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Substrate effects on intrinsic thermal stability and quench recovery for thin-film superconductors

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Two-dimensional conjugate film/substrate conduction equations considering an instantaneous release of energy in the form of a line source, anisotropic thermal conductivity of the film, and Joule heat are employed to investigate substrate effects on the intrinsic thermal stability and quenching recovery of thin-film superconductors. The results show that substrate effects exert significant influences on thermal stability of superconductors, because the higher thermal conductivity of substrates than those of films can cause heat feedback from the substrates. Both critical current densities related to intrinsic stability and recovery are calculated for different ratios of thermal conductivity and thickness of film to substrate, substrate materials, superconductors, Biot numbers, and operating temperatures. © 1996 Elsevier Science Limited

Keywords: substrate effects; thin-film superconductors; quench recovery

Nomenclature

a	Half superconductor width (m)
A	Non-dimensional width
A_r	Aspect ratio, a/d_f (dimensionless)
Bi	Biot number (dimensionless)
c	Specific heat (J/kgK)
d	Thickness (m)
D	Non-dimensional thickness of substrate
e_d	Thermal energy released per unit length (J/m)
E_d	Non-dimensional thermal energy released, $e_d/[2\rho_f c_f(T_c - T_o)ad_f]$
h	Convective heat transfer coefficient (W/m^{2}K)
J	Current density (A/m^2)
J_r	Current density ratio, J_o/J_{co} (dimensionless)
$(J_r)_c$	Critical current density ratio at the point of $(J_r)_{c1} = (J_r)_{c2}$
$(J_r)_{c1}$	Critical current density ratio for intrinsic stability (dimensionless)
$(J_r)_{c2}$	Critical current density ratio for recovery (dimensionless)
k	Thermal conductivity (W/mK)
K_{fs}	Ratio of thermal conductivity, k_{fx}/k_s
K_r	Ratio of thermal conductivity, k_{fy}/k_{fx}
K	Thermal conductivity tensor
q	Heat rate per unit length (W/m)
Q_{rs}	Ratio of heat generation rate to heat rate removed from film, $q_g/(q_{cv} + q_{cs})$
Q_r	Ratio of heat rate transferred by the substrate to the coolant, q_{cs}/q_{cv}
t	Time (s)
T	Temperature (K)
x,y	Coordinates defined in *Figure 1*
X	Transformed x-variable, x/d_f (dimensionless)
Y	Transformed y-variable, $y/(d_f K_r^{1/2})$ (dimensionless)

Greek letters

α	Thermal diffusivity (m^2/s)
σ	Normal state electrical resistivity (Ωm)
θ	Non-dimensional temperature, $(T - T_o)/(T_c - T_o)$
ρ	Density (kg/m^3)
τ	Non-dimensional time, $k_{fy}t/(d^2\ \rho\ C)_f$
ϕ	Instability parameter (dimensionless)

Subscripts

1	Point of intrinsic thermal stability failure
2	Point of stability recovery
c	Critical
co	Critical operation
cv	Convection by coolant
cs	Conduction by substrate
f	Film
g	Generation
max	Maximum
o	Operating
r	Ratio
s	Substrate
x,y	In the x- and y-direction, respectively

One of the most important problems in applying thin-film superconductors is their stability under thermal disturbances, which are most likely to be transient, localized, point heat releases produced by phenomena such as sudden relaxation of dislocations, crystal defects, and other spontaneous processes in superconductors[1,2]. The energy can increase the temperature to the transition level where superconductors change from the superconducting state to the normal resistive state, a phenomenon known as quenching. As local temperatures are raised, critical current densities are reduced, in turn, the current is shared to other undisturbed regions. If the operating current cannot be loaded by these undisturbed regions, Joule heating will occur. To keep superconductors in a superconducting state, it is necessary to provide means for adequate heat transfer after the occurrence of thermal disturbance and Joule heating. Understanding the mechanisms in practical superconducting film/substrate composites that govern their thermal stability is therefore of great importance.

Most previous analyses of thermal stability have been concerned with normal zone behavior after the normal cross-section forms, and are based on consideration of one-dimensional heat conduction along the electric current or in the longitudinal direction[3–8]. If a transient point source releases heat, the cross-sectional area in which the point heat source is located will be quenched, and then will grow into a normal zone along a longitudinal path, if the heat is not adequately transferred to a cooler and/or stabilizer. The normal zone is always the largest at the centre of the length for a longitudinal line source disturbance of any length. This means if the line source is assumed to be a finite length and along the direction of current, the maximum region of heat diffusion on the cross-section would occur in the centre of the line source length. This cross section permits the smallest critical current to pass. In other words, if the current density on this cross-section does not exceed the critical current density, the superconductor can be operated with stability. Nevertheless, once the current density of this cross section exceeds the critical value, the superconductivity will fail on this cross section and form a normal zone. In addition, Joule heat is produced and the normal zone begins to propagate along the longitudinal. Therefore, concern over stability performance of superconductors exhibiting normal-zone behaviour at central cross-sectional locations is important. Flik and Tien[2] defined an intrinsic stability criterion for a normal cross-section of bare film. Intrinsic thermal stability indicates a situation in which a superconductor can carry the operating current without Joule heating for all times after the occurrence of a localized release of thermal energy. Chen and Chu[9] presented a three-dimensional analysis of an instantaneous release of energy by a finite length line source. They also showed that the anisotropy of highly oriented films has a great impact on their thermal stability. Based on previous works, Unal and Chyu[10] further investigated the phenomenon of superconductivity recovery after stability failure and defined the recovery criterion. They provided an explanation of the two distinct types of behaviour in terms of recoverability of superconductivity after quenching. All of these studies analysed the problems of thermal stability by considering only bare films or tapes and neglecting the effects of substrates, which can exert a significant influence on stability. Particularly, when the heat transfer capacity of the substrate differs from that of the coolant, the substrate effects reflect its importance. Seol and Chyu[11] investigated current sharing with the stabilizer and the possible recovery of superconductivity for a one-dimensional composite tape superconductor. A stability criterion was developed based on heat conduction analyses of the current-sharing steady state and quenched steady state. But transversal heat transfer cannot be ignored for large aspect ratios if the region of thermal disturbance is located in a small area.

The present paper investigates the intrinsic thermal stability and quench-recovery of thin-film-superconductor/substrate combinations. Two-dimensional heat conduction equations for instantaneous release of energy by an infinite length of line source at the centre of the cross-section of a superconductor are considered. We present detailed analysis concerning substrates. Two important critical current-density ratios are calculated. The first, $(J_r)_{c1}$, above which intrinsic thermal stability fails and Joule heat is generated, can be determined based on the intrinsic-stability theory[2]. The second, $(J_r)_{c2}$, beyond which recovery is impossible after failure of stability, can be found based on the recovery criterion and the steady-state temperature solution resulting from Joule heating[10]. Our results depict how properties and dimensions of substrates exert significant influence on stability. If a substrate with higher thermal conductivity, which takes more heat away from the superconductor more rapidly than coolant, then simultaneously transfers the heat back because of its the high degree of transverse heat conductivity, this effect will decrease stability.

Analysis

The physical system under consideration is shown in *Figure 1*. A thin-film superconductor of thickness d_f is deposited on a substrate of thickness d_s and their widths are $2a$. An infinite length of line heat source is located at a point (x_o, y_o) of cross-section normal to current direction. At time $t = 0$, a finite amount of thermal energy, e_o, is released. The heat transfer coefficient h between the superconductor and the surrounding coolant is assumed to be constant and uniform for all surfaces in contact with circumstances. The coolant is maintained at the initial or operating temperature T_o. The operating electrical current, I_o, flows through the conductor normal to the xy plane.

