行政院國家科學委員會專題研究計劃成果報告

具時間窗的車輛途程問題之研究

An Investigation of the Vehicle Routing Problem with Time Windows

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一、中文摘要

本研究就顧客具有服務時窗要求之下，物流配送業者如何以多種不同車輛，在最經濟的成本之下完成配送任務進行探討。研究中提出數個具有時窗限制之多車種車輛途程與排程問題啟發式解法。為評估所提解法，我們修正文獻中未曾考慮時窗限制的多車種車輛途程與排程問題之節省啟發式解法以進行比較。以測試結果顯示我們的方法明顯優於文獻中未曾考慮時窗限制的多車種之節省啟發式解法。

關鍵詞：車輛途程問題、時窗限制、多車種

Abstract

This research is concerning the vehicle routing problem with multiple vehicle types and time window constraints. Several insertion-based savings method heuristics are presented. The heuristics are tested on twenty-four 100-customer problems modified from the literature. We did some experiment to demonstrate that heuristics with the consideration of a sequential route construction parameter would yield significantly better solution quality than all other heuristics tested.

Key words: vehicle routing, heuristics, time windows, heterogeneous fleets

二、緣由與目的

The classical fleet size and mix vehicle routing problem (FSMVRP) is a problem of simultaneously determining the composition and routing of a heterogeneous fleet of vehicles in order to service a pre-specified set of customers with known delivery demands from a central depot. Since the temporal aspect of vehicle routing problems has become increasingly important in realistic applications, this study extends the classical FSMVRP by imposing time window constraints on the customers and the central depot. The time window constraints considered in this paper constitute "hard" constraints. That is, a vehicle can not visit a customer beyond a specified latest starting service time and must wait if it arrives too early at a customer location. Clearly, the FSMVRP with time windows (FSMVRPTW) can also be regarded as a generalization of the classical vehicle routing problem with time windows (VRPTW).

We now state our problem in detail as follows. Let G = (V, A) be a directed graph with node set $V = N \cup \{0\}$ and arc set $A = \{(i, j) | i \in V, j \in V, i \neq j\}$, where $N = \{1, 2, \ldots, n\}$ denotes the customer set, and node 0 denotes the central depot. With each node $i \in V$ is associated a demand q_i, a service time s_i and a time window (a_i, b_i) except that q_0 and s_0 are zero. A distance matrix $[d_{i,j}]$ and a travel time matrix $[t_{i,j}]$ defined on the arc set are known. Moreover, we have a given number of vehicle types with known fixed costs and known capacities. Each type of vehicles is assumed to be available with infinite supply. The objective of the FSMVRPTW is to minimize the sum of the vehicle fixed costs and routing costs such that the following constraints are satisfied. (1) Each route begins and ends at the central depot. (2) Each customer in N is visited exactly once without violating the time window constraints. (3) The total demand of all customers served on a route can not exceed the capacity of the vehicle assigned to that route.

One perhaps may argue that the total cost seems to be an inappropriate sum of long-term and short-term costs. In fact, this surrogate cost should not result in any confusion if one really catches what he intends to plan. Depending on the planning purpose, the total cost approach can be justified as follows.

Case (a): Regard the FSMVRPTW as a short-term (or daily) issue.

A distribution manager might be interested in minimizing total cost for a daily service with known customer demands. In here we may treat the fixed cost term as operational related vehicle fixed costs, not the vehicle capital/purchasing costs. For example, servicing customers by using an owned fleet, we perhaps need to consider the following vehicle dependent fixed costs such as fixed dispatching costs, payment in driving different types of vehicles for drivers, and fees for the need of on-truck workers when using larger vehicles. Note that sunk costs such as purchasing/depreciation cost, insurance cost and tax should be excluded when one plans a routing schedule for a daily service. That is, costs considered here are not from an accounting viewpoint. Moreover, even if in the extreme case no vehicle related fixed cost is considered, one may simply set the vehicle fixed cost term mentioned in the paper to zero.

