Transactions Letters

Adaptive Two-Stage GSC-Based PIC Detection for Time-Varying MIMO Channels

Yinman Lee, *Member, IEEE,* and Wen-Rong Wu, *Member, IEEE*

***Abstract*— Adaptive parallel interference cancellation (PIC) has been recently proposed for the signal detection in multiple-input multiple-output (MIMO) systems. However, it suffers from error propagation when operated in time-varying channels. In this letter, an adaptive two-stage PIC with the minimum variance (MV) criterion is proposed to solve the problem. Adaptation with the MV criterion is realized with a decision feedback generalized sidelobe canceller (DFGSC). In the first-stage cancellation, a special structure involving dual DFGSCs is developed. All adaptation operations are implemented with the least-mean-square (LMS) algorithm. Simulations show that the proposed adaptive PIC detection can significantly outperform the conventional adaptive PIC detection in time-varying MIMO channel environments.**

***Index Terms*— Multiple-input multiple-output (MIMO), parallel interference cancellation (PIC), least-mean-square (LMS) algorithm, generalized sidelobe canceller (GSC).**

I. Introduction

In recent years, much attention is paid to the development of multiple-input multiple-output (MIMO) systems. With multiple antennas at both the transmitter and the receiver, the spectral efficiency of a communication system can be increased dramatically [1]. A successive interference cancellation (SIC) approach, known as the vertical Bell Laboratories layered space-time (V-BLAST) system, is commonly used to achieve a substantial portion of the Shannon capacity for MIMO channels [2]. However, the V-BLAST algorithm requires high computational complexity, and the ordering operation inherent in the SIC structure often increases the processing delay and restricts the use of adaptive realization. Generally, there is still no efficient way for the V-BLAST system to work in time-varying channel environments.

Lately, parallel interference cancellation (PIC) detection schemes were proposed for the signal detection in MIMO systems [3]-[6]. In contrast to SIC, PIC detects different data symbols from different transmit antennas in parallel and it is generally implemented with a multistage structure. It has the advantages of low computational complexity and low processing delay. Since PIC does not require the ordering operation, it is more adequate for adaptive implementations. The conventional adaptive PIC can provide satisfactory performance when the channel is static or varies only slightly, but its performance can be significantly degraded in ordinary changing environments. This is due to the error propagation effect inherent in the multistage PIC structure.

In this letter, we propose a new adaptive PIC scheme to improve the MIMO detection performance, especially in time-varying environments. Our emphasis is on the two-stage PIC throughout the letter. The optimization is based on the minimum variance (MV) criterion [7]. The MV detector can be realized adaptively with the generalized sidelobe cancellation (GSC) structure [8]. However, the conventional adaptive GSC suffers from the problems of slow convergence and lack of robustness. Recently, a decision feedback generalized sidelobe canceller (DFGSC) has been proposed and the problems inherent in GSC are solved successfully [9]. Here, we extend the use of the DFGSC. We employ a dual-DFGSC structure and propose an adaptive DFGSC-based PIC for the MIMO signal detection. This can effectively outperform the conventional adaptive PIC under time-varying channels. All adaptations are based on the simple yet efficient least-mean-square (LMS) algorithm. This will keep the overall computational complexity at a low level and make the proposed scheme feasible for real-world applications. Convergence analysis in time-varying environments is also provided. Simulation results confirm that the proposed adaptive PIC detection can perform significantly better than the conventional adaptive PIC detection in changing channel environments.

Throughout the letter, we use the superscripts $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ to denote conjugation, transposition, and Hermitian transposition, respectively. Also, the operators $\mathrm{Dec}\{\cdot\}$, $\mathrm{E}\{\cdot\}$, $\|\cdot\|$, $\mathrm{Diag}\{\cdot\}$, and $\mathrm{Tr}\{\cdot\}$ represent decision, statistical expectation, two-norm, diagonal matrix construction, and matrix trace operations, respectively.

II. MIMO Signal Model and Parallel Interference Cancellation (PIC)

Consider a wireless communication system with M antennas at the transmitter and N antennas at the receiver, assuming

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Y. Lee is with the Graduate Institute of Communication Engineering, National Chi Nan University, Puli, Taiwan, R.O.C. (e-mail: ymlee@ncnu.edu.tw).

W.-R. Wu is with the Department of Communication Engineering, National Chiao Tung University, Hsinchu, Taiwan, R.O.C. (e-mail: wrwu@faculty.nctu.edu.tw).

