行政院國家科學委員會補助專題研究計畫成果報告

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※ **本質維度應用於訊號與影像分析** ※
※ Intrinsic dimension computation applied to signal ※
※ and image analysis ※
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計畫主持人：羅佩禎
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行政院國家科學委員會專題研究計畫成果報告
本質維度應用於訊號與影像分析
Intrinsic dimension computation applied to signal and image analysis

計畫編號：NSC 89-2213-E-009-186
執行期限：89 年 8 月 1 日至 90 年 7 月 31 日
主持人：羅佩禎　國立交通大學　電機與控制工程學系
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一、中文摘要

本研究報告提出一高效能演算法則，用於估測腦電波訊號之本質維度，以量化其整體複雜度。除了針對多通道腦電波之分析外，本質維度亦可用於影像材質紋理的分析與描述。我們已研發出幾種不同方法，以適用於多通道腦電波和影像資料的複雜度分析。研究結果顯示，本方法在大幅減少運算複雜度情況下，亦得以獲得可靠之維度估測。

關鍵詞：腦電波、非線性動態、維度分析、本質維度、複雜度指標、紋理影像。

Abstract

In this research report, we present an efficient method and algorithm for estimating the intrinsic dimension for quantifying the global waveform complexity in EEG (electroencephalograph). In addition to the multi-channel EEG analysis, the intrinsic dimension may be feasible for texture description in image processing. We have developed different methodologies for analyzing both the multi-channel EEG and the image data. The results of this research work demonstrate that the computational complexity is highly reduced without sacrificing the reliability of dimensional estimation.

Keywords: Electroencephalograph (EEG), nonlinear dynamics, dimensional analysis, intrinsic dimension, complexity index, texture image.

二、緣由與目的

The multi-channel EEG signal is characterized by its spatial and temporal features. In the past two decades, the advance in nonlinear dynamics enabled researchers to interpret the brain waves from an alternative viewpoint. Dimensional analysis provides a way to quantify the global waveform complexity of the multi-channel EEG data. A number of methods have been proposed to estimate the EEG dimension. They include the correlation dimension, fractal dimension, information dimension, etc.

Nonetheless, tools from nonlinear dynamical theory used for EEG analysis, such as the dimensional computation and Lyapunov exponent estimation, mostly suffer from the problems of computational inefficiency and bias from implementing parameters (Lo and Principe, 1989). Thus they are not feasible for long-term monitoring or practical use. Since evaluating the correlation dimension involves slope computation using linear regression, an appropriate linear scaling region needs to be determined first. The estimation procedure is an indirect approach. Thus it is not feasible for the long-term, multi-channel EEG analysis.

The topological or intrinsic dimensionality of a point set was introduced to characterize the classifiers in the field of pattern recognition (Fukunaga and Olsen, 1971; Pettis et al., 1979; Verveer and Duin, 1995; Bruske and Sommer, 1998). The fundamental idea of its application to EEG analysis is that the intrinsic dimensionality in some sense reflects variation of data structure in a data set. Accordingly, we adapt the technique for the multi-channel EEG analysis to explore the spatial-temporal characteristics in EEG. To improve the computational efficiency, methods for determining the intrinsic dimension were mostly based on the local approaches, that is, quantifying the local features in small regions.

When applying to the long-term biomedical signals, computational efficiency and processing effectiveness are of important consideration. Thus the authors develop a direct-computation algorithm, based on the *K*NN approach, to analyze the multi-channel EEG. The algorithm avoids the problem of sophisticated computation strategy usually involved in indirect approach like the correlation dimension estimation. The mathematical basis is similar to that given in (Fukunaga and Flick, 1984). The derivation and implementation procedures are somewhat different. The computed intrinsic dimension reflects the global complexity of the spatio-temporal feature. It is accordingly named as the "complexity index($\mathcal{C}$)".