Case (b): Regard the FSMVRPTW as a mid-term (or long-term) issue.

Clearly, when a company intends to

purchase/lease a fleet of vehicles for future services, then the fleet size and mix problem becomes a mid-term planning issue. Customer attributes such as demands, time windows or others for a planning period are usually with uncertainties. How to estimate or forecast the possible needs is not the purpose of this paper and they will be assumed as given data. Once we view the FSMVRPTW as a mid-term issue, the primary purpose should be the finding of the best fleet composition; a routing schedule determined under this situation appears to be useless and an exact routing plan with known demands should be rescheduled for each day. Considering routing and scheduling as factors in mid-term planning is to possibly help in determining an appropriate heterogeneous fleet of vehicles for future services. For mid-term planning purpose, the costs such as purchasing/depreciation cost, insurance cost and tax mentioned in case (a) now have to be considered as a fixed cost term for each specific type of vehicles. A linkage between the aspects of short-term and mid-term should not be ignored as one makes a planning decision. Thus, a surrogate cost should not raise the question of an inappropriate sum of mid-term cost and short-term cost (or daily cost).

In consideration of the possible strengths and weaknesses of the methods for the FSMVRP and the VRPTW, we developed several insertion-based savings heuristics for solving the FSMVRPTW. Throughout the paper, 'vehicle' and 'route' will be used interchangeably.

According to the literature[1-4], heuristic methods for FSMVRP can be summarized as follows. (1) Adaptations of the Clarke and Wright savings algorithm[1]. (2) The giant tour partitioning approach[1] (route first-cluster second). (3) The matching based savings heuristics[2]. (4) The generalized assignment based heuristic[3]. (5) The sophisticated improvement based heuristic[4]. (6) composite heuristics[5]. Recently, Salhi and Sari[6] first proposed a multi-level composite heuristic for the multi-depot FSMVRP.

METHODOLOGY FOR THE FSMVRPTW

It is clear that a route may contain either only one customer or more than one customer. Let TYPE-I and TYPE-II represent the sets of routes containing only one customer and containing at least two customers, respectively. For an arbitrary directed TYPE-II route, say (0-*f*-…-*g*-0), we will call (*f*-…-*g*) *a generalized customer* and (*g*-…-*f*) *a reversed generalized customer*.

For the traditional savings algorithms based on the Clarke-Wright formula[8], any two routes to be combined must fall into one of the following three cases: (a) TYPE-I, TYPE-I; (b) TYPE-I, TYPE-II; (c) TYPE-II, TYPE-II. Knowing that insertion-type algorithms yield better solution quality in solving the VRPTW, we instead of the combining operation of CW-based algorithms by using the insertion point of view. Usually, all links of each route are potential insertion positions for a standard customer. Therefore, a good idea to solve the FSMVRPTW is that for a standard customer, a generalized customer, or a reversed generalized customer we should compute their possible resulting savings with respect to **each link** of other routes.

Our savings formulae are extensions of the formulae proposed by Golden et al.[1] for solving FSMVRP (details can refer to their paper.)

The feasibility conditions and modified savings formulae for solving the FSMVRPTW, and the improvement process are developed.

Feasibility conditions

To simplify the analysis, we assume that the travel time matrix $[t_{i,j}]$ satisfies the triangle inequality. Without loss of generality, the depot with no time window and each customer with zero service time are assumed.

Note that vehicle departure times from the central depot are decision variables. We will assume that initially the first customer on each constructed route is serviced at the earliest possible time. After the complete vehicle schedules have been created, we can compute the actual departure time for each vehicle from the central depot by eliminating any unnecessary waiting time.

Regarding the capacity feasibility condition for a pair of routes under consideration, we only require that the sum of their total demands can not exceed the largest vehicle capacity.