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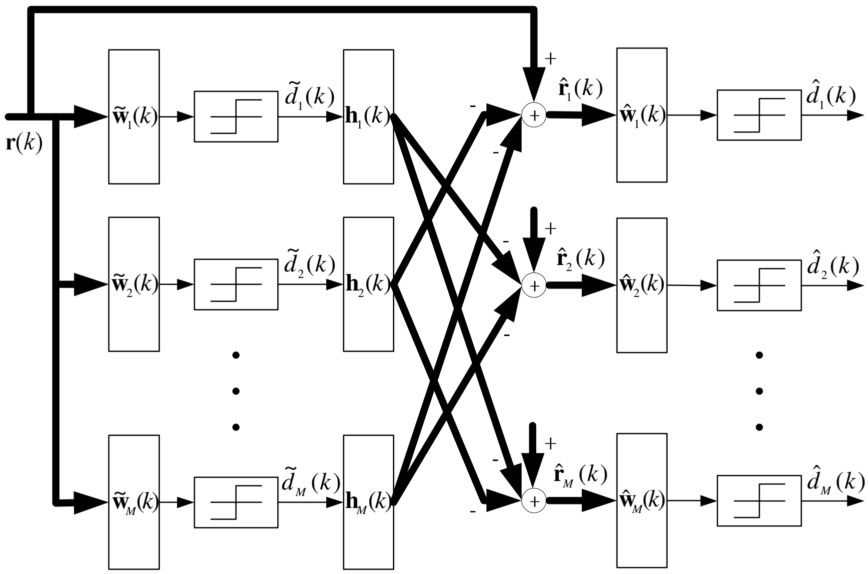

Fig. 1. General two-stage PIC detection scheme for MIMO systems.

$M \leq N$. We define $h_{nm}(k)$ as the flat channel response from the transmit antenna m to the receive antenna n at time instant k, with $1 \leq m \leq M$ and $1 \leq n \leq N$. The M transmitted data symbols at time instant k can be collected into an $M \times 1$ vector, denoted as $\mathbf{d}(k) = [d_1(k) \;\; d_2(k) \;\; \cdots \;\; d_M(k)]^T$. Let $\mathbf{h}_m(k)$ be a vector containing the channel response for $d_m(k)$, i.e.,

$$\mathbf{h}_m(k) = [h_{1m}(k) \;\; h_{2m}(k) \;\; \cdots \;\; h_{Nm}(k)]^T. \tag{1}$$

Then the complete $N \times M$ channel matrix for the transmission of $\mathbf{d}(k)$ can be presented as $\mathbf{H}(k) = [\mathbf{h}_1(k) \;\; \mathbf{h}_2(k) \;\; \cdots \;\; \mathbf{h}_M(k)]$. The data symbols received by all the N antennas yield an $N \times 1$ signal vector $\mathbf{r}(k)$, given by

$$\mathbf{r}(k) = \mathbf{H}(k)\mathbf{d}(k) + \mathbf{n}(k) \tag{2}$$

where $\mathbf{n}(k)$ is an $N \times 1$ complex Gaussian noise vector with zero mean and equal variance in each dimension. In addition, all the noise components are assumed to be independent.

To detect the transmitted data symbols, the multistage PIC is applied. As mentioned previously, we focus on the development of a two-stage structure. The block diagram of a general two-stage PIC detection scheme for MIMO systems is plotted in Fig. 1. As shown, the estimate for the mth element of $\mathbf{d}(k)$ in the first-stage output is expressed as $\tilde{d}_m(k) = \mathrm{Dec}\{\tilde{\mathbf{w}}_m^H(k)\mathbf{r}(k)\}$, where $\tilde{\mathbf{w}}_m(k)$ is a first-stage $N \times 1$ weight vector for the estimation of $d_m(k)$. Conventionally, to avoid noise enhancement, $\tilde{\mathbf{w}}_m(k)$ is optimized through the minimum mean-squared-error (MMSE) criterion [10]. We define the MMSE cost function (in the decision-directed mode) for the optimization of $\tilde{\mathbf{w}}_m(k)$ as

$$\min_{\tilde{\mathbf{w}}_m(k)} \;\; \mathrm{E}\{|e(k)|^2\} = \min_{\tilde{\mathbf{w}}_m(k)} \;\; \mathrm{E}\{|\tilde{d}_m(k) - \tilde{\mathbf{w}}_m^H(k)\mathbf{r}(k)|^2\}. \tag{3}$$

It is well understood that the optimum solution for (3) involves a matrix inversion, which is not desirable in real-world applications. This problem becomes more troublesome when channels are time-varying. As an alternative, the LMS algorithm is adopted to obtain the weight vector recursively. The update equation for $\tilde{\mathbf{w}}_m(k)$ with the LMS algorithm is given by [11]

$$\tilde{\mathbf{w}}_m(k+1) = \tilde{\mathbf{w}}_m(k) + \mu\mathbf{r}(k)e^*(k) \tag{4}$$

where μ is the step size controlling the convergence rate, and $e(k)$ is an error signal as that given in (3). Also shown in Fig. 1, the input vector to the mth branch in the second stage, denoted as $\hat{\mathbf{r}}_m(k)$, is constructed as

$$\hat{\mathbf{r}}_m(k) = \mathbf{r}(k) - \sum_{j=1, j\neq m}^{M} \mathbf{h}_j(k)\tilde{d}_j(k). \tag{5}$$

