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微共振腔量子光學

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Abstract

For the first time the quantum Langevin equation for a coherently driven cavity with two-level atoms is derived. The theory is consisted with the conservation of commutators. The transmission spectra and squeezed state generation in strong coupling regime are presented.

Since 1980s, the interest and success in measuring reduced quantum fluctuations below shot noise limit in many optical systems make necessary quantum theories in modeling and explaining experimental situations. The reduced noise states are called squeezed states [1]. In solving quantum optical problems in which damping and fluctuations are concerned, there exist two equivalent theoretical techniques: one is the master equation approach [2,3] which describes the time evolution of the density matrix operator, and the second is the quantum Langevin equation approach, which directly describes the dynamics of quantum operators. In this paper, we derive the quantum Langevin equations for solving a fundamental quantum optical system consisting of two level atoms in a high Q optical cavity illustrated in Fig. 1. In determining the magnitudes of Langvin forces, we use the conservation of optical field and atomic commutators. The mathematics involved here is much simpler than the master equation approach and the physical picture is also more straightforward and intuitive. Using these Langevin equations we obtain the squeezing spectra for both the absorptive and dispersive normal mode coupling systems.

The resultant quantum Langevin equations are shown as follows:

$$
\begin{aligned}
\frac{da}{dt} &= -\gamma_c(1+i\varphi)a - igN\sigma^- + \sqrt{2\gamma_c}S_{in} + \Gamma_a(t) \\
\frac{da^+}{dt} &= -\gamma_c(1-i\varphi)a^+ + igN\sigma^+ + \sqrt{2\gamma_c}S_{in}^+ + \Gamma_{a^+}(t) \\
\frac{d\sigma^-}{dt} &= -\gamma_a(1+i\Delta)\sigma^- + i2ga(t)\sigma_z + \Gamma_{\sigma^-}(t) \qquad (1) \\
\frac{d\sigma^+}{dt} &= -\gamma_a(1-i\Delta)\sigma^+ - i2ga^+\sigma_z + \Gamma_{\sigma^+}(t) \\
\frac{d\sigma_z}{dt} &= -\gamma_d\left(\sigma_z + \frac{1}{2}\right) + ig\left(a^+\sigma^- - a\sigma^+\right) + \Gamma_{\sigma_z}(t)
\end{aligned}
$$

Here $\phi = (\omega_c - \omega_L)/\gamma_c$, $\Delta = (\omega_a - \omega_L)/\gamma_a$, g is the photon-atom coupling coefficient, a , a^+ are photon annihilation and creation operators, respectively, and σ^-, σ^+, σ_z are Pauli operators, writing

$$
\sigma^- = c_1^+ c_2 ; \sigma^+ = c_2^+ c_1 ; \sigma_z = \frac{1}{2}\left(c_2^+ c_2 - c_1^+ c_1\right), \qquad (2)
$$

and c_1, c_2 are the annihilation operators of the two states of the atom, $S_{in}(t)$ is the external driving field of frequency ω_L. Γ_a and Γ_{a+} are the Langevin forces for the optical field, and $\Gamma_{\sigma-}$, Γ_{σ^+}, and Γ_{σ_z} for the atoms.

When the noise terms in equation (1) are ignored, the equations become the well-known Maxwell-Bloch equations. From them, one may obtain the deterministic steady state solutions P_0, P^*_0, D_0, and the optical bistability state equation shown as follows:

$$
Y = I\left[\left[1 + \frac{2C}{1+\Delta+I}\right]^2 + \left[\phi - \frac{2C\Delta}{1+\Delta+I}\right]^2\right], \qquad (3)
$$

Here $2C = g^2 N/\gamma_a\gamma_c$ is the cavity cooperativity parameter and $I = |A_0|^2/n_0$ and $Y = |S_{in}|^2/n_0\gamma_c^2$, with $n_0 = \gamma_d\gamma_a/4g^2$ being the saturation intensity on resonance. The optical bistability can be found in steady state transmitted response function I/Y.

When the magnitudes of quantum fluctuation terms are small compared to the deterministic steady state solutions, A_0, P_0, D_0, one can solve the problem by linearizing equation (1). Substituting $a = A_0 + \delta a$, $\sigma^- = P_0 + \delta p$, $\sigma_z = D_0 + \delta d$, into equation (1), we obtain the following matrix form of 5 linear equations:

$$\frac{d\delta A(t)}{dt} = M\delta A(t) + N(t). \quad (4)$$

Here $\delta A(t)$ represents the first order fluctuations of the field and atomic operator variables, M is a constant 5 5 matrix, and N (t) is a δ – function correlated noise vector with zero mean.