A three-dimensional analysis might be more realistic as

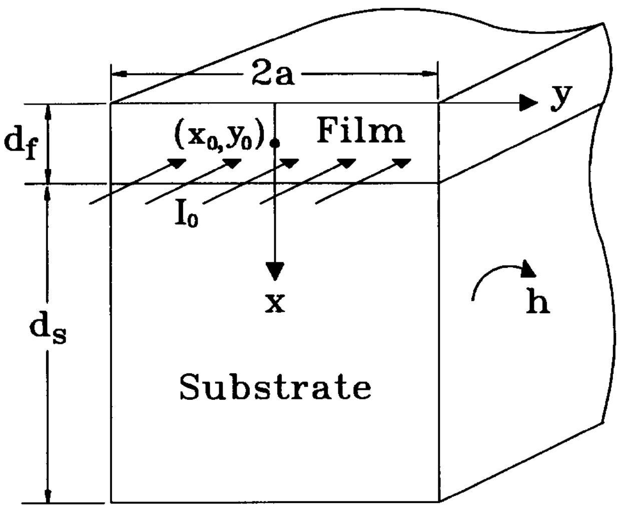

Figure 1 The physical model of a thin-film superconductor deposited on a substrate

a means of considering a release of energy in the form of finite length line source, but since another important purpose of this study (besides stability analysis) is to understand the heat flow through the superconductor to the substrate, a two-dimensional analysis can explain this phenomenon more clearly. Moreover, if the length of the line source is finite, the ignored longitudinal heat conduction in the substrate will induce the underestimation of critical current density, and this underestimation will decrease as the length of the line source increases. Because the position and the form of thermal disturbance are not predictable in reality, we selected the most rigorous physical model to calculate the stability which could obtain the lowest critical current density. Therefore, the position of disturbance is located at the centre of the cross-section where the disturbance is removed into the coolant and the substrate if the capability of heat transfer in the two heat sinks is unknown for initial analysis, and the form of that is assumed to be of infinite length line source. However, a thermal disturbance due to a sudden relaxation of a line dislocation is approximated very well by a line source[2].

Intrinsic thermal stability

Immediately after the instantaneous release of the thermal disturbance, the temperatures of points far away from the origin are not affected by heat diffusion, and the superconductor transfers no heat to the surrounding coolant. The initial condition can be given as

$$\int_{-a}^{a}\int_{0}^{d_f} \rho_f c_f\,(T_f - T_o)dx\,dy = e_d, \quad \text{for } t \rightarrow 0 \tag{1}$$

$$T_f(x,y,t \rightarrow 0) = T_o, \qquad \text{at } (x,y) \neq (x_0,y_0) \tag{2}$$

where ρ_f is the density of the film, and c_f is the specific heat. The properties of the superconductor and substrate are assumed to be constant.

The energy equation governing in this thin-film superconductor and substrate system can be represented by the Fourier equation:

$$\text{for the thin film:} \quad \rho_f c_f \frac{\partial T_f}{\partial t} = \nabla \bullet (\boldsymbol{K}\nabla T)_f \tag{3}$$

$$\text{for the substrate:} \quad \rho_s c_s \frac{\partial T_s}{\partial t} = \nabla \bullet (\boldsymbol{K}\nabla T)_s \tag{4}$$

where $\boldsymbol{K}$ is the conductivity tensor. Most ceramic high-temperature superconductors, such as $YBa_2Cu_3O_7$ with critical temperatures of about 95 K, are highly anisotropic in two main directions and have an orthorhombic crystal structure[12]. The *ab* plane identifies the plane of the CuO layers normal to the *xy* plane, and the *c* axis denotes the direction parallel to the *x* axis. Hagen *et al.*[13] examined single crystals of $YBa_2Cu_3O_{7-\delta}$ at temperatures ranging from 10 to 330K. Their results showed that the thermal conductivity was about four to five times lower in the *c* direction than on the *ab* plane, and that no anomaly existed at the transition temperature T_c. Using two-dimensional Cartesian coordinates, Equations (3) and (4) become:

for the thin film:

$$\rho_f c_f \frac{\partial T_f}{\partial t} = k_{fx}\frac{\partial^2 T_f}{\partial x^2} + k_{fy}\frac{\partial^2 T_f}{\partial y^2}$$

$$0 < x < d_f,\ -a < y < a \tag{5}$$

for the substrate:

$$\rho_s c_s \frac{\partial T_s}{\partial t} = k_s\left(\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2}\right)$$

$$d_f < x < d_s + d_f,\ -a < y < a \tag{6}$$

The associated boundary conditions are:

$$k_{fx}\frac{\partial T_f}{\partial x} = h(T_f - T_o) \qquad \text{at } x = 0 \tag{7a}$$

$$T_f = T_s,\ k_{fx}\frac{\partial T_f}{\partial x} = k_s\frac{\partial T_s}{\partial x}$$

$$\text{at } x = d_f \tag{7b}$$

$$-k_s\frac{\partial T_s}{\partial x} = h(T_s - T_o)$$

$$\text{at } x = d_f + d_s \tag{7c}$$

$$\pm k_{fy}\frac{\partial T_f}{\partial y} = h(T_f - T_o)$$

$$\text{at } y = \pm a,\ 0 < x < d_f \tag{7d}$$

$$\pm k_s\frac{\partial T_s}{\partial y} = h(T_s - T_o)$$

$$\text{at } y = \pm a,\ d_f < x < d_f + d_s \tag{7e}$$

Morelli *et al.*[14] and Hagen *et al.*[13] further showed that the thermal conductivity of 123 phase Y-Ba-Cu-O along the *ab* plane and in the *c* direction remain approximately constant above 77 K. Consequently, we may assume k_{fx} and k_{fy} do not vary with temperature.

The following nondimensional parameters are introduced to obtain a more compact form of the Fourier equation

$$\theta_f = \frac{T_f - T_o}{T_c - T_o}\quad \theta_s = \frac{T_s - T_o}{T_c - T_o}\quad \tau = \frac{k_{fx}t}{d_f^2\rho_f c_f} \tag{8a}$$

$$X = \frac{x}{d_f}\quad Y = \frac{y}{d_f\sqrt{K_r}}\quad \alpha_r = \frac{\alpha_{fx}}{\alpha_s} = \frac{k_{fx}/\rho_f c_f}{k_s/\rho_s c_s} \tag{8b}$$

$$A = \frac{A_r}{\sqrt{K_r}}\quad A_r = \frac{a}{d_f}\quad \mathrm{D} = \frac{d_s}{d_f} \tag{8c}$$

$$K_r = \frac{k_{fy}}{k_{fx}}\quad K_{fs} = \frac{k_{fx}}{k_s} \tag{8d}$$

Substituting Equations (8a)–(8d) into the associated equations, renders an isotropic form of the Fourier equation for thin film

$$\frac{\partial\theta_f}{\partial\tau} = \frac{\partial^2\theta_f}{\partial X^2} + \frac{\partial^2\theta_f}{\partial Y^2}$$

$$0 < X < 1, -A < Y < A \tag{9}$$

$$K_r\alpha_r\frac{\partial\theta_s}{\partial\tau} = K_r\frac{\partial^2\theta_s}{\partial X^2} + \frac{\partial^2\theta_s}{\partial Y^2}$$

$$1 < X < 1 + D, -A < Y < A \tag{10}$$

The boundary conditions become:

$$\frac{\partial \theta_f}{\partial X} = Bi\ \theta_f \qquad \text{at } X = 0 \tag{11a}$$

$$\theta_f = \theta_s,\ K_{fs}\frac{\partial \theta_f}{\partial X} = \frac{\partial \theta_s}{\partial X} \qquad \text{at } X = 1 \tag{11b}$$

$$-\frac{\partial \theta_s}{\partial X} = Bi\ K_{fs}\theta_s \qquad \text{at } X = 1 + D \tag{11c}$$