Note that we treat those data symbols $d_j(k)$ for $j \neq m$ as interference to $d_m(k)$. If the decisions from the first stage are correct, interference from the other transmit antennas can be eliminated in $\hat{\mathbf{r}}_m(k)$, and thus the estimation performance in the second stage can be improved. Similarly, we write the estimate for the mth element of $\mathbf{d}(k)$ in the second-stage output as $\hat{d}_m(k) = \mathrm{Dec}\{\hat{\mathbf{w}}_m^H(k)\hat{\mathbf{r}}_m(k)\}$, where $\hat{\mathbf{w}}_m(k)$ is a second-stage $N \times 1$ weight vector for the estimation of $d_m(k)$. The adaptation of $\hat{\mathbf{w}}_m(k)$ in the second stage can be analogous to that given in (4). However, due to the satisfactory result usually provided in the first-stage processing, the adaptation in the second stage can be omitted. Here, we simply let $\hat{\mathbf{w}}_m(k)$ match to the corresponding channel response in (1), i.e., $\hat{\mathbf{w}}_m(k) = (\mathbf{h}_m^H(k)\mathbf{h}_m(k))^{-1}\mathbf{h}_m(k)$, in which we assume that $\mathbf{h}_m(k)$ is known or can be estimated. We use this way for the second-stage PIC weights throughout the letter. In changing environments, the main problem of the multistage PIC is error propagation. It seriously affects the MMSE-PIC training. In the worst case, the receiver may lose track of the time-varying MIMO channel. When this occurs, the training mode has to be re-initiated.

III. DFGSC-PIC AND ITS ADAPTIVE REALIZATION

A. Review of DFGSC in [9]

In this part, we present the necessary background for the DFGSC. The derivations given below are based on the weights in the first stage of the PIC. Since the structure and the corresponding processing are identical for each data stream in the first stage, we drop the subscript m, and use $\mathbf{w}(k)$ to denote the weight vector, $d(k)$ the transmitted data symbol, $\tilde{d}(k)$ the output decision, and $\mathbf{h}(k)$ the channel response to simplify the notation. The MV criterion is used for optimizing $\mathbf{w}(k)$ [7], i.e.,

$$\min_{\mathbf{w}(k)} \ \mathbf{w}^H(k)\mathbf{R}(k)\mathbf{w}(k), \ \text{ subject to } \mathbf{h}^H(k)\mathbf{w}(k) = 1 \quad (6)$$

where $\mathbf{R}(k) = \mathrm{E}\{\mathbf{r}(k)\mathbf{r}^H(k)\}$ is the input correlation matrix. It is well-known that the GSC is an alternative formulation for the MV criterion. It includes an N-tap signal matched filter $\mathbf{w}_\mathrm{q}(k)$, an $N\times(N-1)$ blocking matrix $\mathbf{B}(k)$, and an $(N-1)$-tap interference cancelling filter $\mathbf{w}_\mathrm{a}(k)$, with $\mathbf{w}(k) = \mathbf{w}_\mathrm{q}(k) - \mathbf{B}(k)\mathbf{w}_\mathrm{a}(k)$. The span of $\mathbf{B}(k)$ is designed to fall into the null space of $\mathbf{h}^H(k)$, and the signal-matching operation is simply $\mathbf{w}_\mathrm{q}(k) = (\mathbf{h}^H(k)\mathbf{h}(k))^{-1}\mathbf{h}(k)$. In [9], a decision feedback operation was introduced to the conventional adaptive GSC to overcome the problems of slow convergence and sensitivity to constraint mismatch. A single-tap filter $w_\mathrm{b}(k)$ was added to the feedback process, and this was referred to as the DFGSC. We use this DFGSC for the adaptive realization of the MV detector. A new cost function for the optimization of both $\mathbf{w}_\mathrm{a}(k)$ and $w_\mathrm{b}(k)$ is given by

$$\begin{aligned}&\min_{\mathbf{w}_\mathrm{a}(k), w_\mathrm{b}(k)} \ \mathrm{E}\{|e(k)|^2\}\\ &= \min_{\mathbf{w}_\mathrm{a}(k), w_\mathrm{b}(k)} \ \mathrm{E}\{|(\mathbf{w}_\mathrm{q}(k) - \mathbf{B}(k)\mathbf{w}_\mathrm{a}(k))^H\mathbf{r}(k)\\ &\qquad\qquad - w_\mathrm{b}^*(k)\tilde{d}(k)|^2\}. \end{aligned} \quad (7)$$

Note that the difference between (7) and the original criterion for the DFGSC is that the time-varying nature is considered in the formulation. For simplicity, we assume that the output decision for the desired data symbol is correct, i.e., $\tilde{d}(k) = d(k)$. Following the procedure in [9], we can derive

$$\mathbf{w}_\mathrm{a,opt}(k) = (\mathbf{B}^H(k)\mathbf{R}(k)\mathbf{B}(k))^{-1}\mathbf{B}^H(k)\mathbf{R}(k)\mathbf{w}_\mathrm{q}(k) \quad (8)$$

$$w_\mathrm{b,opt}(k) = \mathbf{h}^H(k)\mathbf{w}_\mathrm{q}(k) = 1. \quad (9)$$

The performance of the DFGSC can be greatly improved by the feedback operation when compared with that of the conventional GSC. Detailed derivations are omitted here but can be found in [9]. Similar to (4), the LMS update equations for the DFGSC can be expressed as $\mathbf{w}_\mathrm{a}(k+1) = \mathbf{w}_\mathrm{a}(k) + \mu_\mathrm{a}\mathbf{v}(k)e^*(k)$ and $w_\mathrm{b}(k+1) = w_\mathrm{b}(k) + \mu_\mathrm{b}\tilde{d}(k)e^*(k)$, where μ_a and μ_b are the step sizes for $\mathbf{w}_\mathrm{a}(k)$ and $w_\mathrm{b}(k)$, $\mathbf{v}(k) = \mathbf{B}^H(k)\mathbf{r}(k)$ is the input vector for $\mathbf{w}_\mathrm{a}(k)$, and $e(k)$ is the error signal as that given in (7), respectively.