For the cases of (TYPE-I, TYPE-I) and (TYPE-I, TYPE-II), we simply try to insert a standard customer into a link of the other route under consideration. It is not hard to see that we need to check two conditions if a route only containing customer *k* is to be inserted into a candidate link (*i*, *j*).

For the case of (TYPE-II, TYPE-II), more complex operations in checking time feasibility are needed because of the presence of *a generalized customer* and of *a reversed generalized customer*. We first describe the operations for the generalized customer.

In case of considering the reversed generalized customer of an arbitrary route, we need first see whether the corresponding route examined in a reverse direction is still time-feasible. Other operations are similar to the insertion of a generalized customer.

Modified savings formulae

The savings formulas for VRPMVT described previously consider both the spatial costs and the vehicle acquisition costs. However, temporal constraints should not be ignored in solving

VRPMVTTW because route feasibility may be strongly affected by the time windows. To be less myopic, the temporal restriction should be a factor in determining which two routes to be selected. We now describe the modified savings formulas for each of the three different cases in the following.

Case **(1):** the insertion of a TYPE-I route into link $(i^{\mathrm{II}}, j^{\mathrm{II}})$ of a TYPE-II route.

Let k^{I} be the customer of a given TYPE-I route. For cost spent in actual traveling, this operation will reduce cost $2c_{0,k^{\mathrm{I}}} + c_{i^{\mathrm{II}},j^{\mathrm{II}}}$ by paying extra cost $c_{i^{\mathrm{II}},k^{\mathrm{I}}} + c_{k^{\mathrm{I}},j^{\mathrm{II}}}$. Thus, the net savings is $2c_{0,k^{\mathrm{I}}} + c_{i^{\mathrm{II}},j^{\mathrm{II}}} - c_{i^{\mathrm{II}},k^{\mathrm{I}}} - c_{k^{\mathrm{I}},j^{\mathrm{II}}}$. To maintain route feasibility for the future insertions, one possible way is that for customer j^{II} we try to minimize its pushed forward time due to the insertion of customer k^{I}. Therefore, we can compute the temporal opportunity cost by using $D^{new}_{j^{\mathrm{II}}} - D_{j^{\mathrm{II}}}$. In contrast to the CW savings $S_{i,j}$, our modified CW-savings (MS) for VRPMVTTW is given by

$$MS(i^{II}, k^{I}, j^{II}) = w \times (2c_{0,k^{I}} + c_{i^{II},j^{II}} - c_{i^{II},k^{I}} - c_{k^{I},j^{II}}) - (1-w) \times (D^{new}_{j^{II}} - D_{j^{II}}),$$

where $0 \le w \le 1$.

To consider the possible savings in vehicle acquisition costs, we define other modified formulas in a manner similar to those savings formulas for VRPMVT as follows.

Modified Combined Savings (MCS)

$$MCS(i^{II}, k^{I}, j^{II}) = MS(i^{II}, k^{I}, j^{II}) + F(z^{I}) + F(z^{II}) - F(z^{I} + z^{II}).$$

Modified Optimistic Opportunity Savings (MOOS)

$$MOOS(i^{II}, k^{I}, j^{II}) = MCS(i^{II}, k^{I}, j^{II}) + F\big(P(z^{I} + z^{II}) - z^{I} - z^{II}\big).$$

Modified Realistic Opportunity Savings (MROS)

$$MROS(i^{II}, k^{I}, j^{II}) = MCS(i^{II}, k^{I}, j^{II}) + \delta(\tau)F'\big(P(z^{I} + z^{II}) - z^{I} - z^{II}\big)$$

MROS with a route shape parameter λ

$$MROS_{\text{-}\lambda}(i^{II}, k^{I}, j^{II}) = MROS(i^{II}, k^{I}, j^{II}) + w \times (\lambda - 1) \times c_{i^{II},j^{II}}.$$

Case **(2)**: the insertion of a TYPE-I route into a link of another TYPE-I route.