$$\mp\frac{\partial \theta_f}{\partial Y} = \frac{Bi}{\sqrt{K_r}}\ \theta_f \qquad \text{at } Y = \pm A \tag{11d}$$

$$\mp\frac{\partial \theta_s}{\partial Y} = Bi\ K_{fs}\sqrt{K_r}\ \theta_s \qquad \text{at } Y = \pm A \tag{11e}$$

where

$$Bi = \frac{hd_f}{k_{fx}} \tag{12}$$

and the initial condition is of the form

$$\int_{-A}^{A}\int_{0}^{1} \theta_f\ dXdY = 2E_dA \qquad \text{for } \tau = 0 \tag{13}$$

$$\theta_f = 2\delta(X - X_o)\ \delta(Y - Y_o)E_dA$$
$$\text{for } 0 \le X_o \le 1, -A \le Y_o \le A \tag{14}$$

where

$$E_d = \frac{e_d}{\rho_f c_f\ (T_c - T_o)2ad_f} \tag{15}$$

If the thermal disturbance is located at the centre of the cross-section, $(X_o, Y_o) = (0.5, 0)$, the principle of symmetry allows us to consider only half the $X > 0$, $Y > 0$ region as shown in *Figure 1*, and to assume adiabatic boundary conditions on the planes of symmetry. For numerical calculation we prescribe that energy is initially deposited in an area of $4\Delta X \times \Delta Y$, for which the dimensionless form can be described as

$$\theta_f(X,Y,0) = \begin{cases} \theta_d, & 0.5 - \Delta X \le X \le 0.5 + \Delta X,\ 0 \le Y \le \Delta Y \\ 0, & \text{others} \end{cases} \tag{16}$$

where θ_d is a normalized temperature that represents the amount of thermal disturbance. It is defined as

$$\theta_d = \frac{E_dA}{2\Delta X\ \Delta Y} \tag{17}$$

The solution of the conjugate transient heat conduction problem [equations (9)–(17)] stated above is employed to calculate the instability parameter ϕ. Flik and Tien[2] derived the criterion for intrinsic thermal stability of a bare tape or film-type superconductor subjected to an instantaneous linear thermal disturbance, based on the linear relationship between the critical current density and the temperature[1]. This criterion determines the critical operating density ratio value at which superconductivity is maintained and quenching does not occur after thermal disturbance. The critical operating density ratio for intrinsic stability is estimated by

$$(J_r)_{c1} = 1 - \phi_{max} \tag{18}$$

where ϕ_{max} is the maximum value of the instability parameter ϕ. The instability parameter is defined as

$$\phi = \frac{1}{A}\int_{0}^{1}\int_{0}^{A} g(\theta_f)dYdX \tag{19}$$

where

$$g(\theta_f) = \begin{cases} \theta_f, & 0 \le \theta_f \le 1 \\ 1, & \theta_f > 1 \end{cases} \tag{20}$$

The value of ϕ is directly related to the reduction in current-carrying capacity. From this parameter we can examine the stability performance of the superconductor. When the relationship between the current density ratio and the instability parameter

$$J_r \le 1 - \phi(\theta) \tag{21}$$

is satisfied, the superconductor will operate stably at all times during heat diffusion at the dimensionless operating current density ratio J_r, which is determined by the ratio of real current density J_o to critical current density at operating temperature T_o.

We used a finite-difference scheme with an under-relaxation technique to solve this conjugate film/substrate heat-conduction problem. Iterations between energy equations of film and substrate for all grids at each time step were continued until the criterion of convergence, $|(T - T_{iter})/T|_{max}$, within 10^{-4} was met.

Quench recovery

Analysing superconductor quench recovery, we can find two important characteristic parameters: the second critical-current density ratio $(J_r)_{c2}$ and the ratio of heat generation rate to the sum of the substrate conductive and convective cooling rates, $Q_{rs}(J_r)_{c2}$ is a critical value beyond which the stability recovery is impossible, and can be determined by considering the temperature distribution within the superconductor after the failure of intrinsic stability and the onset of Joule heating. For a given operating current density ratio J_r, the intrinsic stability situation is determined by Equation (21). When $\phi = 1 - J_r$, stability will fail and Joule heating is generated. Stability recovery occurs when the heat removed to the substrate and coolant is greater than that generated within the superconductor. In other words, when the instability parameter ϕ is equal to $1 - J_r$ again, the Joule heating has disappeared from the superconductor, and intrinsic thermal stability has recovered. The heat conduction equation for thin-film superconductors can be written based on Equation (9) as

$$\frac{\partial^2\theta_f}{\partial X^2} + \frac{\partial^2\theta_f}{\partial Y^2} + \frac{\sigma(J_rJ_{co}d_f)^2}{k_{fx}(T_c - T_o)} = \frac{\partial\theta}{\partial\tau}$$
$$\text{for } \tau_1 \le \tau \le \tau_2 \tag{22}$$

where τ_1, the time when intrinsic stability fails and $\phi = 1 - J_r$ occurs for the first time, and τ_2, the time when the stability recovers and $\phi = 1 - J_r$ occurs for the second time, are dimensionless times. The third term represents the dimensionless Joule heat for the film. σ is normal-state electrical resistivity and is assumed to be a constant during the period of operation. We determined $(J_r)_{c2}$ from the steady-state solution of energy Equations (22) and (10) with boundary condition Equations (11a–e). This manner for calculating $(J_r)_{c2}$ is the same as that proposed by Unal and Chyu[10]. The numerical method and iterative convergence criterion are similar to those in Equations (9)–(17). However, time is disregarded in this case.

In order to study superconductor stability performance, the heat rate ratio Q_{rs} is defined as

$$Q_{rs} = \frac{q_g}{q_{cv} + q_{cs}} = \frac{\text{heat generation rate per unit-length}}{\text{rates of heat transferred to coolant and substrate per unit-length}} \qquad (23)$$

In our analysis, the heat generated in the film includes initial thermal disturbance energy and Joule heat. q_{cv}, is the convective cooling rate of heat transferred to the surrounding at all of the film boundaries in contact with the coolant, whereas q_{cs} is the rate of heat transferred to the substrate from the film. For $Q_{rs} \cong 1$, the energy is balanced between heat generated in the thin-film superconductor and that conducted to the substrate and coolant. The other important parameter, Q_r, is the ratio of heat transferred by the substrate to the coolant. The value of Q_r can indicate clearly whether the amount of heat conducted to the substrate is greater than that transferred to the coolant or not.

Results and discussion

We first discuss the important parameter $(J_r)_{c1}$, which determines the intrinsic thermal stability criterion. Two main factors related to substrate effects influence this intrinsic critical current density, as shown in *Figures 2–3*. To investigate the effects of these influential factors on the stability, we chose a typical system consisting of a $YBa_2Cu_3O_7$ film deposited on an MgO substrate as a base case. Its properties are as follows: $d_f = 4\ \mu m$, $a = 400\ \mu m$, $k_{fx} = 0.5$ W/mK, $k_{fy} = 2$ W/mK, $\sigma = 1 \times 10^{-4}\ \Omega m$, $J_{co} = 1 \times 10^9$ A/m², $\rho_f = 2720$ kg/m³, $c_f = 180.2$ J/kgK, and $T_c = 95$ K, which is maintained by cooling with liquid nitrogen, where $T_o = 77$ K and $h = 5 \times 10^4$ W/m²K[2,13]. The amount of thermal disturbance, $E_d = 1$ corresponds to a line source of strength $e_d = 2.82 \times 10^{-8}$ J/μm by Equation (15), and located at the centre of intersection. We set $E_d = 20$ for calculation purposes. For substrate MgO: $d_s = 0.5$ mm, $k_s = 485.7$ W/mK, $\rho_s = 3580$ kg/m³, $c_s = 88.7$ J/kgK[15], which does not carry current at all times. Converting the above values into dimensionless variables yields $A_r = 100$, $K_r = 4$, $A = 50$, $D = 125$, $Bi = 0.4$, and $K_{fs} = 0.001$. In this physical model, heat is transferred to both substrate and coolant. Their thermal properties are also important factors that affect the intrinsic thermal stability.