B. Adaptive DFGSC-PIC with Dual-DFGSC Structure

Next, we propose an adaptive DFGSC-based PIC detection scheme for MIMO systems, in which a special dual structure of the DFGSC is applied for adaptive implementations in time-varying environments. The DFGSC is quite robust whenever the channel response changes moderately. The performance degradation can be ignored when it is operated within a short period of time. However, if we want to keep the good performance over a long-term period, $\mathbf{w}_\mathrm{q}(k)$ and $\mathbf{B}(k)$ have to be properly updated. Here, we propose a simple method to do the job. First, define a diagonal matrix as

$$\mathbf{P}(k) = \mathrm{Diag}\{(\mathbf{h}^H(k)\mathbf{h}(k))^{-1}\mathbf{h}(k)\}. \quad (10)$$

This matrix, called the steering matrix, is used to *pre-steer* the look direction of the DFGSC. In other words, the received signal vector is preferably multiplied by the matrix. With this operation, $\mathbf{w}_\mathrm{q}(k)$ will become a time-invariant vector with components of all ones, denoted as $\mathbf{1}$, and $\mathbf{B}(k)$ can be a time-invariant orthogonal matrix, denoted as $\mathbf{B}$. With $\mathbf{B}$ chosen carefully, the computational complexity for signal blocking can be significantly reduced. For the case of $N = 2^l$, where l is any nonnegative integer, a simple choice for $\mathbf{B}$ is the Hadamard matrix excluding the first column. For the case that $N \neq 2^l$, $\mathbf{B}$ can still be designed to achieve low complexity as reported in [12]. With this architecture, only $\mathbf{P}(k)$ has to be updated.

Another problem is how to acquire the up-to-date channel response. With feedback decisions, this can be easily solved using a channel estimator. Let the coefficients of the channel estimator be denoted as $\mathbf{q}(k)$, which is an $N\times 1$ vector. It can be tuned by a new error signal vector $\mathbf{e}_\mathrm{q}(k)$, and the optimization can be written as

$$\min_{\mathbf{q}(k)} \ \mathrm{E}\{\|\mathbf{e}_\mathrm{q}(k)\|^2\} = \min_{\mathbf{q}(k)} \ \mathrm{E}\{\|\mathbf{r}(k) - \mathbf{q}(k)\hat{d}(k)\|^2\}. \quad (11)$$

To have better performance, we use the output decision in the second stage of the PIC, i.e., $\hat{d}(k)$, as the input to the channel estimator. Also note that the estimation exists over all parallel branches. With the assumption of correct decisions, it is not difficult to show that the optimum $\mathbf{q}(k)$, denoted as $\mathbf{q}_\mathrm{opt}(k)$, will be equal to $\mathbf{h}(k)$. The LMS algorithm is used to approach $\mathbf{q}_\mathrm{opt}(k)$ recursively, and the update equation is expressed as $\mathbf{q}(k+1) = \mathbf{q}(k) + \mu_\mathrm{q}\hat{d}^*(k)\mathbf{e}_\mathrm{q}(k)$, where μ_q is the step size for the adaptation. In many cases, the input to $\mathbf{q}(k)$ is a white sequence, which owns the smallest eigenvalue spread of the input correlation matrix. As the convergence rate of the LMS algorithm is inversely proportional to the eigenvalue spread [11], the adaptation of $\mathbf{q}(k)$ is expected to be fast and stable. With the application of the steering matrix and the channel estimator, the adaptive DFGSC can be operated in time-varying environments. However, since $\mathbf{P}(k)$ and $\mathbf{w}_\mathrm{a}(k)$ are connected in series, continuous update of $\mathbf{P}(k)$ may yield always un-convergent $\mathbf{w}_\mathrm{a}(k)$. Since the DFGSC can resist constraint mismatch, there is no need to update the steering matrix continuously; periodic update is more appropriate. There is also one problem associated with periodic update. The abrupt change of the steering matrix will make $\mathbf{w}_\mathrm{a}(k)$ deviate from its optimum state instantaneously. As a result, the performance will be degraded until $\mathbf{w}_\mathrm{a}(k)$ re-converges. To solve this problem, we propose to use a dual-DFGSC structure, as illustrated in Fig. 2, for each branch in the first stage. These two adaptive DFGSCs are operated simultaneously and complementarily. We let $\mathbf{P}(k)$ and $\tilde{d}(k)$ in the first DFGSC be donated as $\mathbf{P}_1(k)$ and $\tilde{d}_1(k)$, and those in the second DFGSC as $\mathbf{P}_2(k)$ and $\tilde{d}_2(k)$. Both $\mathbf{P}_1(k)$ and $\mathbf{P}_2(k)$ are updated periodically in different time instants, and only one of $\tilde{d}_1(k)$ and $\tilde{d}_2(k)$ is selected as the decision passed to the second stage and used as the reference signal

