# 行政院國家科學委員會專題研究計畫成果報

在聲場回授情況下以DSP實現適應性強健主動式電子消音器

DSP Implementation of an Adaptive Robust Active Electronic Silencer in the Presence of Acoustic Feedback

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#### Abstract

An active noise control (ANC) scheme for suppressing duct noise based on the  $H_{\infty}$  control synthesis are proposed. The proposed approach is a hybrid controller that incorporates a feedback compensator. The controller are designed in terms of performance, stability, and robustness by using a general framework of the  $H_{\infty}$  robust control theory. In addition to the fixed controller, the system is further enhanced by introducing an adaptive compensator based on the least-meanalgorithm. square (LMS) Youla parameterization is employed in designing the adaptive compensator so that the resulting system is internally stable. Experimental investigations demonstrate that the proposed methods are effective in suppressing broadband random noises and transient noises in a finite-length duct.

一,摘要

本計畫以H<sub>∞</sub>控制法則為基礎提出消 除管路噪音的主動控制方法。借引進 一回饋補償器於前饋控制器中,以改 善其強健性之不足。控制器的設計是 利用 H<sub>∞</sub>理論下的廣義架構並考慮系 統的性能,穩定性,強健性。在控制 器合成中,可利用廣義架構的便捷將 聲場回授問題合併於控制器設計中。 此外,固定控制器的效果可再由最小 平方法為基礎的適應補償器進一步改 善其性能。藉由尤拉參數化處理內部 穩定性的問題。實驗顯示出本法有抑 制有限長管路中寬頻隨機噪音與暫態 噪音的能力。

## 二,計畫緣由與目的

Active noise control (ANC) techniques have attracted much research attention because it provides numerous advantages over traditional passive methods in attenuating low-frequency

In ANC applications to date, noises feedforward control has been widely used. The filtered-x least-mean-square (FXLMS) method and its variants have been regarded as one of the most important feedforward ANC algorithms Aside from the difficulty of obtaining an upstream correlated reference, acoustic feedback has been a plaguing problem that usually arises in feedforward structure. The anti-sound output to the loudspeaker not only cancels noise downstream by minimizing the error signals measured bv the error microphone but also radiates upstream to the reference microphone Whenever acoustic feedback is present, a positive feedback loop will exist between the canceling loudspeaker and the feedforward microphone, which tends to destabilize the ANC system.

This paper adopts a new approach to tackle the acoustic feedback problem in the ANC application domain. Semiadaptive  $H_{\infty}$  controllers based on a hybrid scheme is proposed. The  $H_{\infty}$ synthesis automatically incorporates the acoustic feedback path into the control design. To alleviate the problem, the hybrid structure is employed to improve its robustness Youla parameterization is employed in the design of the adaptive compensator so that the resulting system is internally stable

三,結果與討論

#### The Fixed $H_{\infty}$ Controllers

The general input-output relation can be

expressed as  

$$\begin{bmatrix} V(z) \\ E(z) \end{bmatrix} = \begin{bmatrix} P_{11}(z) & P_{12}(z) \\ P_{21}(z) & P_{22}(z) \end{bmatrix} \begin{bmatrix} W(z) \\ U(z) \end{bmatrix},$$

$$= P_{\gamma}(z) \begin{bmatrix} W(z) \\ U(z) \end{bmatrix} \quad (1)$$

The rationale of the  $H_{\infty}$  control is to minimize the infinity norm of the transfer function  $T_{vw}(z)$  Hence, the mathematical statement of the optimal  $H_{\infty}$  problem reads

$$\min_{C(z)} \left\| T_{vw}(z) \right\|_{\infty} = \min_{C(z)} \sup_{0 \le \theta < 2\pi} \left\| T_{vw}(e^{j\theta}) \right\|.$$

(2)

However, instead of the optimal solution, only the suboptimal solution can be analytically obtained and this becomes the so-called *standard*  $H_{\infty}$  *problem*: finding C(z) such that  $\|T_{vv}(z)\|_{\infty} < 1$ .

The  $H_{\infty}$  algorithms are by large divided into two classes: the model matching algorithms and the two Riccati equation algorithms. In the study, we adopt the latter approach because it has better numerical properties than the former.

As mentioned previously, acoustic feedback generally causes detrimental effects to the ANC systems. The positive feedback loop must be taken into account in the design of controllers. Indeed, the beauty of the  $H_{\infty}$  general framework lies in the fact that it needs no special treatment insofar as acoustic feedback is concerned.

Although the feedforward controller is effective in broadband noise attenuation, its performance could vary drastically with system perturbations It is then desirable to develop an ANC controller capable of accommodating these detrimental effects. To this end, another feedback we introduce controller to the feedforward system in order to improve the system robustness, while on the other hand the advantages of the feedforward controller are preserved. This is what we term the hybrid structure that has been investigated in some literature The block diagram of the hybrid structure

with acoustic feedback is illustrated in Fig. 1. To find a  $H_{\infty}$  controller, we weight the error signal e(k) by  $W_1(z)$ , the control input u(k) by  $W_2(z)$ , and the downstream sensor output y(k) by  $W_3(z)$ . The input-output relation of the augmented plant corresponding to the hybrid structure can be expressed as

$$\begin{bmatrix} Z_{f}(z) \\ Z_{f}(z) \\ Z_{f}(z) \\ Z(z) \\ E(z) \end{bmatrix}, \quad (3)$$

$$= \begin{bmatrix} W_{f}(z) & -W_{f}(z)P(z) & -W_{f}(z)S(z) \\ 0 & 0 & W_{f}(z) \\ 0 & 0 & W_{f}(z) \\ 0 & W_{f}(z)P(z) & W_{f}(z)S(z) \\ 1 & -P(z) & -S(z) \\ 0 & 1 & F(z) \end{bmatrix}$$

The suboptimal condition of the  $H_{\infty}$  hybrid controller.