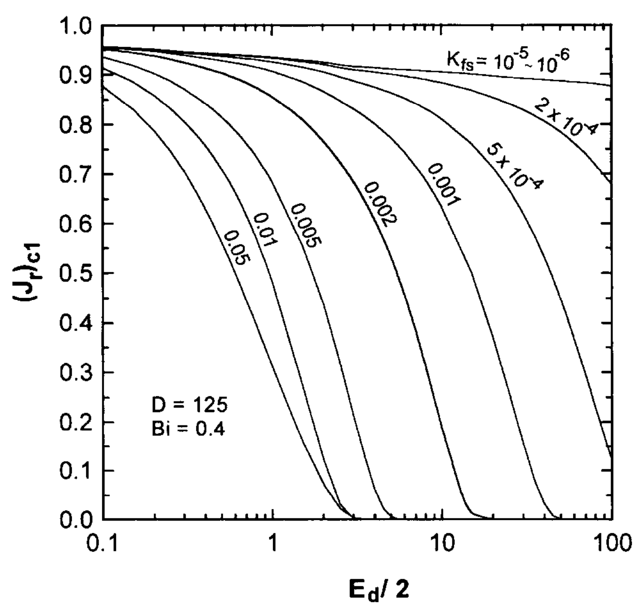

Figure 2 Influence of K_{fs} on $(J_r)_{c1}$ dependence of E_d

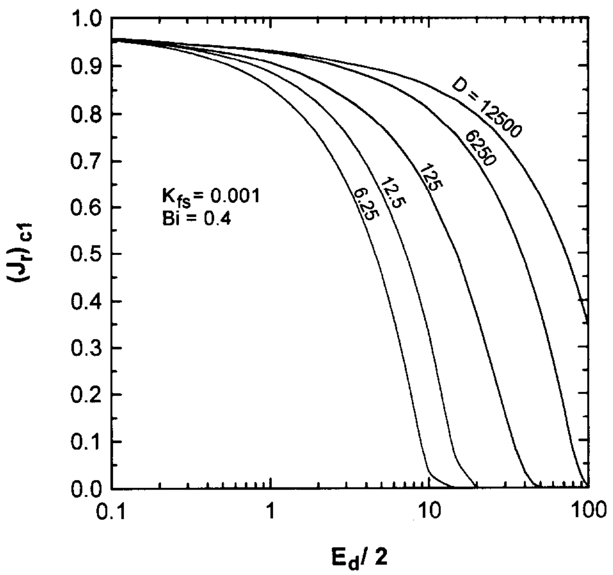

Figure 3 Influence of D on $(J_r)_{c1}$ dependence of E_d

Figure 2 shows the influence of the thermal conductivity ratio, K_{fs}, on the relationship between the disturbance energy, E_d, and the critical current density for intrinsic thermal stability, $(J_r)_{c1}$. It is clear that a large energy release easily induces instability. In other words, higher values of thermal disturbance energy yield higher values of ϕ_{max} and lower $(J_r)_{c1}$. These results are similar to those of bare film cooled only by coolant[10]. The thermal conductivity of the substrate significantly influences $(J_r)_{c1}$, also illustrated in this figure. Note that K_{fs} is the ratio of thin-film superconductor thermal conductivity in the x direction to that of the substrate material. $(J_r)_{c1}$ increases as K_{fs} decreases, indicating that a higher current density is possible without quenching for smaller K_{fs}. A smaller value of K_{fs}, i.e. higher substrate thermal conductivity, obviously promotes $(J_r)_{c1}$ to about 0.9 times the average value for $K_{fs} \leq 10^{-5}$ and $E_d \leq 200$. In this situation the thermal disturbance energy affects stability slightly, and the current density carried in the superconductor approaches J_{co}. If the substrate thermal conductivity is high, the substrate can rapidly transfer heat from the film. When the rate of heat removed from the superconductor is larger than that of the accumulated energy, which causes the formation of normal zones, the superconductor will remain stable. Thus, the effects of the substrate on the thermal stability can be very important and should not be ignored for thin-film superconductors deposited on substrates.

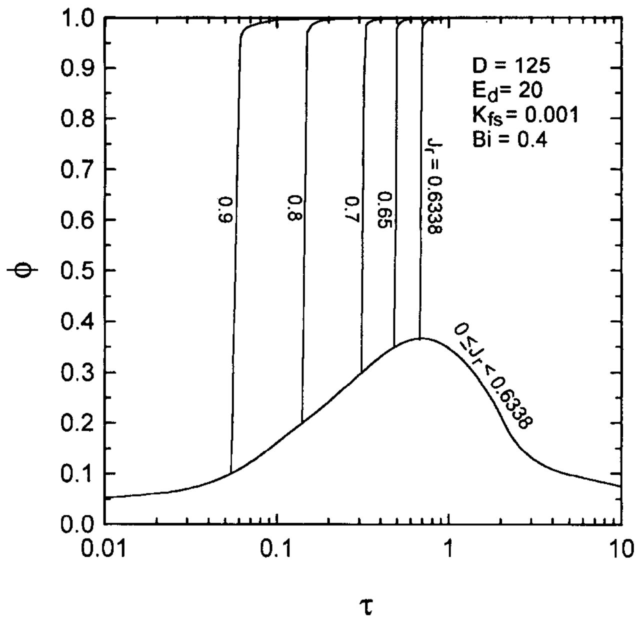

Figure 4 ϕ versus τ with different J_r

Figure 3 depicts the effect of the thickness ratio of substrate to film, D, on $(J_r)_{cl}$ versus E_d. For such an MgO substrate with high thermal conductivity, higher values of D increase $(J_r)_{cl}$, which is favourable for stability. Smaller substrate dimensions induce larger propagative regions of normal zones or higher substrate temperatures by two means. Firstly, the heat transfer capacity of a thin geometry is smaller for a fixed amount of energy and heat dissipation of the substrate alone is less effective. Secondly, the side area of the substrate available to transfer heat to the coolant is decreased, and heat accumulates easily within the substrate. As in previous cases, when E_d is higher than some value for a fixed D, the superconductor cannot carry any current in a stable manner and it will quench even with slight current densities. The value of $(J_r)_{cl}$ approaches zero as E_d reaches 100 in the base case in which D = 125, K_{fs} = 0.001, and Bi = 0.4

