行政院國家科學委員會專題研究計畫成果報告

最佳化樹狀結構濾波器組
Optimal Tree Structure Orthonormal Filter Banks

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一、中文摘要

近來針對訊號之統計資料來設計最佳化樹狀結構濾波器組之研究方向受到了高度重視。最佳化濾波器可由設計集中氏的濾波器組得到，但這些設計不一定能夠真正的用在實際系統上．本計劃研究以樹狀結構為基礎之正交濾波器組之最佳化並發展最佳化設計的演算法則．

關鍵詞：最佳化正交濾波器組、集中式濾波器、樹狀結構濾波器組

Abstract

Recently there has been considerable interest in the design of optimum orthonormal filter banks for a given class of inputs. It has been shown that the ideal solution of the optimum orthonormal filter banks can be obtained by constructing M compaction filters. However these filters are ideal and can not be obtained with finite cost. In this project we have considered optimal orthonormal filter banks with practical tree structure implementations. Also an optimal tree structure building algorithm in the sense of coding gain maximization will be developed.

Keywords: optimal orthonormal filter banks, compaction filters, tree structure implementation

二、緣由與目的

Subband coding has now been one of the most effective data compression techniques. It has been used in various popular standards, e.g., MPEG (motion picture expert group) and NTSC (National Television Standard Committee).

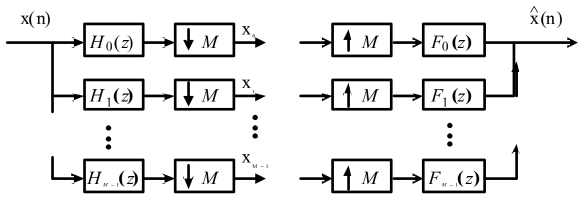

Fig. 1. The M-channel uniform filter bank.

The essential instrument in the implementation of subband coding is the M-channel filter bank as shown in Fig. 1. In the context of filter bank designs, it is of great interest to maximize the coding gain of the filter bank for a given class of input signals. For the case of orthonormal filter banks, it is well known that the coding gain is the ratio of the arithmetic and geometric means of the subband variances [1]. It has recently been shown that this ratio is maximized if the analysis filters are such that the decimated subbands satisfy the so called majorization and decorrelation properties [2]. It has further been shown that these two properties can be satisfied by designing each analysis filter to be an optimum energy compaction filter [3]-[5] for an appropriate partial power spectrum defined from the input [2]. However the filters of the optimal compaction solution are ideal filters that can be realized in practice. In this project we studied the practical tree structure implementations of M-channel optimal orthonormal filter banks. We will also develop a very simple formula for the coding gain computation of tree structured filter banks.

三、結果與討論

Consider the M-channel filter bank in Fig. 1. We assume that the filter bank is orthonormal. With σ_x^2 denoting the input variance and with σ_i^2 denoting the subband variances, the coding gain G is given by

$$G = \frac{\sigma_x^2}{\left(\prod_k \sigma_k^2\right)^{1/M}}$$

In the above coding gain formula, optimal bit allocation is implicit. For a given input power spectral density (p.s.d) $S_{xx}(e^{j\omega})$, the variances σ_i^2 depend only on the analysis filters. If the filters are optimized such that the coding gain is maximized, the filter bank is called optimal.

It has been shown that an M-channel PU filter bank is optimal for a given input if and only if the decimated subbands satisfy the following two properties.

1. The subband processes $x_i(n)$ are uncorrelated.
2. Suppose the subbands have been numbered such that

$$\sigma_0^2 \geq \sigma_1^2 \cdots \geq \sigma_{M-1}^2$$

Then for all ω, we have

$$S_{x_0 x_0}(e^{j\omega}) \geq S_{x_1 x_1}(e^{j\omega}) \cdots \geq S_{x_{M-1} x_{M-1}}(e^{j\omega})$$

In this case, the set of power spectra $\{S_{x_k x_k}(e^{j\omega})\}$ is said to satisfy the majorization property.

