Available online at www.sciencedirect.com

ScienceDirect

Optics Communications 281 (2008) 1997–2001

OPTICS COMMUNICATIONS

www.elsevier.com/locate/optcom

Characterization of one-dimension edge roughness from far-field irradiance at subwavelength-scale precision

Shu-Chun Chu [a,*], Jyh-Long Chern [b]

[a] Department of Physics, National Cheng Kung University, No.1, University Road, Tainan City 701, Taiwan

[b] Department of Photonics, Institute of Electro-Optical Engineering, Microelectronics and Information System Research Center, National Chiao Tung University, Hsinchu 300, Taiwan

Received 19 July 2007; received in revised form 4 December 2007; accepted 14 December 2007

Abstract

It is proposed and numerically verified that the one-dimensional edge roughness of test sample can be characterized by far-field irradiance measurement at subwavelength-scale precision with a constructed aperture playing as an optical ruler. The precision of the proposed scheme of measurement could be better than 3%, even when the edge roughness is in subwavelength-scale. The influence of sample thickness on the proposed measurement scheme is also investigated and considered.

PACS: 42.25.Fx; 07.60.Ly; 42.30.Kq

Keywords: Diffraction; Subwavelength; Measurement

1. Introduction

Subwavelength photonics devices and nanotechnology have great potential in advancing the content of science and technology [1,2]. The fields of subwavelength optics [3] and photonic crystals have continued to progress [4]. Thus, measurements and characterizations of subwavelength scales are important and need to be further developed. Subwavelength measurements can be categorized in two different fields. One is spatial measurement, which measures the subwavelength structure while stationary, i.e., to measure the spatial variation of the test sample from point to point in the precision of subwavelength. The other is temporal measurement, which measures the temporal variation of the test sample in the precision of the subwavelength-scale. Several scanning probe microscope (SPM) skills and applications have been developed to subwavelength spatial measurement [5,6]. Also, advances in biomedical technology have highlighted the importance of subwavelength temporal measurement of bio-particles [7,8]. However, to measure an element using SPM skills, a high degree of care in manipulation must be exercised during measurement to avoid damaging the test sample or the equipment by unintentional carelessness. Measuring the structure in the far-field with optical methods can avoid such problems.

In the past, the extraction of subwavelength information from far-field measurement was generally believed to be very difficult or perhaps impossible. This was because it is commonly stated that the information of the subwavelength details of the illuminated object is carried by evanescent components, which are exponentially attenuated in the course of propagation and thus could not be observed in far-field. However, what we concerned here is the situation that the diffraction aperture is exceeding several wavelengths and the medium of the aperture is homogeneous medium. Recently, however, several achievements have demonstrated that retrieving subwavelength information in the far-field is possible and could be considered to be

* Corresponding author. Tel.: +886 6 2757575; fax: +886 6 274 7995.
E-mail address: scchu@mail.ncku.edu.tw (S.-C. Chu).

0030-4018/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2007.12.032

accomplished in practice. First, Selci and Righini demonstrated experimentally that in the far-field region, when the width of slit exceeds several wavelengths, subwavelength variation can still be detected and its far-field characteristics of integrative intensity and diffracted intensity are consistent with the predictions of diffraction theory, even when the slit variation is in the order of 1/100 of a wavelength [9]. Their experiment demonstrated the possibility of retrieving subwavelength information from far-field characteristics. It has been shown that the subwavelength temporal variation of a simple case, i.e., a one-dimensional subwavelength temporal variation of a rectangular aperture, can be determined from a far-field characteristic, irradiance, with a precision better than 1 nm [10]. Furthermore, detection of subwavelength temporal variation from another far-field characteristic, far-field diffraction pattern, has also been proposed [11]. Detection sensitivity enhancement of a subwavelength temporal signal by using an embedded-aperture interferometer configuration has also been investigated [12]. Usually, temporal and spatial information are correlated in physical objects. It is of interests to express the possible implementation of subwavelength spatial variation in terms of far-field characteristics, and thus to retrieve the subwavelength spatial variation from far-field measurement. In this paper, a slit-like aperture is proposed as an optical ruler, which could be used to identify the spatial edge roughness variation in the precision of subwavelength-scale from only far-field irradiance measurement.