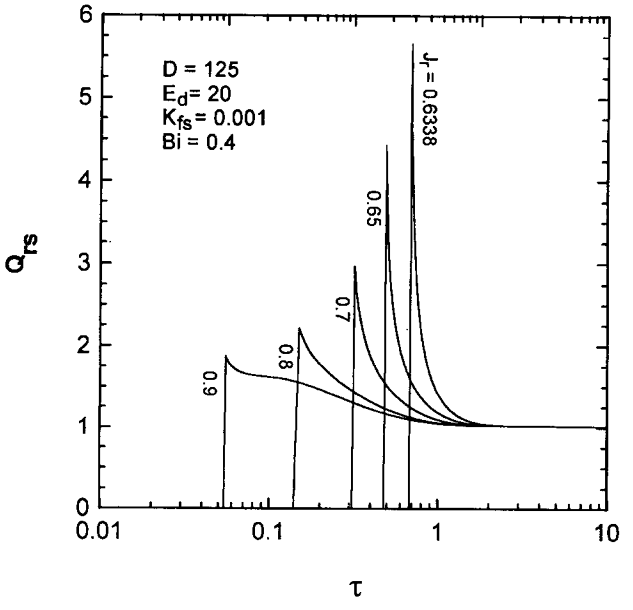

Figure 5 Q_{rs} versus τ with different J_r

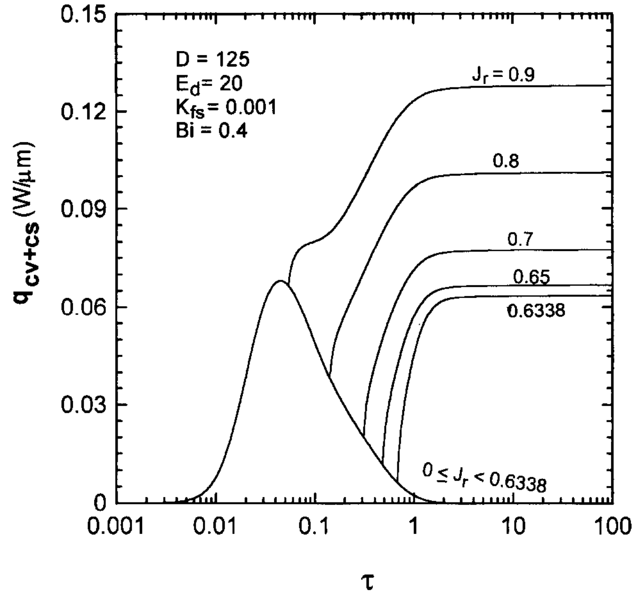

Figure 6 q_{cv} + q_{cs} versus τ with different J_r

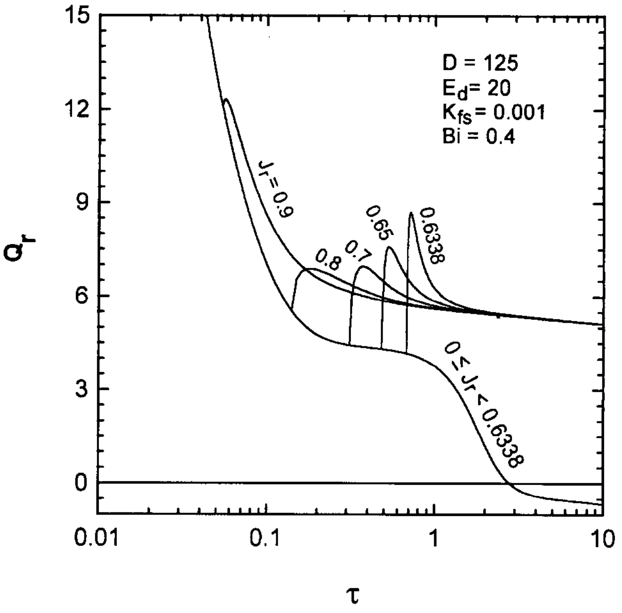

Figure 7 Q_r versus τ with different J_r

Figures 4–7 show the performance results of superconductors subjected to a hypothetical instantaneous disturbance of E_d = 20. Four important parameters, ϕ, Q_{rs}, q_{cv} + q_{cs}, and Q_r, are discussed. Both the conditions of intrinsic thermal stability and quench recovery are considered. These figures show that a superconductor is more likely to be quenched and cannot recover when Joule heating occurs. Using *Figures 4–9* we discuss and explain in detail the influence of the substrates on thermal stability. The values of the intrinsic instability parameter ϕ calculated from Equations (9), (10), and (22) are presented in *Figure 4*. The maximum value of J_r is 0.6337 at τ = 0.7, below this the superconductor will operate stably without Joule heating and intrinsic stability as defined by Equation (21) is always satisfied, as indicated by Q_{rs} = 0 in *Figure 5*. If the operating current density is higher than 0.6337, the superconductor will be quenched immediately as J_r = 0.6338, 0.65, 0.7, 0.8 and 0.9. This phenomenon is similar to the results for bare film or tape presented by Unal and Chyu[10]. We also demonstrate that isotropic thin-film high-T_c superconductor deposited on a higher thermal conductivity substrate, such as MgO, is also either intrinsically stable or irrecoverably unstable under the conditions we assume. In other words, if Joule heat cannot be transferred quickly enough to the substrate and coolant, it will induce irreversible quenching. We also found that higher J_r results in quenching and resistance onset earlier, and that this critical point occurs at $\phi = 1 - J_r$.

Variation in the ratio of heat generated in film to heat transferred to substrate and coolant, Q_{rs}, with τ, is displayed in *Figure 5*. These results demonstrate the influence of J_r on the relationship of Q_{rs} to τ. The value of Q_{rs} increases sharply because of Joule heat generated abruptly as $\phi = 1 - J_r$ is satisfied for a given J_r in film, and reaches the maximum value at once. A maximum Q_{rs} indicates that a minimum percentage of the heat is being transferred from the film. Then Q_{rs} gradually decreases from maximum as time increases until $Q_{rs} = 1$, which represents the energy balance between heat generated in the film and that removed from it. This decrease occurs because little heat is conducted to the boundaries at the onset of electric resistance, but increases over time. Joule heating occurs at $\tau =$ 0.69, 0.49, 0.32, 0.15, and 0.055, respectively, when $J_r =$ 0.6338, 0.65, 0.7, 0.8 and 0.9. $q_{ev} + q_{cs}$ reaches a maximum value for intrinsic thermal stability at $\tau = 0.045$ as *Figure 6* shows. As J_r increases from 0.6338 to 0.9 not only does q_g increase, but $q_{cv} + q_{cs}$ also increases; due to the influence of increasing $q_{cv} + q_{cs}$ overwhelms the increase in q_g, the maximum Q_{rs} decreases as J_r decreases from 0.6338 to 0.9, and that for $J_r = 0.6338$ is the highest value among the five cases. This adverse result, the J_r increase and the maximum Q_{rs} increase, have been shown in previous studies of thermal stability of bare superconductor cooled only by coolant, since the amount of heat, $q_{cv} + q_{cs}$, taken away from the film is small and the heat generated due to resistance, which increases with increasing J_r is quite large.

Figure 6 also shows that more heat is transferred from the film to the substrate and coolant if Joule heating or J_r increases. As time passes, $q_{cv} + q_{cs}$ approaches different constants that are equal to thc Joule heat, q_g, for different J_r. The reason is same as that given for *Figure 5* in which $Q_{rs} = 1$.

Figure 7 displays the variation in the ratio of heat conducted to the substrate to the amount of convection heat transferred to the coolant, Q_r, from film. We can easily see that Q_r remains larger than 1, this means heat drawn out by the substrate is always larger than that taken by the coolant before ϕ_{max} is reached for intrinsic stability or quench onset. When superconductors quench, and time passes, all Q_r approach the same constant of 5, indicating that the heat transferred by the substrate is about 5 times greater than that transferred by the coolant from the upper and lateral surfaces of the film in the steady state. Once Joule heat is generated, Q_r is suddenly promoted because the response of the substrate to heat is faster than that of the coolant. Minimum Q_r is retained at $J_r = 0.8$ for the five cases of different J_r and maximum Q_r is retained at $J_r = 0.9$. When $0 \leq J_r < 0.6338$, during the period of intrinsic thermal stability, the value of Q_r decreases as time increases. The maximum Q_r occurs at the beginning, because the high thermal conductivity of the substrate induces a high heat flux. But Q_r decreases swiftly as time passes, since the heat flows back quickly from the substrate by transverse heat transfer. Therefore, the large amount of heat conducted to the substrate does not mean the superconductor will operate more stably than with no substrate under the film. Calculating the stability of bare superconducting film for same conditions, we find its $\phi_{max} = 0.1602$, smaller than the 0.3663 achieved when using an MgO substrate. In other words, the presence of substrate will induce a larger region of normal zone in the film than no substrate at all will if the substrate thermal conductivity is large. Substrates can have an important influence on the thermal stability.

