行政院國家科學委員會專題研究計畫成果報告

錯置立方體與其一般化

The shuffle cube and its generalization

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一、中文摘要

許多連結網路拓樸在許多文獻中被提出，它們主要的目的都是要連結千百計的處理單元，以共同運作處理特定的工作，這些網路拓樸往往以抽象的圖來表示，其中，點用來表示處理單元，而邊則用來表元處理單元問的網路連。網路拓樸對於連結網路而言可說是非常重要的因素，因為它決定了一個網路的效能，超立方體便是一個非常有名的網路拓樸，然而，就以下的觀點而言它並沒能完全善用它的硬體資源：給定 $N=2^n$ 個點及 $\frac{nN}{2}$ 條連線，建造一個直徑比超立方體直徑(n)更低的網路是可能的。因此，許多變形相繼提出，例如：雙扭立方體、交叉方超立方體、及梅式立方體，這些變形的提出主要都是在降低直徑，但仍保持超立方體的許多良好的特性，這些變形的直徑大約降到原本超立方體的一半左右，精確地說，n 維的超立方體的直徑為 n，而 n 維的雙扭立方體、交叉立方體、及梅式立方體的直徑則均為 $\left\lceil \frac{n+1}{2} \right\rceil$，不過，這樣的改進也犠牲了高度的對稱性。

在我們最近被 Information Processing Letters 期刊所接受的論文中，提出了一個新的超立方體變形，稱之為錯置立方體。n 維的錯置立方體以 SQ_n 表示之，SQ_n 是可以被遞迴地建構出來的，並且已被証明是 n 正則圖形且連通度達到 n，更進一步地，SQ_n 的直徑等於 $\left\lceil \frac{n}{4} \right\rceil + 3$。然而，還有更多其他的拓樸性質尚未研究。

在本計畫中，我們深入研究更多錯置立方體的特性，包含它的寬直徑、有錯漢米爾頓性質、及其一般化。

關鍵詞：連結網路、超立方體、直徑、容錯性、寬直徑、漢米爾頓環路、連通度

Abstract

Many interconnection network topologies have been proposed in the literature for the purpose of connecting hundreds or thousands of processing elements. Network topology is always represented by a graph, where nodes represent processors and edges represent links between processors. Network topology is a crucial factor for interconnection networks since it determines the performance of a network. The hypercube is one of the most popular topologies. However, Q_n does not make the best use of its hardware in the following sense: given N= 2^n nodes and $\frac{nN}{2}$ links, it is possible to fashion networks with lower diameters than the hupercubes's diameter n. Hence, many variations have

been proposed, such as the twisted cube, the crossed cube, and the Möbius cube. The major purpose of these variations is to lower the diameter but to keep many good properties of the hypercube. With these variations, the diameter is improved approximately a factor of 2. More precisely, the diameter of the *n*-dimensional hypercube is n but the diameters of the *n*-dimensional twisted cube, the *n*-dimensional crossed cube, and the *n*-dimensional Möbius cube are $\left\lceil \frac{n+1}{2} \right\rceil$. This is achieved by forfeiting some of the hypercube's high degree of symmetry and redundancy.

In our recent paper, accepted by Information Processing Letters, a variation of the hypercube, called the shuffle cube, is proposed. The *n*-dimensional shuffle cube is denoted by SQ_n. SQ_n, which can be constructed recursively, is proved to be an n-regular graph with connectivity n. Moreover, the diameter of SQ_n is $\left\lceil \frac{n}{4} \right\rceil + 3$. Yet, many other topological properties are not studied.

In this project, we are investigate more topological properties of the shuffle cube. In particularly, we found the wide diameter, fault hamiltonianicity, and the generalization of the shuffle cubes.

Keywords: interconnection networks, hypercubes, diameter, fault-tolerant, wide diameter, Hamiltonian cycle, connectivity

二、緣由與目的

Network topology is a crucial factor for interconnection networks since it determines the performance of a network. Many interconnection network topologies have been proposed for connecting processors in multi-processor systems. The hypercube is one of the most popular topologies [1,15,18,24,25,28,29]. However, the hypercube does not make the best use of its hardware. Hence, many variations have been proposed, such as the twisted cube [6,19,26], the crossed cube [7,13,14], and the Möbius cube [10,11]. The major purpose of these variations is to lower the diameter but to keep many good properties of the hypercube. With these variations, the diameter is improved approximately a factor of *2*.