This situation is special instances of *Case* **(1)** if we let either i or j in above equations be the central depot. For the purpose of being used later, we denote the related savings formulas for this case by substituting the superscripts I and II in the above equations with the respective notations I_1 and I_2. For example, the modified combined savings will be given by

$$MCS(i^{I2}, k^{I1}, j^{I2}) = MS(i^{I2}, k^{I1}, j^{I2}) + F(z^{I1}) + F(z^{I2}) - F(z^{I1} + z^{I2}).$$

Case **(3)**: the insertion of generalized customer $(f^{\mathrm{II}_1} - \ldots - g^{\mathrm{II}_1})$ of a TYPE-II route into link $(i^{\mathrm{II}_2}, j^{\mathrm{II}_2})$ of a distinct TYPE-II route.

For cost spent in actual traveling, this operation will save a quantity of $c_{0,f^{\mathrm{II}_1}} + c_{g^{\mathrm{II}_1},0} + c_{i^{\mathrm{II}_2},j^{\mathrm{II}_2}}$ by paying extra cost $c_{i^{\mathrm{II}_2},f^{\mathrm{II}_1}} + c_{g^{\mathrm{II}_1},j^{\mathrm{II}_2}}$. Thus, the net savings is

$c_{0,f^{\mathrm{II}_1}} + c_{g^{\mathrm{II}_1},0} + c_{i^{\mathrm{II}_2},j^{\mathrm{II}_2}} - c_{i^{\mathrm{II}_2},f^{\mathrm{II}_1}} - c_{g^{\mathrm{II}_1},j^{\mathrm{II}_2}}$.

In contrast to the insertion of a standard customer, the insertion of a generalized customer tends to result in a bigger pushed forward time for customer j^{II_2}. To be on a equal computation basis, we change the temporal opportunity cost used in the above two cases to $(D^{new}_{j^{\mathrm{II}_2}} - D_{j^{\mathrm{II}_2}})/cus^{\mathrm{II}_1}$, where cus^{II_1} is the number of customers that consists of the generalized customer $(f^{\mathrm{II}_1} - \ldots - g^{\mathrm{II}_1})$. Accordingly, the modified CW savings for the case of (TYPE-II, TYPE-II) can be expressed as follows.

$$MS(i^{II2}, f^{II1}, g^{II1}, j^{II2}) = w \times (c_{0,f^{II1}} + c_{g^{II1},0} + c_{i^{II2},j^{II2}} - c_{i^{II2},f^{II1}} - c_{g^{II1},j^{II2}}) - (1-w) \times (D^{new}_{j^{II2}} - D_{j^{II2}})/cus^{II1}.$$

All other modified savings formulas for this case can be obtained in a way similar to those formulas in *Case* **(1)**. For example, the modified combined savings is given by

$$MCS(i^{II2}, f^{II1}, g^{II1}, j^{II2}) = MS(i^{II2}, f^{II1}, g^{II1}, j^{II2}) + F(z^{II1}) + F(z^{II2}) - F(z^{II1} + z^{II2}).$$

Composite improvement scheme

As mentioned previously, several solutions built in the construction phase are recorded and passed to the improvement phase. It is known that a worse solution can be reached during an improving process for methods such as simulated annealing and tabu search in order to avoid trapping in a local minimum. With the same consideration, our composite improvement scheme consists of the following two major [redacted] during the improving process. Computational [redacted] demonstrate that our insertion-based [redacted] strategy is encouraging. The introduction of parameter η into our heuristics has significantly improved the solution quality.

In the paper, performance evaluation is through the comparisons of solution results for different heuristics. Justifying them by using a tight lower bound to the FSMVRPTW would be more meaningful. From a practical viewpoint, other variants of the FSMVRPTW are also worth studying further. The model presented in the paper has been extended in order to satisfy more practical needs such as multiple uses of vehicles, limited private fleets with an outside carrier option, vehicle accessibility to customer location and transfer points. The extended version is now acting as a core module of a

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