行政院國家科學委員會專題研究計畫成果報告

以逐段線性技術求解混和整數規劃

A Global Optimization Algorithm for Mixed Integer Polynomial Programming Problems by Piecewise Linearization Techniques

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一、中文摘要

許多管理上的最佳化問題，都屬於混和整數多項式規劃問題，本專案計畫提出以逐段線性法求解此問題以一全域最佳解。此類問題目標式和限制式會包含帶有多次方的乘積項，決策變數可能是連續變數或整數，本法線性化技術是來自分段規劃法，先將多項式轉換成一連串絕對值項總和，再將這些絕對值項一一線性化，以將一個混和整數多項式規劃問題轉換成線性混和整數規劃問題。本專案計畫所提出的方法可求得問題的近似全域最佳解。本專案計畫的研究步驟分為四個段：

(1) 以連串的絕對值關係式表達逐段線性函數，並就其特性再加以分類。

(2) 用分類線般的特性，配合所發展的線性化技巧，解決各類逐段線性函數的最佳化問題。

(3) 將一非線性目標規劃或非線性整數規劃的問題轉換成分段式規劃的問題。

(4) 針對轉換後的逐段線性模式，發展出求取近似全域最佳解的技巧。

本研究將發展混和整數多項式規劃問題的演算法，並從例子測試與驗證本法與傳統方法的解題效果。

關鍵詞：多項式目標規劃法，混和整數規劃法，分段式規劃法，逐段線性函數

Abstract

Many management optimization problems can be formulated as a mixed integer polynomial programming (PP) problems. This project proposes a new method to solve a PP problem in which the objective function and the constraints contain product terms with exponents and decision variables could be continuous or integral. A linear programming relaxation is derived for the problem base on piecewise linearization techniques by first convert a polynomial term into the sum of absolute terms which are then linearized by goal programming techniques. A PP program is finally transformed into a linear mixed 0-1 program. The proposed method could reach a solution closing to a global optimum. The steps of solving PP problems are as

follows:

(1) Express a piecewise linear function by a series of absolute terms. Classifying the patterns of the line segments on the piecewise functions.
(2) Solve the optimization problem with various types of piecewise linear functions.
(3) Convert a PP problem into a separable programming problem. The converted separable program is then transformed as the optimization problem with piecewise linear functions.
(4) Solve the transformed piecewise linear programs to obtain approximately global optima.
(5) Some examples will be tested to compare the computational efficiency between proposed algorithm and current PP methods.

Keywords Polynomial Goal Programming, Mixed Integer Programming, Separable Programming, Piecewise Linear Function

二、緣由與目的

Signomial programming (SP) problems occur frequently in engineering design and management. Currently there are three approaches for solving a SP problem: 1. Geometric Programming (Beightler, 1976); 2. Reformulation Linearization Technique (Sherali and Tuncbilek, 1998); and 3. Multilevel Single Linkage Technique (Rinnooy and Timmer, 1987). A shortcoming of the first approach is that it can only find a local optimum for a program with high degree of difficulty. The inadequacy of the second approach is that it can only treat a SP problem where all exponent values of variables should be integral. While approach three often requires to solve a huge amount of nonlinear programs based on various starting points. In addition, these three approaches can only handle SP problems containing continuous variables. This study proposes an approximate algorithm to solve a mixed integer signomial program. Follows are the advantages of the proposed algorithm, compared with above three approaches: 1. It will converge to a solution as close as desired to a global optimum; 2. It can treat a SP problem where the exponents of variables could be real values; 3. It can solve a mixed integer SP problem as well as a SP program with continuous variables.

三、結論

This paper proposes an algorithm for solving mixed integer signomial program to find a solution closing to a global optimum. The proposed method first approximately converts a signomial term into the sum of absolute terms. These absolute terms are then linearized by employing the goal programming techniques. A mixed integer signomial program is finally transformed into a linear mixed 0-1 program solvable for finding a solution closing to global optimum. However, the accuracy of the above linearization procedure largely depends upon choosing proper break points for each variable. As the number of break points in a convex function (or a concave function) is increased, the number of deviational variables (or 0-1 variables) in approximating program also increased. There may have several ways for generating the break points in the linearization procedure, thereby tightening its representation at the expense of an increase in variable size. This poses a question of compromise that needs to be resolved. This issue is open to investigation. Here we present the basic machinery and techniques for solving a mixed integer signomial program. Further investigation necessary to understand of how best to utilizing this approach.

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