行政院國家科學委員會專題研究計畫成果報告

橢圓偏光儀之異向晶體量測

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一、中文摘要

本計畫除了延續前計畫利用石英將入射角的偏差更正後量測一單軸異向晶體 YVO4. 本計畫量測此晶體的正常及其異常折射率外並量得該晶體的光軸位置,結果與廠商規格相合。

關鍵詞：橢圓偏光儀，雙折射晶體

Abstract

This report continues our previous work on measuring the optical properties of a uniaxial crystal. We corrected the beam deviation using a Quartz crystal and measure the ordinary and extraordinary refractive indices of YVO4 and compared with vendor's specifications. We also measured the orientation of its optical axis.

Keywords: Ellipsometry, birefringence

二、Introduction

According to our previous study [1], we are able to measure the incident beam deviation in a rotating element ellipsometry by a quartz crystal. Because this calibration, we are able to construct a system to measure the optical axis and refractive indices of a uniaxial crystal by two sheet polarizers. Besides the ordinary and extraordinary refractive indices, we also applied this technique to measure the angle between the normal to the cleavage plane and optic axis of a Yttrium Or-thovanadate (YVO4).

In this study, we proved that this PSA ellipsometry not only can measure the isotropic material it also can be used as rotating element ellipsometry for measure the optical properties of uniaxial crystals.

三、Theory, experiment and result

The reflected χ_r and incident χ_i polarization states are related by [2]

$$\chi_r = \frac{(r_{sp}/r_{ss}) + \chi_i}{(r_{pp}/r_{ss}) + (r_{ps}/r_{ss}) \cdot \chi_i} \qquad (1)$$

where r_{xy} is the Fresnel reflection coefficient for the parallel (p, i.e. x) and perpendicular(s, i.e. y) polarizations. The analytical expressions of these Fresnel reflection coefficients for uniaxial crystals in the Appendix. The complex pseudoreflectance ratio was defined [2] as for anisotropic media, while in general ρ is defined as [3]

$$\rho = \tan\psi e^{i\Delta},$$

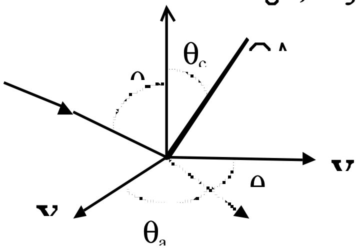
$$\tan^2\psi = \left|\frac{\chi_i}{\chi_r}\right|^2$$

thus

(2)

Since the cross terms vanish in an isotropic medium, tan ψ [3] equals $|r_{pp}/r_{ss}|$, which is the conventional expression for the ellipsometric parameter. The reflection geometry for a uniaxial crystal is shown in Fig. 1.

Figure 1. The reflection geometry: θ_i is the incident angle, xy plane is the



reflecting face of the crystal, zx plane is the incident plane, the z axis is the normal line. OA is the optical axis of the crystal.

A simple model for anisotropic crystals was proposed by Aspnes [4]: the measured ellipsometric parameters for a particular θ_a equal those of the effective isotropic sample whose refractive index is given by its dielectric tensor projection onto the sample surface along the incident direction. This implies that

$$\tan^2 \psi = \frac{{}^{2}{}_{rp}}{{}^{2}{}_{rs}} \tag{3}$$

where I_{rp} represents the reflected intensity parallel to the incident plane and I_{rs} represents the reflected intensity perpendicular to the incident plane, for P = 45°, i.e. χ_i=1. According to equation (3), one can obtain tan ψ simply by measuring the reflected intensities I_{rp} and I_{rs}. If the optical axis of a nonabsorbent uniaxial crystal is parallel to the reflection surface, i.e. θ_c= 90°, then the ellipsometric parameter ψ can be characterized by a twofold symmetry with respect to θ_a, the azimuthal angle. Since we were only interested in determining the AI in a PSA ellipsometry, such as shown in Fig. 2, we simulated the ellipsometric parameter function $\psi(\theta_a)$ for a uniaxial crystal with n_o and n_e as its ordinary and extraordinary refractive indices, respectively. Furthermore, we assume the optical axis of the sample crystal is parallel to the reflection surface so as to obtain the twofold symmetry for comparison.

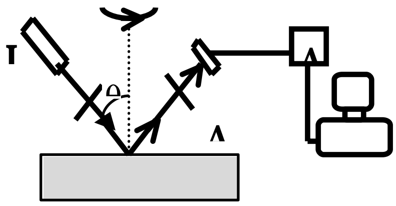


Fig. 2 A schematic set-up of the PSA ellipsometer: L, light source (He-Ne laser); P, polarizer; A, analyzer; D, detector.

The function $\psi(\theta_a)$ is simulated for $\chi_i = i$, i.e. P = 45°, and optimized [5] by $\chi_i = -i$, i.e. P = -45°, to eliminate the error caused by the misalignment of the polarizer, according to equation (3), one can obtain

$$\tan \psi = \left[\left(\frac{I_{rp}}{I_{rs}} \right)_{P=45^o} \left(\frac{I_{rp}}{I_{rs}} \right)_{P=-45^o} \right]^{1/4}. \tag{4}$$

The measured and simulated values are compared in Fig. 2

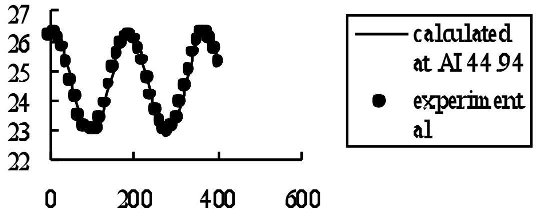


Fig. 3 ψ verses θ_a : YVO4 θ_c=136.01° for YVO4 with n_o=1.9929 and n_e=2.2154, θ_i = 44.94° (line:工 calculated •: measured) and θ_a = 7.24° .

