

行政院國家科學委員會專題研究計畫成果報告

三階碼在同步展頻通訊系統的應用

Investigation of Trinary Coding in Synchronous Spread Spectrum Systems

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一、中文摘要

本計畫提出三階碼應用在同步展頻通訊系統以得到豐富的碼集合及更富彈性的頻譜分配。我們更提出一個改良的三階 Walsh code，其效能比通用的二階 Walsh code 更好。

關鍵詞：展頻、分碼多工、

Abstract

Trinary sequence is proposed, which provides an alternative approach to build orthogonal spreading code sets for synchronous spread spectrum systems. In synchronous channels, perfect orthogonality is kept in the proposed code sets with more flexibility in code construction and spectrum planning compared with binary coding. A modified Walsh code based on trinary codes is proposed, which has a superior performance than that of the popular binary Walsh code.

Keywords: Spread Spectrum, CDMA,

二、緣由與目的

In conventional direct sequence spread spectrum (DS-SS) systems the orthogonal

spreading codes, e.g. Walsh codes, are binary with amplitudes belong to $\{\alpha, -\alpha\}$. Here we propose trinary orthogonal spreading codes with chip numbers taking arbitrary integer values $K (K \geq 2)$ and amplitudes belong to $\{\alpha, 0, -\alpha\}$. This allows flexible choices on spreading code sets and processing gains in designing DS-SS systems. For example, when the total bandwidth available for spread spectrum communications are distributed to several system operators, the spectrum allocation and utilization could sometimes be restricted due to the specific processing gains of binary coding, i.e. $2^l (l=1,2,3\dots)$. The trinary coding can alleviate this restriction and provides more flexibility for spectrum planning and code construction with the help of zero-amplitude chips. To the authors' best knowledge, there was no report on trinary coding for synchronous DS-SS systems so far.

三、結果與討論

3.1 Performance Analysis

Consider a synchronous DS-SS system with k active users. The input data vector is $\mathbf{x}=(x_1, x_2, \dots, x_k)$ where $x_i \in \{1, -1\}$, and each user is assigned with a spreading waveform $c_i(t)$ ($i=1,2,3,\dots,k$), which is restricted within one symbol duration T and composed of a

$c_i(t)$ ($i=1,2,3,\dots,k$), which is restricted within one symbol duration T and composed of a spreading sequences of L chips, i.e.

$$c_i(t) = \sum_{m=0}^{L-1} a_m^{(i)} p(t - mT_c) \quad (1)$$

where $\{a_0^{(i)}, a_1^{(i)}, \dots, a_{L-1}^{(i)}\}$ is a spreading sequence with $a_m^{(i)} \in \{1, 0, -1\}$ and $p(t)$ is the unit rectangular pulse of T_c duration and $T_c = T/L$.

The i 'th sampled output signal during one symbol interval T can be formulated as [2]

$$y_i(t) = x_i \sqrt{E_c} \int_0^T c_i(t)^2 dt + \sum_{j \neq i}^k x_j \sqrt{E_c} \int_0^T c_i(t) c_j(t) dt + \int_0^T n(t) c_i(t) dt \quad (2)$$

where E_{c_q} is the non-zero energy per chip of the q 'th user and $n(t)$ is additive white Gaussian noise. In the synchronous system, the second term (mutual interference) is zero and the variance of the last term is $\frac{N_0}{2} \int_0^T c_i(t)^2 dt$, where $\frac{N_0}{2}$ is the two-sided power spectral density of white Gaussian noise.

