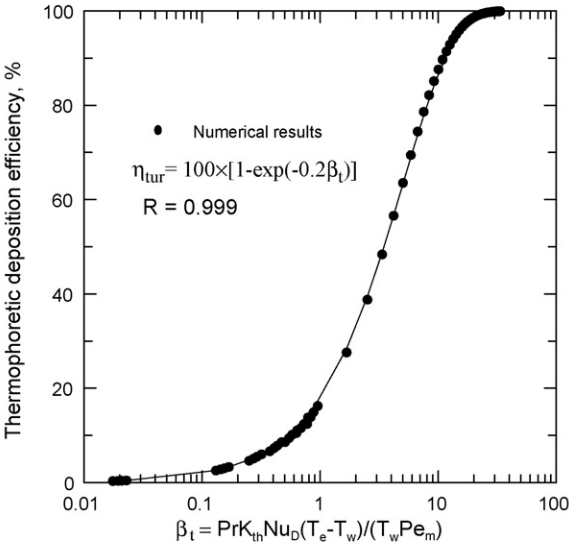

Fig. 1. Thermophoretic deposition efficiency as a function of thermophoretic parameter β_{t} in the turbulent tube flow.

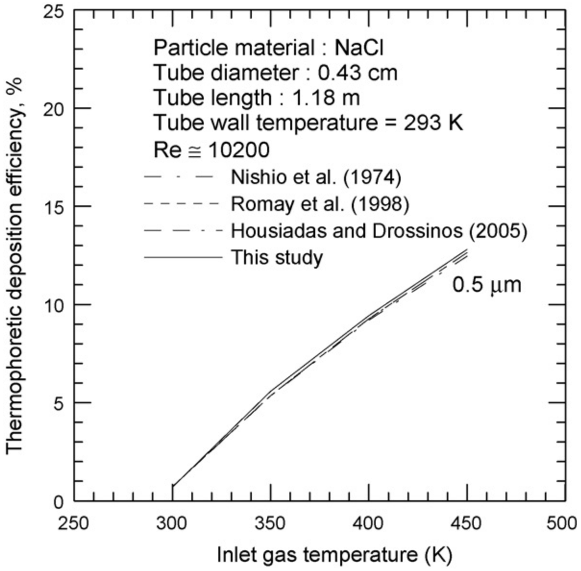

Fig. 2. Comparison of theoretical predictions (Re = 10,200) for particles of 0.5 μm in diameter.

The Lagrangain particle tracking methodology was used in this study. The required turbulent velocity and temperature fields follow power law instead of calculating with CFD code (Housiadas and Drossinos, 2005). It was found that the thermophoretic deposition efficiency can be predicted accurately

Table 1
Theoretical expressions of the thermophoretic particle deposition efficiency

Romay et al. (1998)	$\eta_{\mathrm{tur}}(\%) = 100 \times \left\{1 - \left[\frac{T_{\mathrm{w}}+(T_{\mathrm{e}}-T_{\mathrm{w}})\exp(-\pi D_{\mathrm{t}}hL/\rho_{\mathrm{g}}QC_{\mathrm{p}})}{T_{\mathrm{e}}}\right]^{PrK_{\mathrm{th}}}\right\}$
Nishio et al. (1974)	$\eta_{\mathrm{tur}}(\%) = 100 \times \left\{1 - \exp\left(-\frac{\rho_{\mathrm{g}}c_{\mathrm{P}}K_{\mathrm{th}}\nu(T_{\mathrm{e}}-T_{\mathrm{w}})}{k_{\mathrm{g}}\bar{T}}\left(1-\exp\left(\frac{-4hL}{u_{\mathrm{m}}\rho_{\mathrm{g}}c_{\mathrm{p}}D_{\mathrm{t}}}\right)\right)\right)\right\}$
Housiadas and Drossinos (2005)	$\eta_{\mathrm{tur}}(\%) = 100 \times \left[1 - \left(\frac{T_{\mathrm{w}}}{T_{\mathrm{e}}}\right)^{PrK_{\mathrm{th}}}\right]$
This study	$\eta_{\mathrm{tur}}(\%) = 100 \times \left\{1 - \exp\left(-0.2\frac{PrK_{\mathrm{th}}Nu_{\mathrm{D}}}{Pe_{\mathrm{m}}}\frac{T_{\mathrm{e}}-T_{\mathrm{w}}}{T_{\mathrm{w}}}\right)\right\}$

in turbulent tube flow as compared to theoretical expressions available in the literature.