Recently, Li et al.[27] proposed another variation of the hypercube, called the shuffle cube. The *n*-dimensional shuffle cube is denoted by SQ_n. SQ_n, which can be constructed recursively, is proved to be an *n*-regular graph with connectivity *n*. However, the diameter of SQ_n is $\left\lceil \frac{n}{4} \right\rceil + 3$. Yet, many other topological properties is not studied. In this project, we are going to investigate more topological properties of the shuffle cube. In particularly, we are interesting in the wide diameter, fault hamiltoniancity, and the generalization of the shuffle cubes.

The wide diameter, proposed by

Hsu [20], of an interconnection network topology is an indicator for the network performance. It arises from the study of routing, reliability, randomized routing, fault tolerance, and other communication protocols in parallel architecture and distributed computer networks. Assume that $G=(V,E)$ is a graph with connectivity κ. It follows from Menger's theorem that there are κ internally *vertex-disjoint* (abbreviated as *disjoint*) paths between any two vertices. Given any two vertices u,v of G, let $C(u,v)$ denote the set of all κ disjoint paths between u and v. Each element of $C(u,v)$ consists of κ disjoint paths. For any element $x \in C(u,v)$, let $l(x)$ denote the longest length among the κ disjoint paths of x. Then $d_\kappa(u,v)$ is defined as $\min_{x\in C(u,v)} l(x)$. And the *wide diameter* of G, denoted by $D_\kappa(G)$, is defined as $\max_{u,v\in V} d_\kappa(u,v)$.

The ring structure is important for distributed computing. It is useful to construct a hamiltonian cycle or ring structure in the network. The faulty network is practically meaningful because node faults and link faults may happen when a network is used. A graph G is k-hamiltonian if G-F remains hamiltonian for every $F \subset V(G) \cup E(G)$ with $|F| \le k$. Obviously, $k \le \deg(G)-2$. We call the graph optimal fault hamiltonian if $k = deg(G)-2$.

In this project, we want to show that the wide diameter of the n-dimensional shuffle cube is $D(SQ_n)+C$, where $D(G)$ is the diameter of graph G and C is a constant. And we also want to show that the shuffle cube is optimal fault hamiltonian graph.

三、 結果與討論

我們的研究方法如下：

一:收集文獻:透過各學校圖書館、研討會及 Internet 上的各資料庫如交通大學的數位圖書館，收集關於超立方體(Hypercube)及其變形相關問題的資料，以充分了解 Hypercube 及其變形的特性。

二:研讀，討論及報告:透過定期的研讀、討論並安排人員報告所收集之資料及論文，比較文獻內容，藉此找出錯置立方體的? 直徑及容錯的性質。

三:系統模擬及理論推導:設計軟體模擬尚未解決之問題，求出其基本的特性，並推導出其理論架構並加以學術證明。

四:成果發表:將研究成果撰寫成論文，並發表於國內外知名學術期刊及研討會，並將此計劃執行過程心得，在教學上與學生分享，讓教學與研究能相輔相成。

本研究計劃最後的成果如下：

1. 求出了錯置立方體(Shuffle cube)的? 直徑(Wide diameter)並發表在 Proceedings of the IASTED International Symposia, Applied Informatics, Symposium 2, Networks, Parallel and Distributed Processing, and Applications, pp. 402-407。
2. 錯置立方體有容錯漢米爾頓特性(Fault-tolerant Hamiltonian)，我們也將結果發表在 Proceedings of the 19th Workshop on Combinatorial Mathematics and Computational Theory, pp. 110-119。

四、計畫成果自評

在計畫主持人及共同主持人的帶領下，我們研讀、報告，分析所收集的資料，使用了各種方法，使我們對超立方體，錯置立方體及其他超立方體的變形有清楚的了解。

在本計劃中，我們已成功地求出錯置立方體的一些特性，包含？直徑，另外我們也對錯置立方體找出建構容錯漢米爾頓環的方法，這些結果也都發表在國外的會議論文之中，顯示也爲這領域的專家們所接受，現行世界上已有很多超級電腦的架構是利用到超立方體，若能將之改爲錯置立方體的結構，相信效能及容錯能力必能更上一層。我們也希望這些相關問題的研究能提升有關領域之應用拓展.。

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