This paper is organized as follows. The basic scheme of the constructed aperture roughness measurement is illustrated in Section 2. The sample thickness influences are considered in Section 3. The simulation results and discussions are provided in Section 4. The conclusion is presented in Section 5.

2. Constructed aperture roughness measurement system

Referring to Fig. 1, a constructed aperture measurement system behaving as an optical ruler was proposed to retrieve the edge roughness of the test sample. The diffraction aperture Σ was constructed by a slit-like aperture and the margin of the test sample, where the width of the slit-like aperture along the η direction was denoted as b. A monochromatic plane wave of wavelength λ was assumed to be orthogonally illuminated onto the constructed aperture Σ. The margin of the test sample was situated relative to the straight margin of the slit-like aperture in a base width a_0. The dimensions of a_0 and b were both above several wavelengths for satisfying the assumptions of the scalar diffraction theorem. The front view of the constructed aperture was shown in Fig. 1b. A detector with the size $W \times W$ was positioned behind the aperture $\sum$ at a distance Z_0 in the far-field region, or by introducing a focal lens just behind the aperture $\sum$, situated the detector at the focal plane of the lens. Thus the diffraction pattern on the detector plane could be evaluated by the Fraunhofer diffraction [13]. The overall power collected by the detector was denoted as P_z.

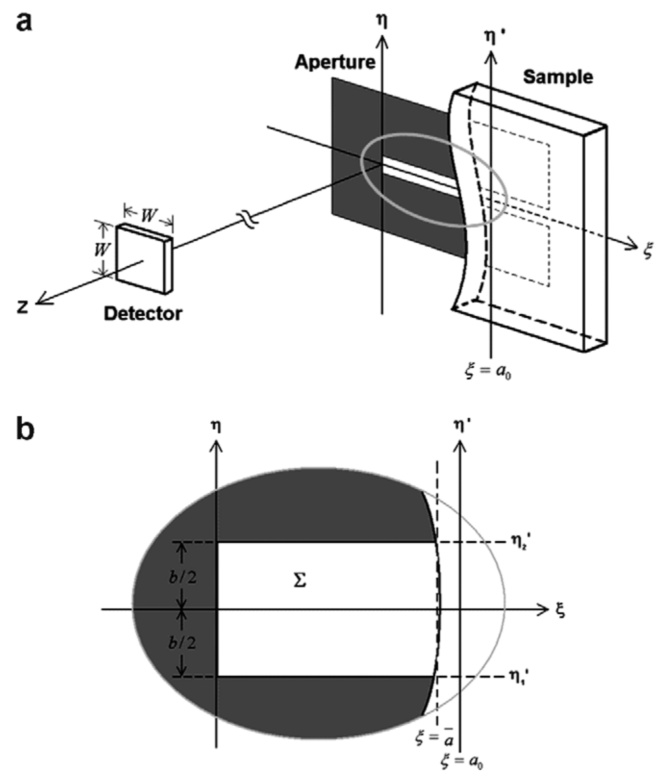

Fig. 1. Schematic diagrams of (a) constructed aperture edge roughness measurement system, (b) a front view of the constructed aperture of the system. The detail description of the figure could be found in Section 2, paragraph 1.

To illustrate the main idea of the proposed scheme, we recalled that while the width deviation from the base width a_0, Δa, of a rectangular aperture in a dimension $a_0 \times b$, was in a scale of subwavelength, the aperture width deviation Δa could be retrieved from a physical parameter, i.e., the derivative intensity $\frac{dP_z}{da}\big|_{a=a_0}$ in a precision better than 1 nm [10] by the following approximation, i.e.,

$$\Delta a \cong \Delta P_z \bigg/ \left(\frac{dP_z}{da}\bigg|_{a=a_0} \right), \tag{1}$$

where ΔP_z is the deviation of the overall power comparing to that of the base width a_0, $P_z(a_0)$, i.e. $\Delta P_z = P_z(a) - P_z(a_0)$. The key idea of the proposed measurement is that, while the width of the optical ruler b is small enough as compared to the spatial variation scale of the edge roughness T_d, the margin of the test sample could be estimated as a straight surface. The width of the constructed aperture (or say the averaging width) was denoted as $\bar{a}$, which could be retrieved by far-field irradiance measurement. The procedures are stated as below.