2.2. Comparison of theoretical prediction with experimental data

In our previous study (Lin and Tsai, 2003), the thermophoretic particle deposition efficiency for fully developed laminar tube flow is derived as a function of the product of the Prandtl number and thermophoretic coefficient, i.e. PrK_{th}, and the dimensionless temperature $(T_{\text{e}} - T_{\text{w}})/T_{\text{e}}$ as:

$$\eta_{\text{lam}}(\%) = 78.3\left(PrK_{\text{th}}\frac{T_{\text{e}} - T_{\text{w}}}{T_{\text{w}}}\right)^{0.94} \tag{15}$$

where K_{th} is defined as (Talbot *et al.*, 1980):

$$K_{\text{th}} = \frac{2C_{\text{s}}C}{(1 + 3C_{\text{m}}(2\lambda/d_{\text{p}}))} \times \left(\frac{k_{\text{g}}/k_{\text{p}} + C_{\text{t}}(2\lambda/d_{\text{p}})}{1 + 2(k_{\text{g}}/k_{\text{p}}) + 2C_{\text{t}}(2\lambda/d_{\text{p}})}\right) \tag{16}$$

This formula has been widely used as an interpolation formula for the thermophoretic coefficient in the transition regime $(0.1 < K_n < 10)$ between well-known solutions of K_{th} in the near continuum regime $(K_n < 0.1)$ (Brock, 1962) and in the free-molecule regime $(K_n > 10)$ (Waldmann, 1961).

In this study, an empirical expression to predict the thermophoretic particle deposition efficiency in turbulent tube flow was developed. The experimental thermophoretic deposition efficiencies are in a good agreement with theoretical prediction based on the thermophoretic coefficient of Talbot *et al.* (1980) as the flow Reynolds number equals to 10,200 in turbulent flow regime as illustrated in Fig. 3. It is seen that the thermophoretic deposition efficiency increasing with an increasing inlet gas temperature.

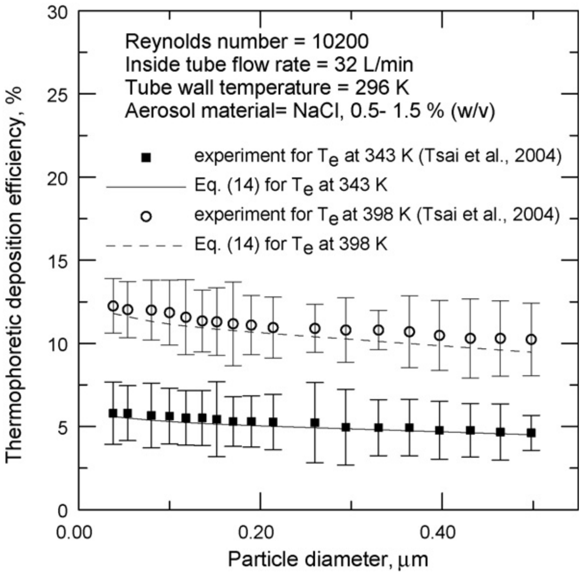

Fig. 3. Comparison of experimental data and theoretical predictions of thermophoretic deposition efficiency derived in this study under turbulent flow condition.

There are a number of current applications that can be modeled as a turbulent aerosol pipe flow, such as particle deposition in automobile exhaust and heat exchanger pipe flow. Particle deposition on pipe wall by thermophoresis can cause undesirable effects, such as reduction of thermal conductivity of heat exchanger pipes. On the other hand, the concept of thermophoresis provides a working principle to fabricate optical fiber in a modified chemical vapor deposition (MCVD) process.

3. Conclusions

The thermophoretic particle deposition in turbulent tube flow was investigated numerically by Lagrangain particle tracking methodology in this study. The predicted thermophoretic deposition efficiencies agree very well with experimental data of Tsai *et al.* (2004). The results of this study show that the inlet gas temperature heavily influences the thermophoretic particle deposition efficiency. Furthermore, an empirical expression to predict the thermophoretic particle deposition efficiency in turbulent tube flow was developed.

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