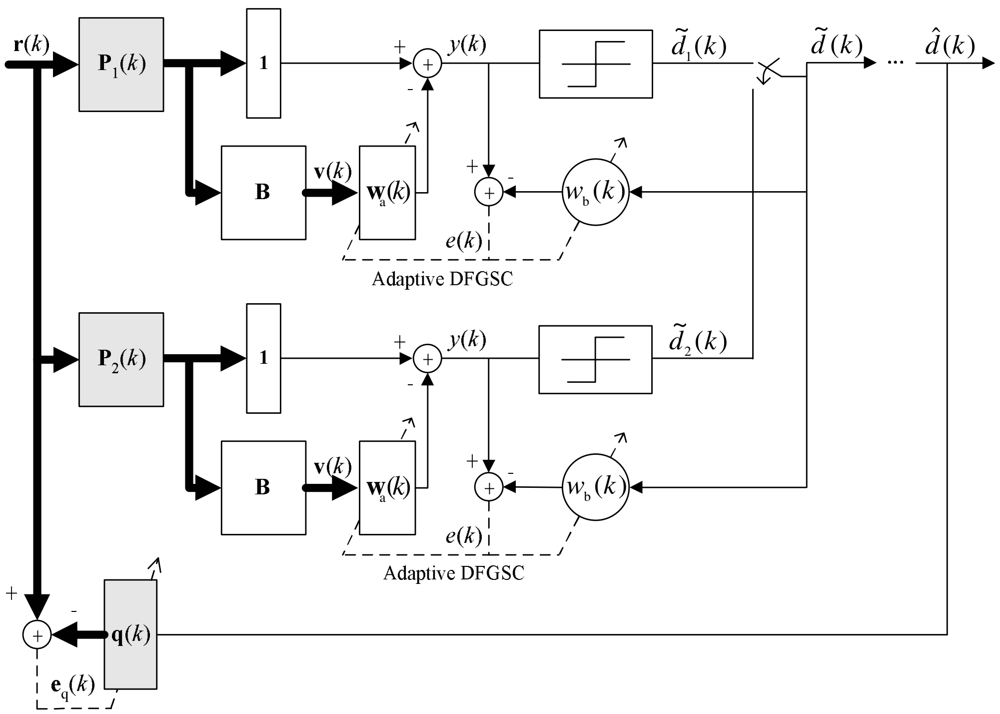

Fig. 2. The proposed dual-DFGSC structure (for the first stage of DFGSC-PIC).

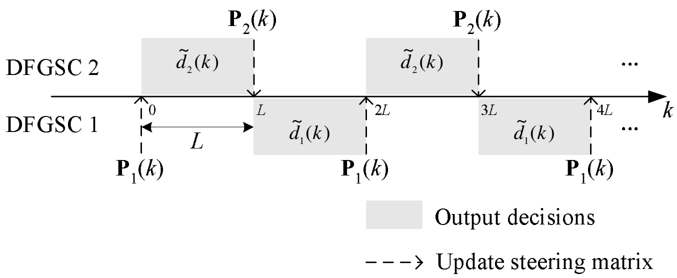

Fig. 3. Update and decision output relation for dual-DFGSC structure.

for the dual-DFGSC adaptation. Let the time origin be zero, L be the update period and $k = 2jL + l$, where j is a nonnegative integer and $0 \leq l \leq 2L - 1$. Then, $\mathbf{P}_1(k)$ and $\mathbf{P}_2(k)$ can be expressed as $\mathbf{P}_1(2jL + l) = \mathbf{P}(2jL)$ and $\mathbf{P}_2(2jL + L + l) = \mathbf{P}(2jL + L)$, where $\mathbf{P}(k)$ is as that shown in (10). Also, $\mathbf{P}_2(l) = \mathbf{P}(0)$ for $0 \leq l \leq L - 1$. The decision passed to the second stage and used for the dual-DFGSC adaptation can be set according to the principle:

$$\tilde{d}(2jL+l) = \begin{cases} \tilde{d}_2(2jL + l), \text{if } 2jL \leq l \leq 2jL + L - 1 \\ \tilde{d}_1(2jL + l), \text{if } 2jL + L \leq l \leq 2jL + 2L - 1 \end{cases}. \tag{12}$$

Fig. 3 illustrates the update and decision output relation for the dual-DFGSC structure. In the next section, we will provide a guideline for the determination of L. As we can see, the dual structure will increase the computational complexity. Fortunately, as mentioned, the structure is only applied to the first stage. The weight vector $\hat{\mathbf{w}}(k)$ for each branch in the second stage only performs the matching operation, i.e., $\hat{\mathbf{w}}(k) = (\mathbf{q}^H(k)\mathbf{q}(k))^{-1}\mathbf{q}(k)$, where $\mathbf{q}(k)$ is the up-to-date channel estimate for the corresponding channel.