四、Conclusion and discussion

The angle (θ_c) between the normal to the cleavage plane and the optic axis of YVO4 crystal at θ_i = 44.94° was obtained to be 136.01° and θ_a (Fig. 1) to be 7.24± 0.01°, as shown in Fig. 3, while θ_c was specified as 135° from the vendor (CASIX). In addition to determining the deviation of incident angle in a rotating element ellipsometry, the following three parameters can be obtained by fitting the measured $\tan\psi$ to the analytic solution of uniaxial crystals: the absolute value of n_o, n_e and directions of optical axis (θ_a and θ_c) in the laboratory frame. Since the resolving power of the system can be increased as the incident angle moves closer to the Brewster angle (the reflected intensity at 50° will be about 0.4% of the incident intensity), the system can be improved by using a sensitive detector or a higher power light source. It is our interest to extend the current experimental system to measure a material which consists of both linear and circular birefringence.

五、Acknowledgement

We like to thank NSC for grating the research. This research and last one has been combined and published in Journal of Physics: D [8]

、Appendix

This appendix is cited from reference [6] and [7]. The reflection geometry is shown in Fig. 1. The direction of optical axis is specified by angles θ_a and θ_c relative to the laboratory xyz, if $\vec{c}$ is the unit vector of optical axis, we can express it as

$$\vec{c} = c\alpha\, c\beta\, c\gamma\, c,$$

where $\alpha = \cos\theta_a \sin\theta_c$, $\beta = \sin\theta_a \sin\theta_c$ and $\gamma = \cos\theta_c$. Let the incident wave vector be $c\ \vec{i} + c_1 \vec{c}$, where K= k $n_i \sin\theta_I$, q_1= k $n_i \cos\theta_i$, for a wave number $k = \frac{\omega}{c}$ at incident angle θ_i. According to reference 11, we summarized the Fresnel reflection coefficients for uniaxial crystals of ordinary refractive index $n_o = \sqrt{\varepsilon_o}$ and extraordinary refractive index $n_e = \sqrt{\varepsilon_e}$ as follows;

$$e_{ss} = {}_o c c_1 - c_e\, c A E_y^e - c c_1 - c_e\, c E E_y^o]/ E$$

$$r_{sp} = {}_x n_i k\, c A E_x^e - B E_x^o\, d\, D$$

$$e_{pp} = x c_t\, {}_o c c_1 + c_e\, c E_x^o E_y^e - c c_1 + c_o\, c E_x^e E_y^o\, c]/ E - x$$

$$r_{ps} = {}_x n_i k\, c q_e - q_o\, c E_y^o E_y^e / D$$

The ordinary and extraordinary modes have wave vector normal components q_o, and q_e related to the medium as

$$q_e = c\sqrt{d} - \alpha\gamma K \Delta\varepsilon\, d\, c\varepsilon_o + \gamma^2 \Delta\varepsilon\, c,$$

$$q_o = \varepsilon_o k^2 - K^2, \quad c_t = c_1 + c\ \tan\theta_i$$

where $\Delta\varepsilon = \varepsilon_e - \varepsilon_o$, and

$$d = \varepsilon_o [\varepsilon_e (\varepsilon_o + \gamma^2 \Delta\varepsilon) k^2 - (\varepsilon_e - \beta^2 \Delta\varepsilon) K^2]$$

the corresponding electric field vectors $\mathbf{E}^o$ and $\mathbf{E}^e$ noted as

$$\mathbf{E}^o = N_o(-\beta q_o, \alpha q_o-\gamma K, \beta K) ,$$

$$\mathbf{E}^e = N_e(\alpha q_o^2-\gamma q_e K, \beta \varepsilon_o k^2, \gamma(\varepsilon_o k^2-q_e^2)-\alpha q_e K),$$

where N_o, N_e are the normalization factor, respectively. For simplicity, we also state the collective parameters as follows;

$$= z \frac{}{o} + \frac{}{1} + \tan\theta_i \, n \frac{o}{x} - \frac{o}{z}$$

$$1 = z \frac{}{e} + \frac{}{1} + \tan\theta_i \, n \frac{e}{x} - \frac{e}{z}$$

$$D = z\, q_1 + q_e \, nAE_y^e - z\, q_1 + q_o \, nBE_y^o$$

參考文獻

1. 趙于飛 國科會報告 NSC 87-2112-M-009-032 (1998)
2. Alonso M I and Garriga M 1995 *Appl. Phys. Lett.* **67** 596
3. Azzam R M A and Bashara N M, *Ellipsometry and Polarized light* 1992 (Amsterdam: North-Holland)
4. Aspnes D E 1980 *J. Opt. Soc. Am.* **70** 1275
5. Chao Y F, Lee W C, Hung C S and Lin J J 1998 *J. Phys. D.: Appl. Phys.* **31** 1968
6. Lekner J 1991 *J. Phys. : Condens. Matter* **3** 6121
7. 寇人傑, 預測單軸晶異向性介質之橢圓偏光參數, 國立交通大學光電所 85 碩士論文
8. Y. F. Chao, M.W. Wang and Z. C. Ko (in press: Journal of Physics D: applied physics)