We define the "zero-chip ratio" of a sequence as $\beta = Z/G$, where Z is the total number of zero-amplitude chips and G is the length (e.g. for a sequence $=\{1,0,-1,1,-1\}$, $Z=1$, $G=6$ and $\beta=1/6$). Let β_i be the zero-chip ratio of the i 'th spreading sequence $\{a_m^{(i)}\}$, then

$$\int_0^T c_i(t)^2 dt =$$

$$\int_0^T \left(\sum_{m=0}^{L-1} a_m^{(i)} p(t - mT_c) \right)^2 dt = (1 - \beta_i)T \quad (3)$$

and the bit error rate (BER) of the i 'th user is

$$P_e^{(i)} = Q\left(\sqrt{\frac{E(y_i)^2}{\text{Var}(y_i)}}\right) = Q\left(\sqrt{\frac{2E_c T (1 - \beta)}{N_0}}\right)$$

$$\text{where } Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy. \quad (4)$$

Assume the non-zero energy per chip E_c and the symbol duration T be known exactly,

1. For a fixed E_c , the SNR is proportional to $(1 - \beta)$ and the BER reaches the lower bound when $\beta=0$, i.e. the spreading sequence is binary.
2. For equal transmit powers, the BER performance using trinary spreading codes is the same as that by using binary spreading codes since

$$E_c^T (1 - \beta) = E_c^B \quad (15)$$

where E_c^T and E_c^B represent the chip energies of the trinary (for non-zero chips) and binary spreading codes, respectively.

3.2 Modified Walsh codes

Here we propose a class of modified Walsh codes based on trinary coding, which improves the BER performance of the popular binary Walsh codes. Each N^{th} order modified Walsh code includes 2^N code sequences and is composed of two subsets : the first subset is constructed from the

Kronecker product of the $N-1^{\text{th}}$ Hadamard matrix with the vector $[0,1,1,0]$, i.e, $\text{Kron}(H_{N-1}, [0,1,1,0])$, where H_{N-1} is the $N-1^{\text{th}}$ Hadamard matrix [3]. The second subset is built from the Kronecker product of H_{N-1} with the vector $[1,1,-1,-1]$, i.e, $\text{Kron}(H_{N-1}, [1,1,-1,-1])$. The orthogonality of the modified Walsh codes is guaranteed by the orthogonal property of Hadamard matrix and the orthogonality between the two vectors $[0,1,1,0]$ and $[1,1,-1,-1]$. We set each trinary code sequence the same energy as the binary ones by multiplying the amplitude of codes in the trinary subset by $\sqrt{2}$.

To compare the BER performance under multipath environment, we use the result derived in [4] with the binary spreading waveforms replaced by trinary waveforms, given by

$$P_e^{(i)} = Q\left(\left\{\frac{1}{(1-\beta_i)^2} \sum_{k \neq i}^K \frac{1}{6N_{k,i}^3} \left(\frac{A_k}{A_i}\right)^2 r_{k,i} + \frac{N_0}{2E_i}\right\}^{-\frac{1}{2}}\right) \quad (6)$$

where β_i is the ‘zero-chip-ratio’ for the i^{th} spreading code, A_q is the amplitude of the q^{th} spreading waveform, $r_{k,i}$ is the average interference parameter (AIP) between k^{th} and i^{th} codes, $N_{k,i}$ is the chip number of a code pair after they are partitioned to specific ‘tiny chips’ which is capable of finding the consistent AIP, and E_i is the total energy of the i^{th} spreading code during one symbol duration.

For simplicity we assume that the multipath interference signals have the same power as the desired signal, the average BER of the modified Walsh codes and the binary Walsh codes can be carried out using Eq.(16). As illustrated in Fig. 3, the former is superior to that of the latter and the improvement ratio increases with the energy-to-noise ratio, revealing that the modified Walsh codes could provide a better interference performance than the binary Walsh codes.

四、成果自評

We propose trinary sequence with chip numbers taking arbitrary integer values K ($K \geq 2$). Obviously this provides more flexibility for code design and spectrum planning compared with binary coding. In synchronous transmission channels with equal transmit powers, the same BER performance can be achieved for both trinary and binary spreading code based DS-SS systems. We further propose a class of modified Walsh codes whose BER performance is superior to that of the popular binary Walsh codes.

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