Applying the conservation of correlation matrix $<\delta A(0)\ \delta A^T(0)> = <\delta A(t)\ \delta A^T(t)>$, we derive the noise correlation matrix

$$\begin{aligned} &\langle N(t) N(t') \rangle = B\delta(t - t') \\ &B = -M\langle \delta A(0)\delta A^T(0) \rangle - \langle \delta A(0)\delta A^T(0) \rangle M^T, \end{aligned} \quad (5)$$

where the bracket denotes the reservoir average. The equation above shows the noise correlation matrix is determined directly by $<\delta A(0)\ \delta A^T(0)>$. The noise correlation matrix B is expressed as the follow:

$$B = \left[\begin{array}{cc:ccc} 0 & 2\gamma_c(\bar{n}(T)+1) & 0 & 0 & 0 \\ 2\gamma_c\bar{n}(T) & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & \frac{2\gamma_a}{N} & \frac{\gamma_d P_0}{N} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\gamma_d P_0^*}{N} & \frac{\gamma_d\left(D_0 + \frac{1}{2}\right)}{N} \end{array}\right]. \quad (6)$$

And such obtained noise correlation matrix and Langevin equations are automatically consistent with the field and atom commutators, i.e.

$$\begin{aligned} &[\ a(t), a^+(t')\ ] = \delta(t - t'), \\ &\mathit{and}\ [\ \sigma^{\pm}, \sigma_z\ ] = \mp\sigma^{\pm},\ [\ \sigma^-, \sigma^+\ ] = -2\sigma_z \end{aligned} \quad (7)$$

We have applied this approach to calculate the squeezed spectra $V(M_\theta\ \omega)$ of the quadature field $M_\theta(t)$ of the transmitted light S_{out}, [4] where

$$M_\theta(t) = e^{i\theta}S_{out}(t) + e^{-i\theta}S_{out}^+(t), \quad (8)$$

and

$$V(M_\theta, \omega) = \int_{-\infty}^{\infty} e^{-i\omega\tau}\langle M_\theta^+(t+\tau), M_\theta(t)\rangle d\tau, \quad (9)$$

We note $V(M_\theta\ \omega)$ =1 for a coherent state and squeezing occurs for $V(M_\theta\ \omega) < 1$ and perfect squeezing corresponds to $V(M_\theta\ \omega) = 0$.

For minimum squeezed states, $V(M_\theta\ \omega) < 1$ and $V(M_\theta\ \omega) \circ V(M_{\theta + \pi/2}, \omega) = 1$.

In Fig. 2, we plot the transmission spectrum and the squeezing spectrum of the absorptive case. Similar spectra for the dispersive case are shown in Fig.3. The parameters for both cases are listed in the figure captions. The normal mode splitting ($g\sqrt{N}$) is clearly shown in the transmission spectra. The splitting is caused by the strong coupling of the field and atom, i.e. $g\sqrt{N} >> \gamma_\chi, \gamma_\alpha$. The spectrum depicts the coherent exchange of excitation between the cavity field and the atomic polarization. The presence of this dynamic process suggests its possible use for the generation of squeezed states, which is verified in our simulation. We found that a significant degree of squeezing is peaked around the coupling frequency $g\sqrt{N}$ [5]. About 13 dB squeezing is shown in Fig. 3(b). However, the states are not the minimal squeezed states as those generated by the parametric wave mixing processes. These squeezing spectra Fig.4 (a) are compared with those calculated from the master equation approach as shown in the Fig. 4 (b). [6]. The agreement confirms the validity of our method.

Reference:

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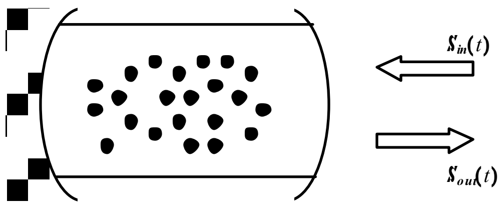

Fig.1 Scheme of the composite system with N two-level atoms coupled to a single mode cavity with a coherent driven field S_{in}