Figure 8 shows the dimensionless temperature profiles for different positions of Y at $\tau = 0.7$, when $\phi = \phi_{max}$. From this figure, the heat flow direction can be decided by the temperature profile in some position. As $Y = 0$, 5, 10, the heat transferred from film to substrate, but Y in other positions show the heat flows in the opposite direction, while at this time the net heat flux is still greater than zero and $Q_r \sim 4.2$ in *Figure 7*. When $\tau = 2.8$ is arrived at, the Q_r begin to change to negative values. This means that net heat flux between the film and the substrate has begun to flow back from the substrate. This phenomenon can be observed in *Figure 9*, which presents the temperature profiles for different dimensionless time τ at $Y = 0$. Because the thermal disturbance is located at $(X_o, Y_o) = (0.5, 0)$, in this section of $Y = 0$ a maximum heat flux exists and indicates the direction of net heat flow. When $\tau \geq 4$, the θ of the film for every position of X is smaller than that of the substrate, which means that heat flow is fully from substrate to film.

Figures 10–12 show a comparison between $(J_r)_{c1}$ and $(J_r)_{c2}$ for various parameters. Unal and Chyu[10] developed the stability behaviour criterion from $(J_r)_{c1}$ and $(J_r)_{c2}$. If $(J_r)_{c1} < (J_r)_{c2}$, three characteristic regions in terms of superconductor stability and recovery have been identified: when $0 \leq J_r \leq (J_r)_{c2}$, the superconductor is always stable despite thermal disturbances; when $(J_r)_{c1} < J_r \leq (J_r)_{c2}$, the superconductor can recover stability after quenching; when $J_r > (J_r)_{c2}$, superconductivity has failed. If $(J_r)_{c1} \geq (J_r)_{c2}$, two characteristic regions can be identified: when $0 \leq J_r \leq (J_r)_{c1}$, the superconductor is stable in spite of the heat disturbance; when $J_r > (J_r)_{c1}$, superconductivity can never be recovered after quenching. According to the definition as stated above, a plot including both $(J_r)_{c1}$ and $(J_r)_{c2}$, such as

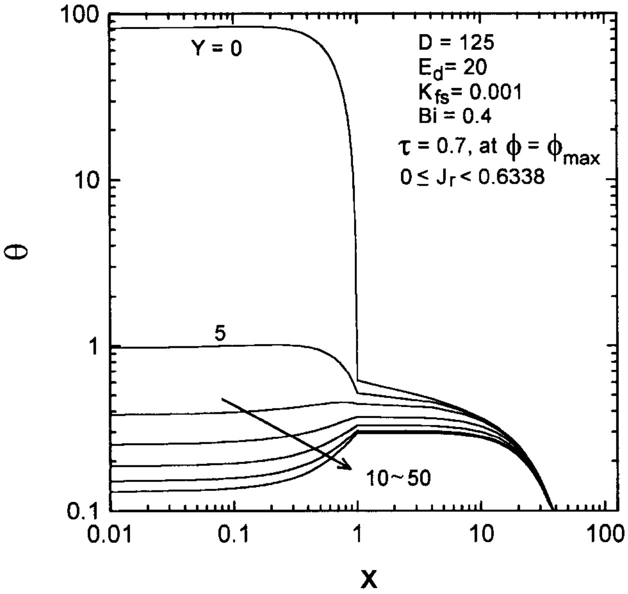

Figure 8 θ versus X across film and substrate with different Y at $\tau = 0.7$ for ϕ_{max} occurrence

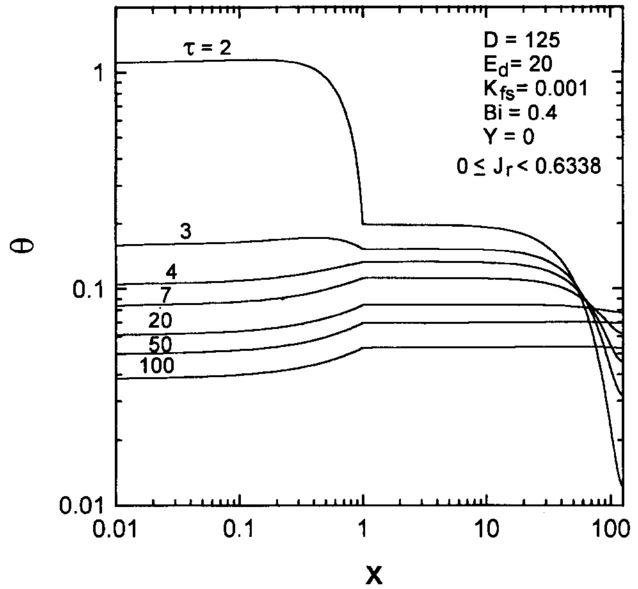

Figure 9 θ versus X across film and substrate with different τ at $Y = 0$

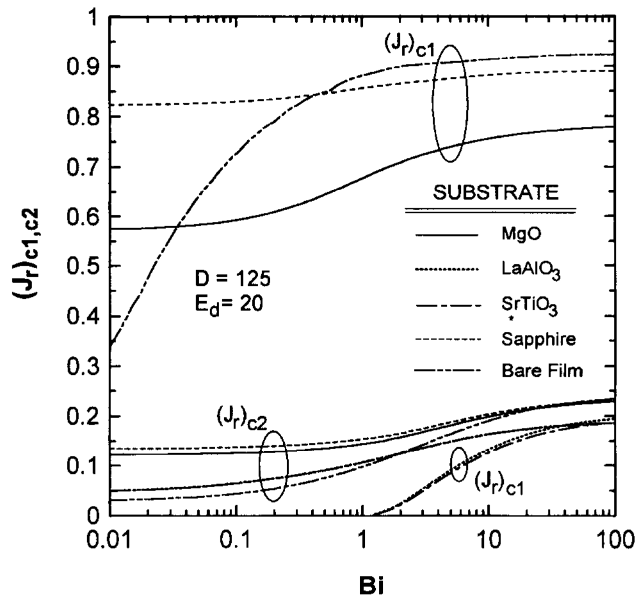

Figure 11 $(J_r)_{c1}$ and $(J_r)_{c2}$ with different Bi for YBCO deposited on four substrates and bare film

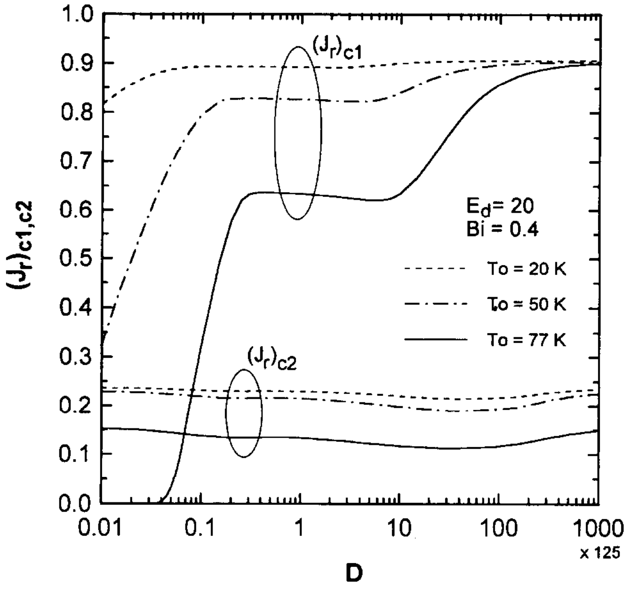

Figure 10 $(J_r)_{c1}$ and $(J_r)_{c2}$ with different D for YBCO at operating temperatures of 20, 50, and 77K

