行政院國家科學委員會專題研究計畫成果報告

衰變資料分析之無母數迴歸研究

計畫編號：NSC 90-2118-M-009-016-

執行期限：89 年 8 月 1 日至 91 年 7 月 31 日

主持人：洪志真　交大統計所

一、中文摘要

為了節省時間及成本，工程師常使用逐步應力加速衰變試驗來研究高可靠度產品的壽命。我們針對逐步應力加速衰變試驗資料，提出一個無母數迴歸加速衰變試驗模型。假設應力的大小只會影響產品衰變的速度，並不會影響產品衰變的路徑。利用加速因子與應力間的關係，我們將逐步應力加速衰變試驗模型轉換成最大應力下的加速衰變試驗模型，並提出一個演算法來估計加速因子。最後，藉由 Shiau, Chien, and Chang (2002)的作法來推估產品的壽命分布。我們將此方法應用在一組利用發光二極體的加速衰變試驗資料及已知的加速因子產生的『虛擬』逐步應力加速衰變試驗資料上，結果相當不錯。此外，我們也利用已知函數來模擬出逐步應力加速衰變試驗資料，一方面驗證模型的合適度，另一方面也驗證推估壽命分布的方法。結果也不錯。

關鍵詞：無母數迴歸、衰變資料、加速衰變試驗、壽命分佈、隨機過程

Abstract

In recent years, for reducing the experimental time and costs, engineers gradually adopt the step-stress accelerated degradation tests (SSADT) in the study of lifetime distributions of highly reliable products. In this study, we propose a nonparametric regression stochastic process model for SSADT data. We assume that different stress levels only affect the degradation rate of the product characteristic, but not the degradation trend. Given a set of acceleration factors, SSADT data can be transformed into the form of usual accelerated degradation test (ADT) data under the largest stress level. With the relationship between the acceleration factors and the stress levels, we propose an algorithm to estimate the acceleration factors. Then, we can obtain the lifetime distribution of the product under usual use by applying the method proposed by Shiau, Chien, and Chang [14]. To illustrate the proposed method, we apply the method on a set of SSADT data, which is generated from a set of real ADT data of an LED (light emitting diode) product with given acceleration factors. To study the effectiveness of the method, we further simulate two sets of SSADT data with known functions in order to evaluate the method and the resulting lifetime distribution estimate. The results are promising. The proposed nonparametric regression estimation method offers a flexible alternative to the usual parametric models for analyzing SSADT data.

Keywords: Nonparametric regression、Accelerated degradation tests、Step-stress accelerated degradation tests、Lifetime distribution、Stochastic process

二、緣由與目的

New advanced technologies, greater market competition, higher customer expectations, and much more quality improvement efforts have been contributing to the high reliability of many products. Therefore, how to assess the lifetime distributions of these highly reliable products has become a challenging task. In such circumstances, accelerated tests are widely used to shorten the product life or hasten the

degradation of the product performance. The aim of accelerated degradation tests is to obtain inferences on product life or performance under usual use in a reasonable length of time by appropriate modeling and analysis.

For some situations that the failure time of the product cannot be observed in a short period of time, it may be possible to measure the physical degradation of the product as a function of time in terms of certain product quality characteristics, for instance, "light intensity" of light emitting diode (LED) products. The data obtained by measuring such quality characteristics over time are called degradation data.

In order to reduce the time and costs, degradation data are often obtained by accelerated degradation tests (ADT) or step-stress accelerated degradation tests (SSADT). ADT is an experiment under a constant stress level, while SSADT uses different stress levels at different experimental time intervals. If there are n test items for each of the m stress levels, then the total number of test items for the whole ADT experiment is mn, while the SSADT experiment would only need n test items. Hence the SSADT experiment is more economical than the ADT experiment. This is particularly true at the beginning of the product development, because usually there may not be enough prototypes to perform ADT. SSADT experiment can help overcome this problem.

Almost all the ADT or SSADT analyses use parametric regression models to infer the product life under usual use. Nelson [10] provided a fairly thorough survey on ADT. Lu and Meeker [6] estimated the life distribution with a nonlinear mixed effects model for degradation data. Meeker and Escobar [7] reviewed recent research in accelerated testing. Meeker, Escobar, and Lu [9] presented methods for analyzing ADT data. Meeker & Escobar [8] presented many up-to-date statistical methods for analyzing reliability data. Nelson [10, Chapter10 and 11] described basic ideas on step-stress accelerated degradation tests models. Tseng and Wen [15] described a method for assessing the reliability of an LED product with a set of SSADT data and performed a sensitive analysis of the proposed model.