First, we derive the derivative intensity $\frac{dP_z}{da}\big|_{a=a_0}$ of an aperture varying in one-dimension with only one side, in a correspondence to the constructed measurement system here. It is for the reason that while measuring the edge roughness, the test sample will be moved in the η direction. Because the edge roughness is varied spatially from point to point, the width of the constructed aperture Σ, a, will be changed as the sample is moved. The constructed aperture will vary only on one side and the derivative intensity of measured power, $\mathrm{d}P_z/\mathrm{d}a$, could be deduced as to be shown below.

The exact form of the diffraction pattern on the detector U can be derived from Fraunhofer diffraction, and the intensity is $|U^2|$. The overall power collected by the detector is the integration of the intensity over all the complete detector area, which is also the function of the varying aperture width a,

$$P_z(a) = 4\left(\frac{\lambda z_0}{\pi}\right)^2 f_p(W/2,a) f_p(W/2,b), \tag{2}$$

where the function $f_p(X,a) = \int_0^X \frac{\sin^2[kax/2z_0]}{x^2} = \frac{ka}{2z_0} Si(\frac{kaX}{2z_0}) - \sin^2(\frac{kaX}{2z_0})/X$. $k = \frac{2\pi}{\lambda}$ is the wave number, and Si is the sine-integral function. The derivative intensity is the derivation of overall power P_z over the varying aperture width a, which can be derived as

$$\frac{\mathrm{d}P_z}{\mathrm{d}a} = 8\frac{Z_0}{k} f_p(W/2,b) Si\left(\frac{kaW}{2Z_0}\right). \tag{3}$$

The numerical value of the derivative intensity $\frac{\mathrm{d}P_z}{\mathrm{d}a}|_{a=a_0}$ can be evaluated from Eq. (3), and can be substituted into Eq. (1) to retrieve the margin position of the test sample as $\bar{a} = a_0 + \Delta a$.

3. Analysis of the thickness effect of test sample

In Section 2, the constructed aperture is estimated as an ideal planar aperture. However, in a real situation, the test sample will have a thin thickness d inevitably. As shown in the Fig. 2, the actual constructed aperture Σ' will have an inclination angle, $\phi = \arctan(d/a_0)$, to the incident plane wave. The introducing power deviation on the detector D cannot be neglected as comparing to the power deviation caused by the constructed aperture width variation in the subwavelength-scale. In other words, a solution to recover the influence caused by the sample thickness has to be considered. Referring to Fig. 2, considering that a plane wave passes through a plane rectangular aperture Σ, the diffraction optical field on the detector D behind the aperture in the far-field distance Z_0 can be evaluated by Fraunhofer diffraction, i.e.,

$$U_p(x,y) = \frac{C_0(x,y)}{\lambda Z_0} \int\int_\Sigma U_0(\xi,\eta) \times \exp\left[-j\frac{2\pi}{\lambda Z_0}(x\xi + y\eta)\right] \mathrm{d}\xi \mathrm{d}\eta, \tag{4}$$

where for an aperture, $U_0(\xi,\eta) = 1$ and $C_0(x,y)$ is the phase term. Next, considering the situation that the constructed aperture Σ' has an inclination angle ϕ to the incident plane wave, the diffraction optical field on the plane orthogonally behind the aperture Σ' in the far-field distance Z_0 can be evaluated as an oblique plane wave incident on a plane aperture Σ', i.e.,

$$U_T(x',y) = \frac{C_0(x',y)}{\lambda Z_0} \int\int_{\Sigma'} U'(\xi',\ \eta) \times \exp\left[-j\frac{2\pi}{\lambda Z_0}(x'\xi' + y'\eta)\right] \mathrm{d}\xi' \mathrm{d}\eta, \tag{5}$$