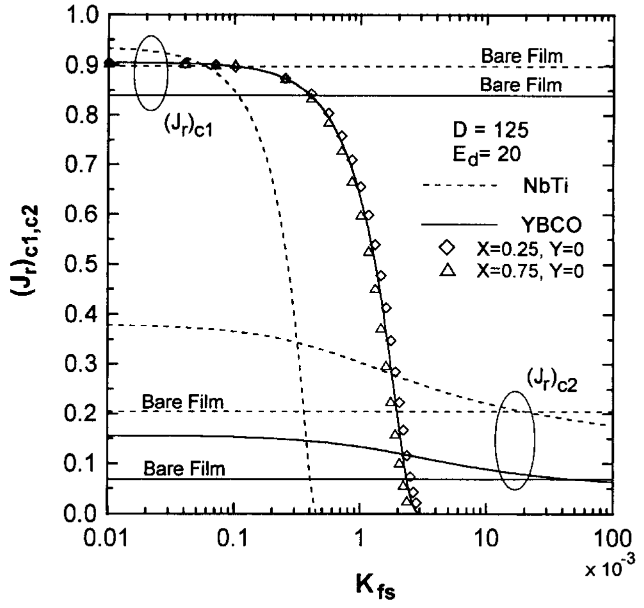

Figure 12 Comparison of $(J_r)_{c1}$ and $(J_r)_{c2}$ with different K_{fs} for YBCO and NbTi

Figures 10–12, is helpful. In these figures, only the $(J_r)_{c1} > 0$ region is meaningful (when $(J_r)_{c1} = 0$, intrinsic thermal stability is impossible for any trifling current). These figures show that $(J_r)_{c1}$ is greater than $(J_r)_{c2}$ for most conditions, except for LaAlO$_3$ and SrTiO$_3$ substrates in *Figure 11*. By contrast, it is very unlikely that $(J_r)_{c1} \leq (J_r)_{c2}$ and $(J_r)_{c1} \neq 0$.

At $T_o = 77$ K, $(J_r)_{c1} \geq (J_r)_{c2}$ is satisfied for $D \geq 7.83$, and $(J_r)_{c1} > (J_r)_{c2}$ for $D < 7.83$. $(J_r)_{c1} = (J_r)_{c2} = 0.1407$ occurs at $D = 7.83$. This result agrees with that in *Figure 4* when $D = 125$ and only two regions are present. In *Figure 11*, the substrate properties have a great impact on $(J_r)_{c1}$, which makes $(J_r)_{c1}$ either larger or smaller than $(J_r)_{c2}$ for a particular substrate. $(J_r)_{c1} \geq (J_r)_{c2}$ is satisfied at different Bi for three cases: YBCO/MgO, YBCO/sapphire, and bare YBCO film. This result of bare film is in agreement with that estimated by Unal and Chyu[10], which demonstrate that the operation of such a high-T_c superconductor film under the assumed conditions is either intrinsically stable or irrecoverably unstable. Whereas it is clear that this conclusion cannot be applied to a YBCO/substrate system in which the substrate thermal conductivity is small, because of $(J_r)_{c1} \leq (J_r)_{c2}$ for LaAlO$_3$ and SrTiO$_3$. This result is fully different from that of ignoring substrate effects. Therefore, substrate effects may be an important factor to decide whether a thin-film superconductor can recover superconductivity after quenching or not. *Figure 12* shows the values K_{fs} are 2.254×10^{-3} and 3.243×10^{-4} at $(J_r)_{c1} = (J_r)_{c2}$, respectively, for YBCO: $(J_r)_c = 0.12$ and NbTi: $(J_r)_c = 0.34$. K_{fs} are 0.001, 0.0269, 0.0273, and 4.42×10^{-4} for MgO, LaAlO$_3$, SrTiO$_3$, and sapphire, respectively. The results for these four substrates in *Figure 12* correspond with those in *Figure 11* for $Bi = 0.4$.

The effects of the operating temperatures T_o are demonstrated in *Figure 10*. The properties of the film and substrate are listed in *Table 1*. Lower operating temperatures cause an increase in critical current densities, $(J_r)_{c1}$ and $(J_r)_{c2}$, because the thermal diffusivity of the substrate is promoted by the decrease in operating temperature. This means that the higher thermal diffusivity of the substrate quickly takes heat away from the film, which reduces the instability of the superconductor and increases the current density tolerance value. *Figure 10* also shows that $(J_r)_{c1}$ is influenced strongly by the substrate thickness, but variations in $(J_r)_{c2}$ are slight as substrate thickness increases from $D = 1.25$ to 125,000. There is a range around $D = 125$ where the variation of $(J_r)_{c1}$ is particularly small. The same phenomenon also exists at three different operating temperatures. As D was increased to 125,000 in our study, the $(J_r)_{c1}$ approached a constant value of 0.9 for three different operating temperatures, indicating that if the substrate thickness is large enough, the critical current density for intrinsic thermal stability is high and is almost independent of operating temperature or superconductor and substrate properties. From these results, we can propose that the substrate thickness must be at least 37.5 times as large as that of the film to ensure stable operation at higher critical current density for a YBCO/MgO system immersed in liquid nitrogen. Since the $(J_r)_{c2}$ is the steady-state solution, it is approximately independent of D and the effect of substrate thickness can be ignored.

Taking the most common substrates used for YBCO film: MgO, $SrTiO_3$, $LaAlO_3$, and sapphire, the influences of different substrates on thermal stability were analyzed. *Figure 11* shows the effects of four substrates and bare film on $(J_r)_{c1}$ and $(J_r)_{c2}$ at different Bi. The properties of the substrates are listed in *Table 1*. Both critical current densities, $(J_r)_{c1}$ and $(J_r)_{c2}$, for sapphire are the highest, those of MgO second, those of $LaAlO_3$ and $SrTiO_3$, which very nearly approximate each other, are lowest. The main factor leading to different critical current densities for different substrates is that a high thermal conductivity conducts heat more rapidly to the coolant, and thus decreases the film temperature and the growth of normal zones. The properties of substrates and coolant have only a diminutive effect on $(J_r)_{c2}$, because $(J_r)_{c2}$ represents the ability of bearing Joule heat for a maximum J_r, which has a more attractive relationship to the properties of the superconductor than to the environment. A comparison of the properties of $LaAlO_3$ and $SrTiO_3$ shows that the effects of heat capacity are slight on $(J_r)_{c1}$ and $(J_r)_{c2}$, because the difference in their heat capacities is about 100 times, while thermal conductivities are almost equal. Their $(J_r)_{c1}$ are very close to each other and the differences cannot be observed graphically, and their $(J_r)_{c2}$ have only a small difference.

The variation in Bi from 0.01 to 100, which slightly affected the $(J_r)_{c1}$ and $(J_r)_{c1}$ of all samples, except for bare film and film deposited on both substrates $LaAlO_3$ and $SrTiO_3$ which have small thermal conductivity. For bare film the $(J_r)_{c1}$ and $(J_r)_{c2}$ increase to 0.58 and 0.2, respectively, and both values are higher than those of other samples since the heat transfer capacity of the coolant becomes an important factor for films with no substrate. When the substrates have poor heat conduction, such as $LaAlO_3$ and $SrTiO_3$, if the Bi is small, the superconducting film cannot carry any current. Therefore, the assumption that substrate effects can be ignored is inappropriate and significantly overestimates the $(J_r)_{c1}$. Besides, when $Bi \geq 0.5$ and 0.032, respectively, for sapphire and MgO, $(J_r)_{c1}$ is overestimated, but is underestimated below these Bi values. Consequently, the $(J_r)_{c1}$ of films on MgO substrates is higher than that of bare film at $Bi = 0.4$ in liquid nitrogen. According to this result, the assumption of substrates ignored is still inappropriate for high substrate thermal conductivity.