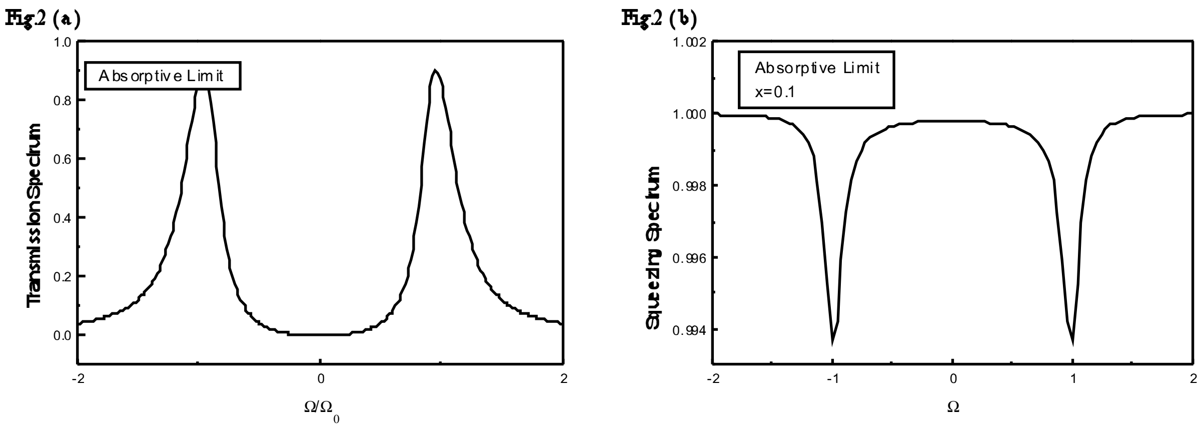

Fig. 2 (a) Plot of transmission spectra in absorptive limit. (b) Plot of squeezing spectra in absorptive limit. $\Delta=\varphi=0$,C=100,$\gamma_c=2$,$\gamma_a=0.5$,g N=14.14,x=0.1

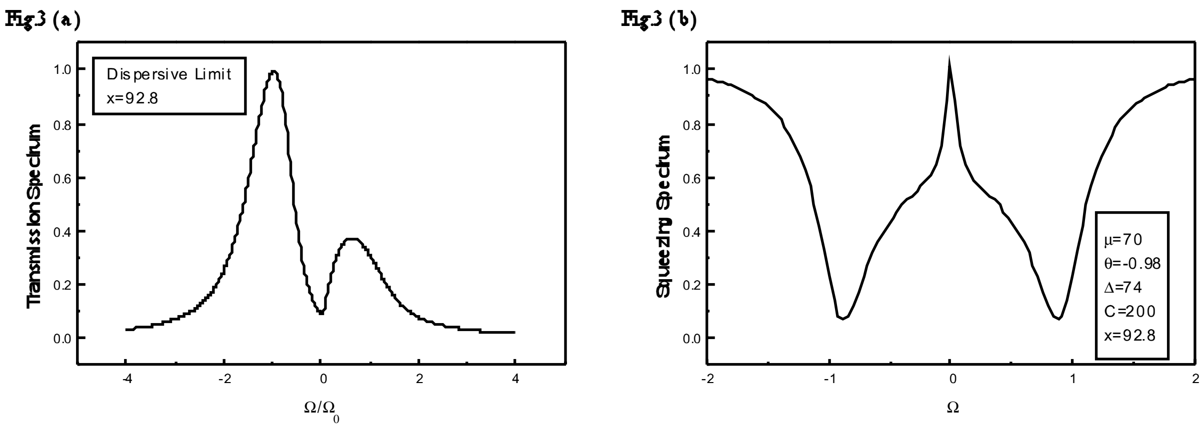

Fig. 3 (a) Plots of transmission spectra in dispersive limit. (b) Plot of squeezing spectra in absorptive limit.$\varphi=-0.98$, $\Delta=74$,C=200,$\gamma_c=70$,$\gamma_a=0.5$,g N=118.32.

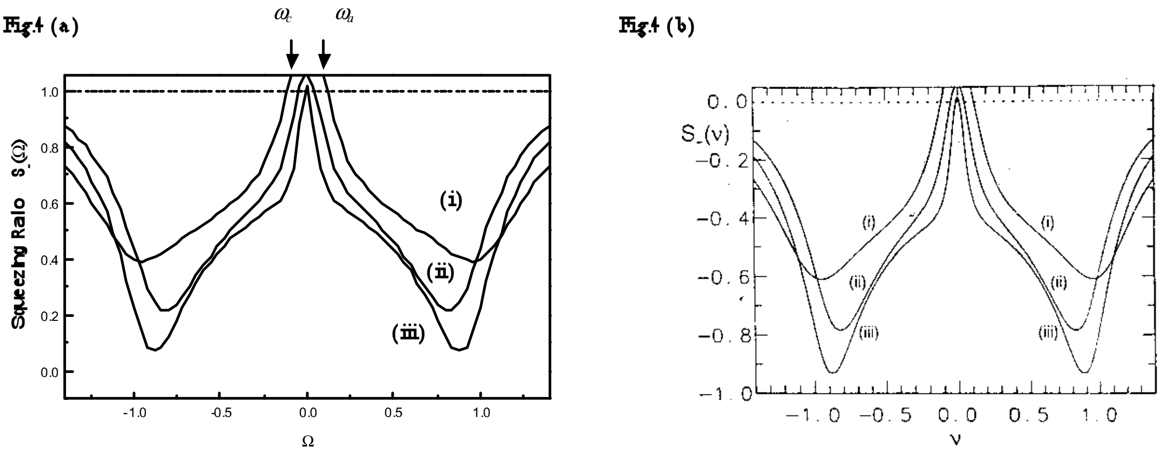

Fig.4 Comparison of the theoretical results using two equivalent quantum approaches. (a) our results (b) Orozco *et a* [6]. 1987