三、結果與討論

As previously demonstrated (Lo and Principe, 1989), the EEG trajectory forms a strange attractor (Hentschel and Procaccia, 1983). Let $\mathbf{X}=\{\boldsymbol{X}_i\}_{i=1}^{N}$ be the points on the EEG trajectory, where X_i is an n-dimensional point constructed from the n-channel EEG signals. For instance, X_i = (F3(i), Fz(i), F4(i)) represents a point on the 3-dimensional space, whose coordinates (degrees of freedom) are brain electrical potentials recorded from sites F3, Fz, and F4, respectively. Derivation of the equation for computing the complexity index was illustrated in details in (Lo and Chung, 2000). In summary, for each point in the set **X** (e.g., X_i), we first compute the K- and (K+1)-nearest neighbor (KNN and (K+1)NN) distances, $d_{K\mathrm{NN}}$ and $d_{(K+1)\mathrm{NN}}$. It follows that

$$₵ = \frac{1}{K}\left(\frac{E\{d_{(K+1)\mathrm{NN}}\}}{E\{d_{K\mathrm{NN}}\}} - 1\right)^{-1} \qquad (1)$$

which provides the way to evaluate the complexity index.

One advantage of the method is its easy implementation of the algorithm by following the above equation. The algorithm first determines a 2-dimensional $N\times N$ array **D** with its elements d_{ij}, i, j=1, …, N representing the inter-point distances. Then elements in each row are sorted according to their magnitudes. The resulting $N\times N$ array **D'**=[d'_{ij}] i, j=1, …, N has its elements in each row arranged from the smallest (j=1) to the largest (j=N) number. Thus the Kth element of the ith row of **D'** is the $d_{i,K\mathrm{NN}}$. (i.e., $d_{i,K\mathrm{NN}}$= d'_{iK}) The algorithm actually compute the $E\{d_{K\mathrm{NN}}\}$ by averaging all the elements in the Kth column of **D'**. Likewise, the $E\{d_{(K+1)\mathrm{NN}}\}$ can be computed by averaging all the elements in the (K+1)th column of **D'**. The complexity index ₵ is then computed by using eq.(1). Details of efficient implementation of the algorithm were reported in (Lo and Chung, 2000).

The estimated ₵ depends on value of K and length of the evolving trajectory. Note that the ₵ versus K curves fluctuate. Thus the authors average the ₵'s over a moderate range of values of K to obtain the final estimate. Table 1 lists the estimated average ₵ for the model-generated trajectories. The average ₵ is denoted by $\overline{₵}$. As listed in Table 1, the values of K used to obtain $\overline{₵}$ range from 30 to 60. Table 1 also lists the correlation dimension estimated for each model system (Grassberger and Procaccia, 1983). The average complexity index $\overline{₵}$ well approximates the correlation dimension. Evaluating the correlation dimension involves slope computation by linear regression. The result highly depends on the scaling range selected for slope computation. This is the major advantage of the proposed algorithm over the correlation dimension method.

Since the computational time required for ₵ evaluation highly depends on the evolving length N (number of n-dimensional data points), it is a practical consideration to choose the minimum value of N that ensures an adequate coverage of the attractor dynamics. In our study, pronounced saturation of the curves is observed for $N\geq 2500$. Using an N smaller than 1000 results in a severe underestimation of the complexity index. We also observed that the range of values of K showed little effect on $\overline{₵}$ when N was properly selected ($N\geq 2500$). Deviation is within 3% ($\overline{₵}$ =2.01-2.06, compared with 2.06 listed in Table 1). This fact demonstrates that $\overline{₵}$ computation is rather insensitive to a wide range of K. It then allows us to confidently select the range of values of K when evaluating the $E\{d_{K\mathrm{NN}}\}$ in eq.(1).