These degradation data can be considered as so-called functional data. Ramsay and Silverman [11] introduced how to analyze the functional data and used principal components analysis to describe the covariance structure of the data. Shiau and Lin [12] proposed a nonparametric regression accelerated life-stress (NPRALS) model for assessing the mean time to failure of an LED product from a set of ADT data. Shiau, Chien, and Chang [14] further estimated the lifetime distribution by simulating a large number of the degradation paths of the product from the estimated NPRALS model. They also proposed a method to handle data with some censored lifetimes. A comparative study showed that the proposed method outperforms the existing parametric modeling method.

In this study, we extend the previous research to developing a non-parametric regression method for estimating the lifetime distribution of a product from SSADT data.

三、結果與討論

As the result of the project, we have developed a procedure for estimating lifetime distributions from SSADT data by nonparametric regression approach. The first step is to filter out the noise contained in the SSADT data, for which we can apply a smoothing technique to data curves to achieve this objective. Because the SSADT data have different degradation rates for different stress levels, data smoothing needs to be done piecewisely. For smoothing techniques, see [1,2,3,4,5,11,16].

We develop an algorithm to estimate the mean curve of the SSADT data and the acceleration factors. For given acceleration factors, we transform the SSADT data into the form of ADT data under the largest, say, m-th stress level by time scaling. The mean curve under the m-th stress level of the ADT data can be estimated by a smoothing spline. We can estimate the acceleration factors by grid search or downhill simplex method. The

algorithm is described as follows:

1. As a preprocessing step, we smooth the degradation path of the SSADT data first to prevent the experimental noise from getting in the way of estimation.

2. Consider a candidate set of the smoothing parameter *df* (means "effective degrees of freedom"). For each *df*, we will search over the possible domain of acceleration factors. For each candidate df and acceleration factors, first transform the SSADT data to the ADT data of the m-th stress level. Estimate the mean curve of the m-th stress level of the ADT data by spline smoothing. In our study, S function *smooth.spline* is used for computing spline estimates. *df* can be a real number; but for simplicity, we only choose integers.

3. For each *df* considered, minimize the average squared error criterion to estimate the acceleration factors.

4. Choose the smoothing parameter *df* by minimizing the GCV (generalized cross validation) criterion. Then the corresponding acceleration factors and mean curve estimate are the final estimates.

We currently do not have suitable SSADT data at hand to analyze so that we create a SSADT data set from a real ADT data set as described in the following. The ADT data consist of three sets of degradation paths of an LED product collected under three stress levels, 25℃, 65℃, and 105℃, respectively. The quality characteristic is the light intensity, which was measured at 59 time points. Because the values of the initial light intensity are not the same for these test items, the data are standardized so that all the initial values become one. In order to eliminate the noise in the data, we smooth the data before any further data analysis. According to Nelson [10], if temperature is the acceleration variable, then the Arrehenius rate relationship is widely used for temperature-acceleration degradation. Shiau, Chien, and Chang [13] analyzed the data and found that the data fit the Arrehenius relationship very well. The acceleration factors that we choose to generate SSADT data are (0.1, 0.4, 1). According to the SSADT model, we generate SSADT data by interpolation from the smoothed ADT data under 105℃. Consequently, we generate 17 SSADT paths, and for each path the standardized light intensity is computed at 86 time points. The termination time under the three stress levels are at 1680, 6720, and 14534 hours, respectively.

With the generated SSADT data, the resulting estimates of the relative acceleration factors (with respect to the last acceleration factor) are 0.1078 and 0.4121. To make data look more realistic, we perturb the value at each data point with independent Gaussian noise. It is observed that the smoothing step helps in estimating the relative acceleration factors.

In order to have a known true mean function and covariance structure to check the validity of the method, we conduct a simulation study. Let the mean curve under the usual use be $\exp(-.2t)/(1+2t)$. Assume the covariance structure of the curve data can be represented by the first two principal components, $\sqrt{15/8}(1-(.4t-1)^2)$ and $\sqrt{2}\sin(.4\pi t)$. We first generate degradation paths (under usual use) with no error term. Then transform these paths to SSADT paths according to the prescribed relative acceleration factors.

We generate a set of SSADT data, which consists of 30 curves with relative acceleration factors (0.1, 0.5, 1), and each curve is evaluated at 42 time points. The termination time of the three stress levels are 0.04, 0.096, and 0.164. As before, we estimate the relative acceleration factors for this set of data, and obtain 0.0995 and 0.4992, which are very close to the true values. Again, we perturb these paths with some errors. It is found that the estimation of relative acceleration factors of the pre-smoothed data is much better than that of the un-smoothed data, and these estimates of the smoothed data are very close to the true values.