In  $H_{\infty}$  control synthesis, the optimal tradeoff between performance and stability is achieved by tuning the weighting functions  $W_1(z), W_2(z)$ , and  $W_3(z)$  that generally chosen to be low-pass, constant, and high-pass, respectively.

#### The Semi-Adaptive $H_{\infty}$ Controllers

In practical applications, ANC schemes with more flexibility should be sought to further enhance the performance of the system. Nest an adaptive compensator based on LMS algorithm to the original fixed controller. Because of the coupling between the fixed controller and the adaptive compensator, we refer the scheme as the *semi-adaptive* controller.

The semi-adaptive hybrid structure M(z) where M(z) heart of the semi-adaptive design, U(z) heart of the semi-adaptive design, W(z), represents the adaptive compensator that is aimed to *fine-tune* the fixed controllers. With this setting, the model-matching problem can be expressed as

$$\min_{W} \left\| T[P + S \frac{(C_1 + W) - PC_2}{1 - F(C_1 + W) + SC_2}] \right\|,$$
(4)

Note that T(z), acts like a weighting function. In practice, T(z)may not be measurable so that analytical modeling or heuristic guess is needed.

The compensator W(z) must be designed with care under the constraint that the resulting system is internally stable. For a stable SISO plant K(z), the Youla's theorem states that the controllers W(z) that guarantee the internal stability of the closed-loop system can be parametrized as

$$W(z) = \frac{X(z) + M(z)Q(z)}{Y(z) - N(z)Q(z)},$$
(5)

where the parameter Q(z) is some stable, proper, and real-rational function (denoted as  $Q(z) \in RH^{\infty}$ ),

$$K(z) = N(z) / M(z)$$

$$=\frac{-F(z)}{1-C_2(z)F(z)+C_1(z)S(z)}$$

is the coprime factorization of K(z), and  $X(z) \in RH^{\infty}$  and  $Y(z) \in RH^{\infty}$ satisfy the Bezout identity

$$N(z)X(z) + M(z)Y(z) = 1.6$$

If the plant is proper and stable, as it is in our case, one may simply let M(z)=1, N(z)=K(z), X(z)=0, and Y(z)=1. Then the compensator in Eq. (10) turns out to be  $W(z)=\frac{Q(z)}{1-K(z)Q(z)}$ .<sup>(7)</sup>

Substituting Eq. (7) into the minimization problem in Eq. (4) leads to

$$\min_{Q \in RH^{-}} \left\| T[P + S \frac{(1 - KQ)(C_1 - PC_2) + Q}{(1 - KQ)(1 - FC_1 + SC_2) + FQ}] \right\|.$$

(8)

The  $Q_{opt}(z)$ , can be obtained by using the LMS algorithm, in that Q(z) is an FIR filter. With this choice, the requirement in the Youla's parameterization that Q(z) must be proper and stable is automatically satisfied. The update formula for the LMS algorithm is derived as follows. The update equation for the parameter filter Q(k)

$$Q(k+1) = Q(k) - \mu x'(k) e(k), (9)$$

where x'(k) = m(k) \* x(k) is the filtered input signal. After  $\hat{Q}_{opt}(z)$  is obtained from the LMS iteration, the optimal controller  $W_{opt}(z)$  can be recovered by

$$W_{opt}(z) = \frac{Q_{opt}(z)}{1 - K(z)Q_{opt}(z)}.$$
 (10)

As pointed out by the reviewer, the neglected terms in the gradient and the use of truncated impulse response might weaken somewhat the stability, although this is not evidenced in the study.

### 四,計畫成果自評

The semi-adaptive hybrid active noise controller has been developed for

suppressing duct noise. The controller design is based on a general framework of the  $H_{\infty}$  theory that takes into account performance, stability, and robustness of the control system. The  $H_{\infty}$  synthesis procedure automatically incorporates the acoustic feedback path. These ANC system was implemented by using a floating-point DSP. However, it is justified in the experimental results of a finite-length duct obtained by using the semi-adaptive  $H_{\infty}$  hybrid control, the performance has been improved by introducing the feedback compensator and the LMS compensator, where the Youla parameterization is employed in designing the adaptive compensator. Even in the presence of serious acoustic feedback, the proposed method show potential in suppressing not only stationary noises but also transient noises that are commonly encountered in industrial applications.

五,參考文獻

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  六、圖示說明

Figure 1. Hybrid ANC structure including an acoustical feedback path.(a) Experimental setup; (b) control block diagram.

Figure 2. Block diagram of the semiadaptive  $H_{\infty}$  hybrid controller.





\* The oblique part stands for the path of acoustic feedback.

(b)