where $U'(\xi',\eta) = U_0(\xi',\eta)\exp[jk\xi'\sin\phi]$, for an aperture, $U_0(\xi',\eta) = \Psi$, $\Psi = \cos\phi$ is tilt factor and $\exp[jk\xi'\sin\phi]$ is the relative phase of the optical field at the constructed aperture Σ'. Substituting $U'(\xi',\eta)$ into Eq. (5), we have

$$U_T(x',y) = \frac{C_0(x',y)}{\lambda Z_0} \int\int_{\Sigma'} U'(\xi',\eta) \times \exp\left\{-j2\pi\left[\left(\frac{x'}{\lambda Z_0} - \frac{\sin\phi}{\lambda}\right)\xi' + \frac{y\eta}{\lambda Z_0}\right]\right\} \mathrm{d}\xi' \mathrm{d}\eta. \tag{6}$$

With Eqs. (6) and (4), it can be seen that by rotating the detector relative to the constructed aperture in an angle $\phi = \arctan(d/a_0)$, the detector will be parallel to and perpendicularly behind the constructed aperture Σ' at a distance Z_0. The sample thickness induced light-oblique incidence will cause a shift of diffraction pattern. If we replace x' with $x'' + \Delta x'$, $\Delta x' = Z_0 \sin\phi$, we have

$$U_T(x'',y) = \frac{C_0(x'',y)}{\lambda Z_0} \int\int_{\Sigma'} U'(\xi',\eta) \times \exp\left[-j\frac{2\pi}{\lambda Z_0}(x''\xi' + y'\eta)\right] \mathrm{d}\xi' \mathrm{d}\eta, \tag{7}$$

Referring to Fig. 2, and comparing Eqs. (4) and (7), this means that if we position the detector at a new position, the only difference in these two equation is the diffraction aperture width of constructed aperture Σ' that effective aperture width is a/Ψ. The derivative intensity of the detector at a new position will be

$$\frac{\mathrm{d}P_z}{\mathrm{d}a} = 8\frac{\Psi Z_0}{k} f_p(W/2,b) Si\left(\frac{kaW}{2Z_0\Psi}\right), \tag{8}$$

and can be substituted into Eq. (1) to retrieve the margin position of the test sample as $\bar{a} = a_0 + \Delta a$. This means that if we position the detector at a new position, which is:

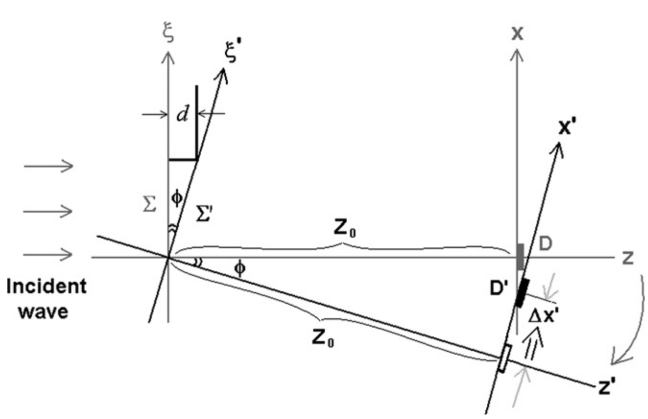

Fig. 2. Schematic diagram of a side view of the constructed aperture edge roughness measurement system. The detail description of the figure could be found in Section 3, paragraph 1.

(i) Rotating the original detector D relative to the aperture in an angle ϕ to be parallel to the constructed aperture Σ', and
(ii) Shifting it in a distance $\Delta x' = Z_0 \sin\phi$ along the direction of $+x'$-axis.

The corresponding derivative intensity of the detector at a new position could be deduced, as shown in Eq. (8), and substituted into Eq. (1) to retrieve the margin roughness of the test sample. To be specific, in $\xi - z$ plane, the center position of the original detector was $(Z_0, 0)$ and that of new detector was $(Z_0\cos\phi + \Delta x\sin\phi,\ Z_0\sin\phi + \Delta x\cos\phi)$. Moreover, while the thickness of the test sample is small compare to the aperture width, tilt factor $\Psi \cong 1$; the influence of Ψ is small and thus can be further neglected.