An alternative method for estimating acceleration factors by the generalized additive model is also studied. This method performs okay but not as good as the first method.

After estimating the relative acceleration

factors and the mean curve of the degradation paths under the m-th stress level, we now can estimate the lifetime distribution of the product under usual use by the following procedure.

(1) Transform each path of the SSADTdata into a path of the ADT data of the largest stress level by the estimated relative acceleration factors.
(2) Apply Shiau, Chien, and Chang [14]'s method to the transformed ADT data to obtain the lifetime estimate.

In the LED example, the goal of estimating the lifetime distribution for an LED product under usual use is achieved by utilizing the Arrhenius rate relationship. If the relationship between the stress level and the acceleration factor is not available for other applications, then including the usual use condition in the experiment is suggested.

四、計劃成果自評

This project has been well executed. A master student was well trained in this area of research. The research result [13] will be submitted for publication to a well-known international journal on Reliability. The new nonparametric regression method proposed under this project is a brand new approach in SSADT data analysis problems. It is believed that this approach will be useful in many other applications.

五、參考文獻

[1] J. Fan, I. Gijbels, *Local Polynomial Modeling and its Applications*, 1996; Chapman and Hall.

[2] P.J. Green, B.W. Silverman, *Nonparametric Regression and Generalized Linear Model: A Roughness Penalty Approach*, 1994; Chapman and Hall.

[3] W. Hardle, *Applied Nonparametric Regression*, 1990; Cambridge University Press.

[4] W. Hardle, *Smoothing Techniques: With Implementation in S*, 1991; Springer-Verlag.

[5] T. Hastie, R. Tibshirani, *Generalized Additive Models,* 1990; Chapman and Hall.

[6] C.J. Lu, W.Q. Meeker, "Using degradation measures to estimate a time-to-failure distribution", *Technometrics*, vol 35, 1993 May, pp 161-174.

[7] W.Q. Meeker, L.A. Escobar, "A review of recent research and current issues in accelerated testing", *Int'l Statistical Review*, vol 61, 1993 April, pp 147-168.

[8] W.Q. Meeker, L.A. Escobar, *Reliability Data Analysis*, 1998; Wiley.

[9] W.Q. Meeker, L.A. Escobar, C.J. Lu, "Accelerated degradation tests: modeling and analysis", *Technometrics*, vol 40, 1998 May, pp 89-99.

[10] W. Nelson, *Accelerated Testing: Statistical Models, Test Plans, and Data Analysis*, 1990; John Wiley and Sons.

[11] J.O. Ramsey, B.W. Silverman, *Functional Data Analysis*, 1997; Springer.

[12] J.-J.H. Shiau, H.-H. Lin, "Analyzing accelerated degradation data by nonparametric regression", *IEEE Transactions on Reliability*, Vol. 48, No. 2, 1999 June, pp 149-158.

[13] J.-J.H. Shiau, C.-Y. Chien, S.-C. Chang, "Estimating Lifetime distributions from ADT data by nonparametric regression", 2002, in revision.

[14] J.-J.H. Shiau, S.-C. Chang, "Estimating Lifetime distributions from SSADT data by nonparametric regression", 2002, in preparation.

[15] S.-T. Tseng, Z. C. Wen, "Step-Stress Accelerated Degradation Analysis for Highly Reliable Products", *Journal of Quality Technology*, Vol. 32, No. 3, 2000, 209-216.

[16] G. Wahba, *Spline Models for Observational Data*, 1990; Society for Industrial and Applied Mathematics.

附件：封面格式

行政院國家科學委員會補助專題研究計畫成果報告

※※※※※※※※※※※※※※※※※※※※※※※※※
※　　　　　　　　　　　　　　　　　　　　　　　　※
※　　　　　**轉換模式的一些半母數推論問題**　　　　　※
※　　　　　　　　　　　　　　　　　　　　　　　　※
※※※※※※※※※※※※※※※※※※※※※※※※※

計畫類別：□個別型計畫　　□整合型計畫
計畫編號：NSC 90-2118-M-009-016
執行期間：90 年 8 月 1 日至 91 年 7 月 31 日

計畫主持人：洪志真
共同主持人：
計畫參與人員：

本成果報告包括以下應繳交之附件：
□赴國外出差或研習心得報告一份
□赴大陸地區出差或研習心得報告一份
□出席國際學術會議心得報告及發表之論文各一份
□國際合作研究計畫國外研究報告書一份

執行單位：國立交通大學統計所

中　華　民　國　91 年 10　月 30　日