Another important result due to substrate effects is that the $(J_r)_{c1}$ and $(J_r)_{c2}$ decrease asymptotically towards different constants with Bi for MgO and sapphire, which have high thermal conductivity. The asymptotical values are minimum for a particular substrate, but not equal to zero. The $(J_r)_{c1}$ are equal to 0.82 and 0.57 for sapphire and MgO substrate, respectively, and the $(J_r)_{c2}$ are about 0.13 for the two substrates. It means that a large thermal conductivity of substrate can carry a large amount of J_r under a particular thermal disturbance even if the coolant cannot transfer any heat by convection.

To understand the influence of the thermal properties of a superconductor itself on stability in superconductor/substrate structures, we made a comparison between critical current densities of low-T_c and high-T_c superconductors on the same substrates. The chosen low-T_c superconductor was NbTi, with the following properties: $k_{fx} = 0.11$ W/mK, $K_r = 1$, $\sigma = 6 \times 10^{-7}$ Ωm, $J_{co} = 1 \times 10^9$ A/m², $\rho_f = 6200$ kg/m³, $c_f = 0.87$ J/kgK, $T_c = 9.6$ K, $T_o = 4.2$ K and $h = 10000$ W/m²K; other data are based on the same conditions as those for YBCO. The Biot numbers are 0.4 and 0.36, respectively, for YBCO and NbTi. The $(J_r)_{c1}$ depends significantly on K_{fs}, as K_{fs} increases or k_s decreases $(J_r)_{c1}$ is reduced abruptly from 0.9 to 0. The same result is also displayed in *Figure 11* and we discuss it for different sub-

Table 1 Characteristic parameters of YBCO, substrates, and coolant at different operating temperatures

T_o (K)	K^a_{fx} (W/mK)	C_f (J/kgK)	Substrate	ρ_s (Kg/m³)	k_s (W/mK)	C_s (J/kgK)	h^b (W/m²K)
77	0.5	180.2^c	MgO	3580^d	485.7^d	88.7^e	5×10^4
			$LaAlO_3$	6520^f	18.6^f	14.1^f	
			$SrTiO_3$	5120^e	18.3^e	181.6^e	
			Sapphire	3990^d	1131^e	60.7^e	
50	0.62	92.33^g	MgO	3580^d	1088.2^d	24.21^e	5×10^4
20	0.37	7.8^g	MgO	3580^d	1444.5^d	1.273^e	5×10^4

[a] From figure provided by Morelli *et al.* (1987)[14]
[b] Filk and Tien (1990)[2]
[c] Butera (1988)[16]
[d] Slack (1962)[17]
[e] Touloukian and Buyco (1970)[18]
[f] Michael *et al.* (1992)[19]
[g] Ferreira *et al.* (1988)[20]

strates. $(J_r)_{c1} = 0$ occurs at $K_{fs} = 0.003$ and 4.5×10^{-4} for YBCO and NbTi, respectively. The values of $(J_r)_{c2}$ for YBCO are about 50% lower than those of NbTi, indicating that recovery is more likely in the latter and the properties of the superconductors are confirmed to exert a larger effect than those of the substrates or coolant shown in *Figures 11* and *12*. The K_{fs} increased from 10^{-5} to 0.1 and $(J_r)_{c2}$ decreased by about half. The effect of K_{fs} on $(J_r)_{c2}$ was not so serious as that on $(J_r)_{c1}$. The thermal conductivity of YBCO is about 5 times higher than NbTi, this factor is the key point in promoting the $(J_r)_{c2}$ of NbTi. This result is the same as that for bare film presented in a previous study[10]. Therefore, we know that in proportion to the thermal conductivity of film decrease, the $(J_r)_{c2}$ will rise. This influence on $(J_r)_{c1}$ is contrary as shown in *Figure 12* where the $(J_r)_{c1}$ of YBCO is higher than that of NbTi for most $K_{fs,}$ except $K_{fs} < 5 \times 10^{-5}$. Note that with a smaller $K_{fs,}$ stability can be sustained at a higher operating J_r. In other words, once stability fails, it is less likely to be recovered because a large amount of Joule heat is produced.

The $(J_r)_{c1}$ and $(J_r)_{c2}$ of bare film are also presented in *Figure 12*. Ignoring the effects of substrates on $(J_r)_{c1}$ results in a highly significant overestimation as $K_{fs} > 4 \times 10^{-4}$ and 6×10^{-5} for YBCO and NbTi, respectively. Two main factors affect the $(J_r)_{c1}$ and intrinsic thermal stability. One is the heat transferred through substrate to coolant, which increases the stability. The other is the heat feedback from the substrate because the thermal conductivity of the substrate is higher than that of the film, and this effect induces an increase in the normal zone. When K_{fs} decreases or the substrate thermal conductivity increases from $(J_r)_{c1} = 0$ situation, not only does the amount of heat transferred to the substrate increase but the amount of heat feedback also increases in a transient process. If the latter influence on stability overwhelms the former influence, the $(J_r)_{c1}$ can abruptly decrease to zero. But the same phenomenon is not found in *Figure 11*, because the transverse heat transfer, which causes the heat feedback, was not considered for the coolant. On the other hand, the substrate effects exert an obvious variation in $(J_r)_{c2}$, while the larger deviation occurs for smaller K_{fs}. From the comparison between the bare films and the films deposited on substrates, we can conclude that the critical current density for intrinsic thermal stability is significantly overestimated by the bare film for the higher substrate thermal conductivity, in contrast to the lower substrate thermal conductivity that the critical current density for recovery is apparently underestimated. The samples of YBCO and NbTi films were deposited on the same MgO substrate, their K_{fs} are about 0.001 and 2.26×10^{-4}, enforcing smaller $(J_r)_{c1}$ and larger $(J_r)_{c2}$ than those for bare film.

The $(J_r)_{c1}$ in different positions of thermal disturbance located for YBCO/substrate are also shown in *Figure 12*. Three different positions are chosen at $(X_0, Y_0) = (0.25, 0)$, $(0.5, 0)$, and $(0.75, 0)$. An increase in the X_o represents a thermal disturbance near the substrate and the substrate effect becomes more evident. The influence of disturbance positions is not obvious for small K_{fs}, but as K_{fs} increases with an increase in the effect of variation of positions until $(J_r)_{c1}$ approaches zero. Moreover, the $(J_r)_{c1}$ has the lowest value at $(X_o, Y_o) = (0.75, 0)$ for a particular K_{fs} because a stronger substrate effect causes a more significant heat feedback.

Conclusions

Intrinsic thermal stability and quenching recovery of thin-film superconductors deposited on substrates have been investigated numerically. The results show high-T_c superconductors are either intrinsically stable or irrecoverably unstable only for high substrate thermal conductivity. The intrinsic thermal stability is influenced strongly by the substrate effects. By contrast, the variation in the critical current density for recovery is slight for most substrate thicknesses and thermal conductivities, but the properties of the superconductors themselves have a significant effect on this critical current density. In proportion to the film thermal conductivity decrease, critical current density will rise. Lower operating temperatures and substrate thermal conductivities cause an increase in both critical current densities. From the viewpoint of heat transfer, sapphire is a better substrate for stably operating superconducting thin films. Ignoring the substrate effects overestimates significantly the critical value for intrinsic stability for higher thermal conductivity ratios of film to substrate, because back-flow of heat from substrates is not considered. The critical value for quenching recovery for $YBa_2Cu_3O_7$ film is about half lower than that for low-T_c: NbTi, indicating that recovery is more favourable in the latter.

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