In this research work, we also established a guideline of selecting implementing parameters for multi-channel EEG analysis. A number of EEG segments under different CNS (central nervous system) dynamics have been investigated. Here we present the results of analyzing EEG data of an epilepsy syndrome called "Benign Partial Epilepsy of Childhood". A 25-channel electrode array with a common linked-ear reference was applied in the recording: {F3, Cz, F4, P3, P4, F7, F8, T5, T6, Fp1, Fp2, O1, O2, Fz, Pz, C3, C4, T3, T4} (north hemisphere) and {IO1, IO2, CB1, CB2, MS1, MS2} (south hemisphere). The signals were digitized by a rate of 200 Hz. We define the state space dimension n as the number of channels. The channel combination indicates the composition of recording sites used in the analysis. It is associated with the set of state variables describing the brain dynamics.

A 5-channel protocol, involving electrode sites Cz, F3, F4, P3, and P4, is used. In our study of the implementing parameter, we found that the estimated ₵ (EEG1: 4.64±0.09, EEG2: 4.57±0.08, EEG3: 4.73±0.09) was fairly consistent in the range of $K\geq 25$. For a small K, poor statistics (due to the small number of n-dimensional points involved) lead to an unreliable estimate. Yet, in consideration of computational efficiency, a small K is preferred.

When viewing the ₵ measure as a relative indicator of the global complexity of EEG waveform patterns, development of the implementing algorithm need not aim at derivation of absolute values. Thus the evolving length N can be much smaller than that required to obtain a convergent estimate, only that it provides an invariant feature descriptor when applied to the long-term EEG monitoring. Figure 1 illustrates the situation. The three curves represent the running measures of $\overline{₵}$, over a 150-second EEG record, using N = 1000 (solid), 1500 (dashed), and 2000 (dotted). In all three cases the moving size is 100 points (0.5 seconds) and the K ranges from 25 to 35. The horizontal axis indicates the beginning time of each running window. The diamonds symbolize the occurrence of focal-sharp-wave events. The three curves basically follow the same up-and-down course. However, we may clearly recognize that increasing the signal duration N results in (time) advance and (magnitude) ascent of the running $\overline{₵}$ curve. The phenomena can be explained as follows. The background EEG has a complexity index between 4.0

and 4.5, which is reduced to 3.0~3.5 when focal-sharp-wave transient occurs. This analysis demonstrates one of the applications of $\overline{\mathcal{C}}$ computation to EEG signal. That is, the running $\overline{\mathcal{C}}$ curve may be used to detect the occurrence of particular events in the EEG monitoring.

Investigation of the effect of state-space dimension (n) is a complicated task since it involves not only the values of n but the composition of electrodes. It is known that, for a deterministic signal, the estimated fractional dimension does not increase with the space dimension (Lo and Principe, 1989). From our study in the past year (Figure 2), we concluded that the set of 13 electrode sites, sparsely distributed over the north-hemispherical (scalp) region, involve sufficient information for quantifying the EEG spatial complexity at this physiological state (Benign Partial Epilepsy syndrome).

In this preliminary study, we have introduced the method of estimating intrinsic dimensionality into multi-channel EEG analysis. The work is not aimed at obtaining the absolute value of intrinsic dimensionality, considering the practical use in EEG analysis. Since dimensionality in a sense characterizes the global waveform complexity, we name it the "complexity index ($\mathcal{C}$)". Evaluation of complexity index is conceptually comprehensible and easily implemented. We have demonstrated the effect of implementing parameters on $\mathcal{C}$ computation for both model systems and multi-channel EEG. Though selection of implementing parameters affects the computed value of $\overline{\mathcal{C}}$, the running feature of $\overline{\mathcal{C}}$ appears to be fairly insensitive to a wide range of values of K and N. Considering the effect of space dimensions n (number of electrodes), the running $\overline{\mathcal{C}}$ curve fluctuates in a coherent manner for $n \geq 5$. Analyzing the 5-dimensional (5-channel) EEG trajectory simply requires as few as 1000 data points (5 seconds) for obtaining invariant running $\overline{\mathcal{C}}$ characteristic. It then resolves the problem of poor temporal resolution and computational inefficiency often encountered in dimensional analysis. According to the analysis of four different electrode arrays, the background EEG has an estimated $\overline{\mathcal{C}}$ in the range of $4.0 \leq \overline{\mathcal{C}} \leq 4.5$, which drops to $3.0 \leq \overline{\mathcal{C}} \leq 3.5$ at the occurrence of the events (focal-sharp-wave activities). The preliminary results presented in this paper demonstrate that this method has the potential value to quantification of EEG spatial correlation and identification of EEG pattern transition.