4. Simulation verifications

The feasibility of the proposed edge roughness measurement will be numerically demonstrated as below. We first set the edge roughness of test sample, and use a base width $a_0 = 50$ μm to evaluate the base overall power $P_z(a_0)$. The width of the optical ruler, b, is chosen as 6 μm, which is about 10 times the size of the light source wavelength, 632.8 nm that considered here. The width of the detector W is also 50 μm, and the detector is placed behind the aperture at a distance of 30 cm. While moving the test sample, the corresponding overall power variation will be substituted into Eq. (1) to retrieve Δa, the deviation from edge position to the base width a_0. The exact edge position of the test sample is simply the sum of the base width and the deviation Δa.

Without loss of generality, two edge roughness profiles, i.e., sine variation $a(\eta) = a_0 + \alpha \times \sin(2\pi\eta/T_d)$ and quasi-periodic variation $a(\eta) = a_0 + \frac{\alpha}{2} \times \sin(2\pi\eta/T_d) + \frac{\alpha}{2} \times \sin(2\sqrt{2}\pi\eta/T_d)$ are used to simulate the edge roughness, and the thickness of the test sample was taken as one wavelength. The amplitude of the edge roughness fluctuations, α, is set to 10 nm, and T_d is the spatial variation scale of the surface roughness.

The simulation process is summarized here. The edge form of constructed aperture is varying as the pre-setting function, $a(\eta)$. The corresponding collected power on the detector D' is evaluated numerically, and then substitute the overall power variation ΔP_z into Eq. (1) to get the width deviation Δa from base width a_0, then we get the estimated edge roughness form: $\bar{a} = a_0 + \Delta a$ and compare it with the pre-setting center roughness value of the constructed aperture. The results of the proposed edge roughness measurement of different spatial scale T_d are shown in Fig. 3. Three different spatial variation scales, $T_d = 2b$, $5b$ and $10b$, are used to explore the feasibility and the precision limitation of the proposed measurement. The deduced results of two edge roughness forms are shown separately in Fig. 3a and b by comparing the width deviation Δa to that of the pre-setting value, and the maximum error percentage of the results are shown in the figures. The pre-setting edge roughness is plotted in thin lines as reference, spatial variation scale $T_d = 2b$ is plotted in gray thin lines, $T_d = 5b$ is plotted in thick lines, and $T_d = 10b$ is plotted in dot lines.

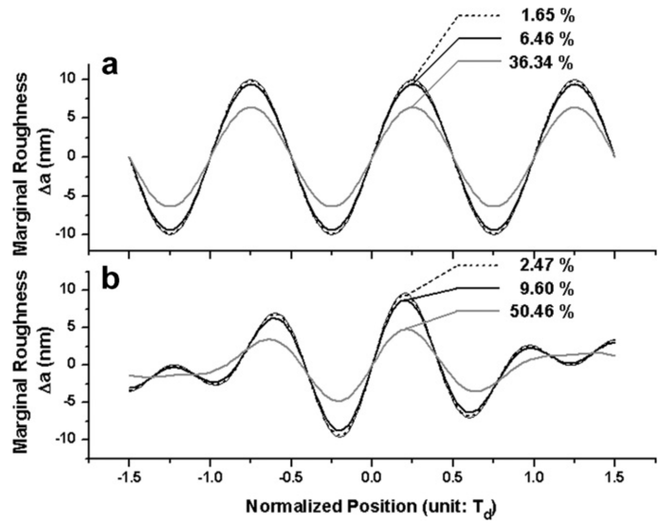

Fig. 3. The retrieving results of two different edge roughness profiles: (a) sine variation. (b) quasi-periodic variation.The detail description of the figure could be found in Section 4, paragraph 3.

It can be seen that one-dimensional edge roughness can be measured by far-field irradiance measurement with the proposed approach. The relation between the width of slit-like aperture b and the spatial variation scale of the edge roughness T_d determine the measurement precision of the proposed scheme. While the width of the optical ruler is 1/5 of the spatial variation scale T_d, the maximum error of the proposed edge roughness measurement will exhibit a percentage below 10%. If the width of the optical ruler is 1/10 of the spatial variation scale T_d, the maximum error percentage will be further reduced to below 3%. From our simulation results, the measuring precision of roughness varying in an amplitude 10 nm is better than 1 nm, while the width of optical ruler is smaller than 1/5 of the roughness spatial variation scale T_d.