IV. Convergence Analysis

In this section, we will analyze the convergence behavior of the proposed DFGSC operated in time-varying channel environments. We assume that each coefficient in the MIMO channel varies independently according to Jakes' model and the channel response $\mathbf{h}(k)$ in (1) is normalized (made possible by automatic gain control in practice). Recall that the correlation function of $\mathbf{h}(k)$ in this situation can be modeled as $r_{\mathrm{hh}}(\tau) \triangleq \mathrm{E}\{\mathbf{h}^H(k)\mathbf{h}(k - \tau)\} = \mathfrak{J}_0(2\pi f_{\mathrm{d}} T_{\mathrm{s}} \tau)$ [13], in which τ is the time lag, f_{d} is the Doppler frequency and T_{s} is the symbol duration, and the function $\mathfrak{J}_0(\cdot)$ is the zeroth-order Bessel function of the first kind. We assume that the channel variations are small in a short period of time and so the channel vector can be described by a random walk process as

$$\mathbf{h}(k+1) = \mathbf{h}(k) + \boldsymbol{\eta}(k) \tag{13}$$

where $\boldsymbol{\eta}(k)$ denotes a white noise vector with the variance σ_η^2 calculated by $N\sigma_\eta^2 = \mathrm{E}\{\| \ \mathbf{h}(k) - \mathbf{h}(k+1) \ \|^2\}$. Thus, $\sigma_\eta^2 = (2 - 2r_{\mathrm{hh}}(1))/N$. For notation simplicity, we drop the time index for $\mathbf{w}_{\mathrm{q}}(k)$ and $\mathbf{B}(k)$ since they are constants during an update period. Let $\mathbf{B} = [\mathbf{b}_1 \ \ \mathbf{b}_2 \ \ \cdots \ \ \mathbf{b}_{N-1}]$ and $\mathbf{w}_{\mathrm{a,opt}}(k) = [w_1(k) \ \ w_2(k) \ \ \cdots \ \ w_{N-1}(k)]^T$. Due to the decision feedback operation and small channel variations, the desired data symbol can be assumed to be totally eliminated in the optimization. Also, the leakage in the output of $\mathbf{B}$ is small and can be ignored. We first consider a noiseless environment with only one interference, in which $\mathbf{h}(k)$ denotes the channel vector for that interference. With perfect interference cancellation, we have

$$\mathbf{w}_{\mathrm{q}}^H \mathbf{h}(k) = \mathbf{w}_{\mathrm{a,opt}}^H(k) \mathbf{B}^H \mathbf{h}(k). \tag{14}$$

At time instant $k+1$, interference from $\mathbf{h}(k+1)$ is also perfectly cancelled. We will have

$$\mathbf{w}_{\mathrm{q}}^H \mathbf{h}(k+1) = \mathbf{w}_{\mathrm{a,opt}}^H(k+1)\mathbf{B}^H\mathbf{h}(k+1). \quad (15)$$

Substituting (13) and (14) into (15), we obtain

$$\sum_{n=1}^{N-1} w_n^*(k)\mathbf{b}_n^H\mathbf{h}(k) + \mathbf{w}_{\mathrm{q}}^H\boldsymbol{\eta}(k) = \sum_{n=1}^{N-1} w_n^*(k+1)\mathbf{b}_n^H(\mathbf{h}(k)+\boldsymbol{\eta}(k)). \quad (16)$$

Now, assume that the variation term $\mathbf{w}_{\mathrm{q}}^H\boldsymbol{\eta}(k)$ in (16) can be evenly cancelled by each weight element in $\mathbf{w}_{\mathrm{a,opt}}(k)$. We then have

$$w_n^*(k)\mathbf{b}_n^H\mathbf{h}(k) + \frac{1}{N-1}\mathbf{w}_{\mathrm{q}}^H\boldsymbol{\eta}(k) = w_n^*(k+1)\mathbf{b}_n^H(\mathbf{h}(k)+\boldsymbol{\eta}(k)) \quad (17)$$

for $1 \le n \le N-1$. After some manipulation, we can obtain the following approximation:

$$w_n^*(k+1) \simeq w_n^*(k) + \frac{1}{N-1}\frac{\mathbf{w}_{\mathrm{q}}^H\boldsymbol{\eta}(k)}{\mathbf{b}_n^H\mathbf{h}(k)}. \quad (18)$$