In image analysis, complexity index can be applied to the texture images for characterizing the structural complexity. Consider the texture images shown in Figure 3. The estimated complexity indexes are 5.99 (D28), 6.64 (D29), 5.38 (D65), and 5.73 (D55), respectively. Apparently, better structural regularity results in lower value of complexity index.

Table 1 Average complexity index estimated for five model systems

	Complexity Index $\overline{\mathcal{C}}$	$N / \Delta t$	range of K	Correlation Dimension	$N / \Delta t$
Rabinovich-Fabrikant model	2.22	8,000 / 0.05	30 – 60	2.19	15,000 / 0.25
Lorenz model	2.06	8,000 / 0.05	30 – 60	2.05	15,000 / 0.25
Rossler model	1.86	8,000 / 0.05	30 – 60	***	
Zaslavskii map	1.74	15,000	30 – 60	~1.50	25,000
Hénon map	1.25	15,000	30 – 60	1.25	15,000

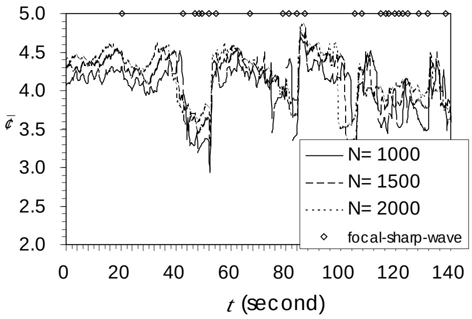

Figure 1 The running $\overline{\mathcal{C}}$ for a 150-sec EEG record, using 3 window sizes (N=1000, 1500, 2000).

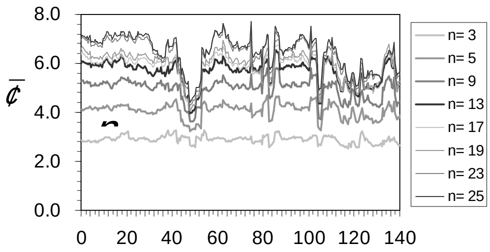

Figure 2 The running $\overline{\mathcal{C}}$ curves for different numbers of electrodes (dimensions, n).

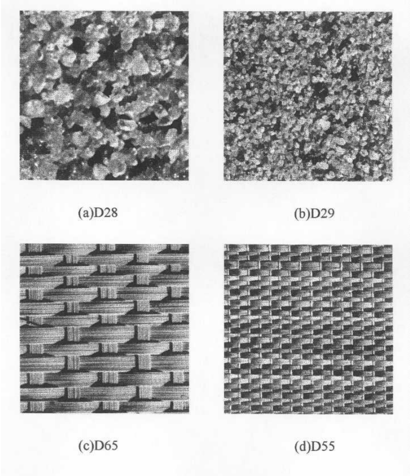

Figure 3 Texture images.

四、計畫成果自評

Results of this research study have been published in two papers. The paper published in *Biometrical Journal* (Lo and Chung, 2000) reported the strategies and considerations of determining the implementing parameters. The second one published in *IEEE Transactions on Biomedical Engineering* (Lo and Chung, 2001) presented an efficient algorithm for computing the complexity index. This research project has been conducted well in agreement with the proposal. In addition, the efficient algorithm will be applied to our future research in the CNS dynamics under meditation.

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