5. Conclusions

In conclusion, a characterization scheme to retrieve the edge roughness in a precision of subwavelength-scale by far-field irradiance measurement of a constructed aperture configuration has been numerically demonstrated be having a reasonable error ratio below 3%. Better results can be obtained by choosing an optical ruler with a shorter width. Besides, we should note that while using a shorter wavelength, the restriction of optical ruler width could be reduced. It means that by using an optical ruler with a shorter wavelength: (1) a higher precision can always be achieved for measuring the same sample, and (2) the restriction of the spatial variation scale of the test sample will be released. A further impact message contained in out results is we provided an evidence of the possibility to retrieve spatial subwavelength-scale information from far-field measurement.

On the other hand, the limitation of the propose roughness measurement should be addressed. First, in this preliminary study, what we addressed is a very thin sample with finite thickness or samples having same edge roughness in the finite thickness, thus in such finite region the edge roughness could be estimated as a same value to be retrieved. Secondly, the proposed roughness retrieving scheme is based on the assumption of the scalar diffraction theory, i.e., what we considered is that constructed aperture is exceeding several wavelengths and the medium of the constructed aperture is homogeneous medium. Indeed, while the diffraction structure is not large enough comparing with incident light wavelength or when the diffraction structure is active medium, the interaction between the light and diffraction edge could not be ignored. Thus for predicting the diffraction pattern, scalar theory is not enough and a rigorous treatment is needed [3]. Furthermore, it should be noted that the proposed measurement is still workable even when the fluctuation amplitude of the edge roughness, α, is increased to the value of one wavelength. And, the detector width optimization [10] or the embedded-aperture interferometer configuration [12] could be further implemented to increase the detection sensitivity. The extension of the proposed measurement to retrieve the whole two-dimensional surface roughness by far-field measurement is on the progress and will be further published elsewhere.

Acknowledgement

This work was supported in part by a grant from the National Science Council of Taiwan, R.O.C., under contract No. NSC 96-2112-M-006-019-MY3.

References

[1] Z. Bomzon, G. Biener, V. Kleiner, E. Hasman, Opt. Lett. 27 (2002) 188.
[2] T. Thio, K.M. Pellerin, R.A. Linke, H.J. Lezec, T.W. Ebbesen, Opt. Lett. 26 (2001) 1972.
[3] W.L. Barnes, A. Dereux, T.W. Ebbesen, Nature 424 (2003) 824.
[4] P.-T. Lee, J.R. Cao, S.-J. Choi, Z.-J. Wei, J.D. O'Brien, P.D. Dapkus, Appl. Phys. Lett. 81 (2002) 3311.
[5] L.S.C. Pingree, M.C. Hersam, M.M. Kern, B.J. Scott, T.J. Marks, Appl. Phys. Lett. 85 (2004) 344.
[6] K. Nakamoto, C.B. Mooney, S.-I. Kitamura, Jpn. J. Appl. Phys. 43 (2004) 2686.
[7] B. Niggemann, T.L. Drell IV, J. Joseph, C. Weidt, K. Lang, K.S. Zaenker, F. Entschladen, Exp. Cell Res. 298 (2004) 178.
[8] S. Cooper, BMC Cell Biol. 5 (2004) 35. <http://www.biomedcentral.com/1471-2121/5/35>.
[9] S. Selci, M. Righini, Opt. Lett. 27 (2002) 1971.
[10] S.-C. Chu, J.-L. Chern, Opt. Lett. 29 (2004) 1045.
[11] S.-C. Chu, J.-L. Chern, JOSA A 22 (2005) 335.
[12] S.-C. Chu, J.-L. Chern, JOSA A 22 (2005) 1600.
[13] J.W. Goodman, Introduction to Fourier Optics, second ed., McGraw-Hill, New York, 1996, p. 74.