From (18), we can write $\mathbf{w}_{\mathrm{a,opt}}(k+1) = \mathbf{w}_{\mathrm{a,opt}}(k) + \boldsymbol{\omega}(k)$, where $\boldsymbol{\omega}(k)$ is the process noise vector with its elements defined as the second term in (18). Therefore, we see that the optimum weight vector $\mathbf{w}_{\mathrm{a,opt}}(k)$ can also be modeled as a random walk process. Note that $\mathrm{E}\{|\mathbf{w}_{\mathrm{q}}^H\boldsymbol{\eta}(k)|^2\} = \sigma_\eta^2$ and $\mathrm{E}\{|\mathbf{b}_n^H\mathbf{h}(k)|^2\} = 1/N$. Thus, the variance of each component in $\boldsymbol{\omega}(k)$ is $N\sigma_\eta^2/(N-1)^2$. For the scenario of multiple interfering streams, (14) and (15) can be modified to include multiple channel vectors. With the derived random walk process, we can formulate the adaptation of $\mathbf{w}_{\mathrm{a}}(k)$ as a time-varying system identification problem. Let $\mathbf{v}(k) = \mathbf{B}^H\acute{\mathbf{H}}(k)\mathbf{i}(k)$, where $\acute{\mathbf{H}}(k)$ is the channel matrix excluding the column corresponding to the desired data symbol, and $\mathbf{i}(k)$ is the vector consisting of interfering data streams. For this interference only system, the input to the system is $\mathbf{v}(k)$ and the output of the system is $\mathbf{w}_{\mathrm{a,opt}}^H(k)\mathbf{v}(k)+z(k)$, where $z(k)$ is white noise with zero mean and variance σ_z^2. The filter $\mathbf{w}_{\mathrm{a}}(k)$ is then used to identify $\mathbf{w}_{\mathrm{a,opt}}(k)$. Following the analysis procedure in [11, Ch. 14], we can readily find the optimum step size for $\mathbf{w}_{\mathrm{a}}(k)$ as $\mu_{\mathrm{a,opt}} \simeq \sqrt{\mathrm{Tr}\{\boldsymbol{\Omega}\}/(\sigma_z^2\mathrm{Tr}\{\mathbf{R}_{\mathbf{v}}\})}$, where $\mathbf{R}_{\mathbf{v}}$ is the correlation matrix of the input vector $\mathbf{v}(k)$, and $\boldsymbol{\Omega}$ is the covariance matrix of $\boldsymbol{\omega}(k)$ given by $\boldsymbol{\Omega} = (N\sigma_\eta^2/(N-1)^2)\mathbf{I}$, with $\mathbf{I}$ being an identity matrix. After that, we can discuss the determination of the update period L. The period should be long enough for $\mathbf{w}_{\mathrm{a}}(k)$ to converge, and short enough for constraint mismatch to remain small. The settling time of the LMS algorithm is proportional to the average time constant [11], which is approximated as $\tau_{\mathrm{av}} \approx 1/(2\mu_{\mathrm{a}}\lambda_{\mathrm{av}})$, where λ_{av} is the average eigenvalue for the underlying correlation matrix, i.e., $\mathbf{R}_{\mathbf{v}}$ in our scenario. As a rule of thumb, we choose two times of τ_{av} for a good trade-off between the convergence of $\mathbf{w}_{\mathrm{a}}(k)$ and the mismatch experienced by the DFGSC.

V. Simulation Results and Conclusions

Computer simulations are conducted to demonstrate the effectiveness of the proposed adaptive two-stage DFGSC-PIC detection scheme for time-varying MIMO channels. We first show the learning curves of the output signal-to-interference plus noise ratio (SINR). The parameters used are listed in Table 1. Transmission is over independent time-varying flat Rayleigh fading channels. Assume that the channel response is perfectly known after initial training. For comparison, we also consider a two-stage MMSE-PIC detector. The same channel tracking mechanism is used for the MMSE-PIC, and the step sizes in all LMS adaptations are chosen to optimize the performance. Fig. 4 shows the learning curves of the output SINR in the first and second stages of the DFGSC-PIC and the MMSE-PIC. For the DFGSC-PIC in the first stage, it is clear that the update of the steering matrix does not affect the learning behavior. The output SINRs in both stages remain unchanged. This shows that our DFGSC-PIC is quite robust in the time-varying MIMO channel. In the same figure, we observe that the MMSE-PIC cannot track this changing environment. The SINR values for both stages of the MMSE-PIC degrade continuously due to error propagation.

We next show the symbol-error-rate (SER) performance for the environment described previously. We compare the DFGSC-PIC with the MMSE-PIC and the V-BLAST system with parameters updated blockwisely [14]. The V-BLAST sys-

TABLE I
Parameters used in simulations

Parameters	Values
Normalized Doppler frequency ($f_{\mathrm{d}}T_{\mathrm{s}}$)	5×10^{-4}
M	2
N	4
Modulation	16 quadrature amplitude modulation (16-QAM)
SNR	20 dB
$\mu_{\mathrm{a,opt}}$	6.0×10^{-3}
L	$2\tau_{\mathrm{av}} \simeq 17$
Frame size	1024

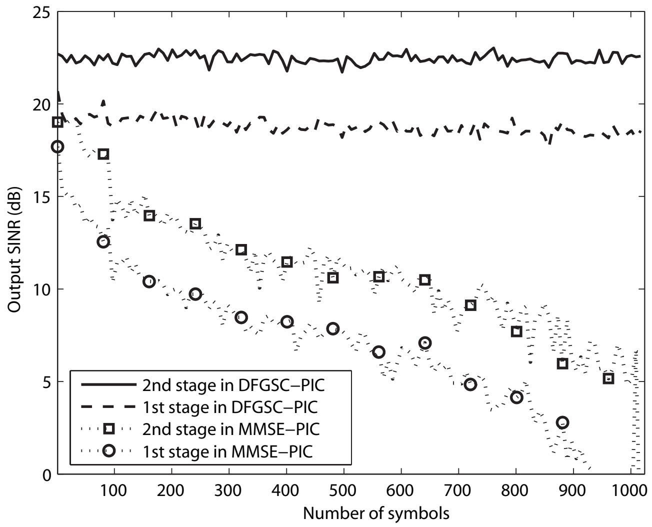

Fig. 4. Learning curves of PIC output SINR with $f_{\mathrm{d}}T_{\mathrm{s}} = 5 \times 10^{-4}$ and SNR = 20 dB.