For a fixed input power spectral density, a filter bank satisfies these two properties has been successfully constructed.

The coding gain of a tree structured filter bank can be expressed in terms of the coding gains of the member filter banks. For example, the coding gain G of the two-level tree structure (Fig. 2) is related to the coding gain, G_0, of the first level FB and the coding gains, G_1 and G_2, of the second level FB by

$$G = G_0 \sqrt{G_1 G_2}$$

Equivalently, we can express the coding gain G_i in dB and obtain

$$G = G_0 + \frac{1}{2}(G_1 + G_2) \quad \text{(dB)}$$

This result can be generalized to tree structured filter bank of more than two levels with member filter banks of more than two channels. For example, suppose FB_1 in the second level has M channels and a further

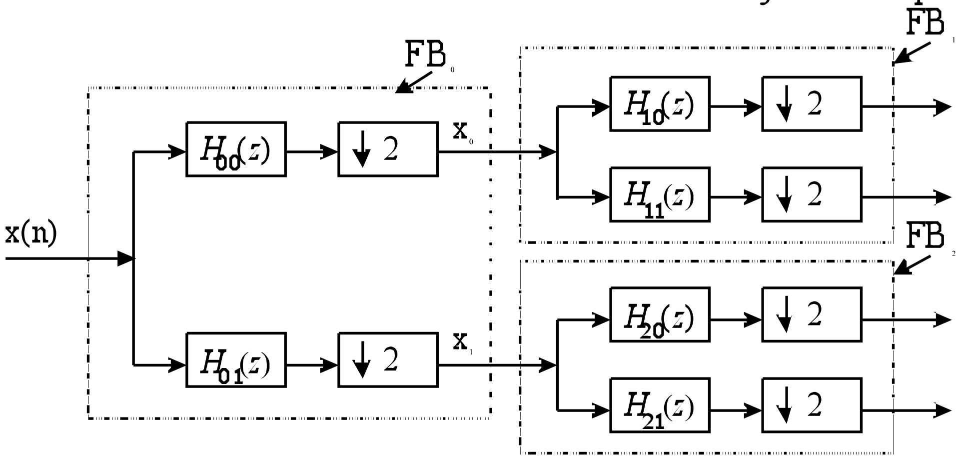

Fig. 2. A two-level tree-structured filter bank.

split is introduced to each subband. Let these M filter banks have coding gain $G_{3,0}$, $G_{3,1}$, … and $G_{3,M-1}$. Then following a similar procedure we can show that the coding gain of the three-level tree structured filter bank is given by

$$G = G_0 + \frac{1}{2}\left(G_1 + G_2\right) + \frac{1}{2M}\sum_{k=0}^{M-1} G_{3,k} \quad \text{(dB)}$$

The generalization to the more general L levels can be carried out in a similar way.

With the above result, we can build the tree structure by considering one level at a time until we reach the given complexity budget, similar to the greedy algorithm. Using such a procedure the member filter bank in the tree structure is optimal for its input power spectral density. However this, in general, does not give us the maximum coding gain for the given tree. It is a sub-optimal design. But it is guaranteed that further splits always provide additional gain as each member filter bank is orthonormal and has coding gain greater or equal to one. Although the overall tree structure may not be optimal, we can observe the following property for the terminal filter banks (member filter banks that do not have further split in their subbands): A terminal filter bank does not affect the coding gains of other filter banks in the previous levels. So to maximize the coding gain of the tree structured filter bank, it is necessary that each terminal filter bank be optimal for its input power spectral density.

However the class of tree structured orthonormal filter banks is only a subset of orthonormal filters even when M is a power of 2. Not every orthonormal filter bank has a tree structure implementation. The use of tree structured orthonormal filter banks does not in general yield the maximum coding gain achievable by orthonormal filter banks.

五、參考文獻

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