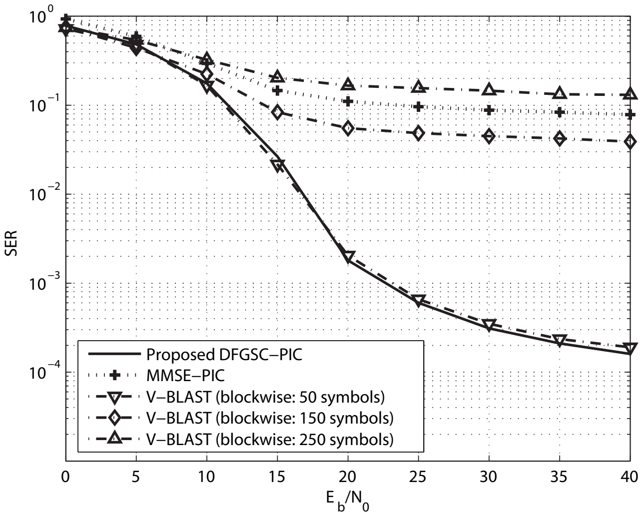

Fig. 5. SER performance for various SNR values with $f_{\rm d}T_{\rm s} = 5 \times 10^{-4}$.

tem re-calculates the detection order and weight vectors with the MMSE criterion in the beginning of each block. Hence, the smaller the block length is, the better the performance is expected, but the higher the computational complexity it requires. Usually, the V-BLAST system can perform very well provided that the channel is exactly known. However, if the V-BLAST system only updates in a blockwise manner in time-varying channel environments, its performance will be significantly degraded, especially when the update interval is large. In general, the computational complexity for calculating both the detection order and weight vectors is on the order of $O(M^3 + M^2N)$ [15]. While the DFGSC-PIC is more complex than the MMSE-PIC, the computational complexity of both adaptive PIC schemes keeps on the order of $O(MN)$. Fig. 5 shows the resultant SER against different signal-to-noise ratio (SNR) values. We observe that the performance of the MMSE-PIC becomes poor, but our DFGSC-PIC still performs quite well. For the V-BLAST system to perform similarly to the DFGSC-PIC, the block length should be shortened to around 50.

From these results, we conclude that the proposed DFGSC-PIC can perform very well in time-varying channel environments and provide a good trade-off between performance and computational complexity.

REFERENCES

[1] G. J. Foschini and M. J. Gans, "On the limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 315–335, Mar. 1998.

[2] G. D. Golden, G. J. Foschini, R. A. Valenzuela, and P. W. Woniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture," *Electron. Lett.*, vol. 35, no. 1, pp. 14–16, Jan. 1999.

[3] W. H. Chin, A. G. Constantinides, and D. B. Ward, "Parallel multistage detection for multiple antenna wireless systems," *Electron. Lett.*, vol. 38, no. 12, pp. 597–599, June 2002.

[4] H. Dai, A. F. Molisch, and H. V. Poor, "Downlink capacity of interference-limited MIMO systems with joint detection," *IEEE Trans. Wireless Commun.*, vol. 3, no. 2, pp. 442–453, Mar. 2004.

[5] S. M. Tseng and H. L. Lee, "An adaptive partial parallel multistage detection for MIMO systems," *IEEE Trans. Commun.*, vol. 53, no. 4, pp. 587–591, Apr. 2005.

[6] F. C. Zheng and A. G. Burr, "Signal detection for orthogonal space-time block coding over time-selective fading channels: a PIC approach for the $\mathcal{G}_i$ systems," *IEEE Trans. Commun.*, vol. 53, no. 6, pp. 587–591, June 2005.

[7] S. Shahbazpanahi, M. Beheshti, A. B. Gershman, M. Gharavi-Alkhansari, and K. M. Wong, "Minimum variance linear receivers for multiaccess MIMO wireless systems with space-time block coding," *IEEE Trans. Signal Processing*, vol. 52, no. 12, pp. 3306–3312, Dec. 2004.

[8] L. J. Griffiths and C. W. Jim, "Alternative approach to linear constrained adaptive beamforming," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 27–34, Jan. 1982.

[9] Y. Lee and W. R. Wu, "A robust adaptive generalized sidelobe canceller with decision feedback," *IEEE Trans. Antennas Propagat.*, vol. 53, no. 11, pp. 3822–3832, Nov. 2005.

[10] J. G. Proakis, *Digital Communications*, 4th Ed. New York: McGraw-Hill, 2001.

[11] S. Haykin, *Adaptive Filter Theory*, 4th Ed. Englewood Cliffs, NJ: Prentice-Hall, 2001.

[12] J. Q. Trelewicz, J. L. Mitchell, and M. T. Brady, "Vectorized transforms in scalar processors," *IEEE Signal Processing Mag.*, vol. 19, pp. 22-31, July 2002.

[13] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.

[14] Q. Sun and D. C. Cox, "Training-based channel estimation for continuous flat fading BLAST," in *Proc. IEEE ICC 2002*, Helsinki, Finland, pp. 325–329, June 2002.

[15] J. Benesty, Y. Huang, and J. Chen, "A fast recursive algorithm for optimum sequential signal detection in a BLAST system," *IEEE Trans. Signal Processing*, vol. 51, no. 7, pp. 1722–1730, July 2003.