行政院國家科學委員會專題研究計畫成果報告

克爾透鏡鎖模雷射之非線性動力學研究
Studies on Nonlinear Dynamics of Kerr-Lens Mode-Locking Lasers

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一、中文摘要

基於線性穩定分析，我們定義一般雷射腔之穩定因子。因為物理系統趨向於高穩定狀態，由克爾鎖模共振腔對鎖模和連續光操作的相對穩定性當選擇條件，我們決定的最佳自啟動克爾鎖模的範圍與實驗報導之結果相符。

關鍵詞：線性穩定分析、克爾鎖模、雷射、非線性動力學

Abstract

The generalized stability factor of a general resonator was obtained from its iterative map based on the linear stability analysis. Since a physical system tends to stay with high stability, the preferred resonator for Kerr-lens mode-locking is determined by the relative stability between Kerr-lens mode-locking and cw operations. With this criterion the preferable Kerr-lens mode-locking regions agree with the previous experimental self-starting regions.

Keywords: Linear Stability Analysis, Laser, Nonlinear Dynamics, Kerr-Lens Mode-Locking,

二、簡介

In recent years, an approach borrowed from the nonlinear dynamics has been used to study the dynamics of laser resonators [1-3]. By constructing the iterative maps from the beam parameters, the dynamics is very sensitive to nonlinear effect in some special cavity configurations within the geometrically stable region.

In a Kerr-lens mode-locking (KLM) resonator, which is a well-known nonlinear resonator for femtosecond pulse generation, the self-focusing effect within a Kerr medium modifies the cavity mode profile [4,5] to introduce a self-amplitude modulation (SAM). Although it was believed that the KLM Ti:sapphire lasers could not start without initial perturbation because of too low nonlinear SAM, several groups had reported self-starting KLM Ti:sapphire lasers without perturbation [6-8]. The self-starting KLM lasers can be achieved by a carefully analytical cavity design [9,10] to optimize dynamic loss modulation for hard aperturing [6] or dynamic gain modulation for soft aperturing [7,8].

Because the Kerr parameter, the beam power over the critical power of self-trapping [11], can be used to distinguish the laser resonators operating at KLM or cw in the spatial domain [9-12], we will construct two-dimensional map corresponding to curvature and spot size. The eigenvalue of the map represents the variant rate of the dynamic system against a small perturbation.

Comparing the eigenvalues between KLM and cw operations, we can define the relative stability and obtain the preferred configurations for KLM resonators.

三、原理

Consider a dynamic system evolving with an n-dimensional state vector **y** governed by

$$\dot{y} = F(y;k), \quad y \in R^n, \qquad (1)$$

where k is the dynamical parameter. Using the linear stability analysis, the time evolution of a small perturbation on the state vector, $u = \delta y$, at the fixed point can be written as [13]

$$\dot{u} = [D_y F(y;k)]u + O(|u|^2), \qquad (2)$$

where $D_y F$ is the derivative of F to state vector y and $O(|u|^2)$ denotes the order of norm of u on R^n to the second power. As a result, the dynamic stability at the fixed point is simply determined by solving Eq. (2). This is equivalent to calculate the eigenvalues of the Jacobian matrix at the fixed point for a system governed by an iterative map. The system is dynamically stable when all the moduli of eigenvalues are less than unity and it is unstable if at least one of them is greater than one. Therefore, the stability of the system determined by the dynamic stability of the map is governed by the largest modulus of the eigenvalues. The largest modulus of the eigenvalues, presented as χ is defined as the stability factor of the dynamic system. When the dynamic system $F(y;k)$ is subjected to a small increment of dynamical parameter from k to k+δk, the time evolution of small perturbation on the state vector becomes

$$\dot{u} = [D_y F(y;k)]u + \left\{\frac{\partial[D_y F(y;k)]}{\partial k} + \sum_i \frac{\partial[D_y F(y;k)]}{\partial y_i}\frac{\partial y_i}{\partial k}\right\}\delta k\, u + O(|u|^2) \qquad (3)$$

Similarly, the stability of the system with dynamical parameter k+δk is determined by solving Eq. (3) for the field or calculating equivalently the eigenvalues of the Jacobian matrix for the map at the fixed point.

Moreover, it is worth to note that the χ represents the convergent (or divergent) rate of the system against a small perturbation. The value of χ is the smaller, the mode is the more stable. Thus, the relative stability between the systems with a small successive increment of k can be defined by

$$\gamma(k) \equiv \frac{\delta\chi}{\delta k} = \frac{\chi(k+\delta k) - \chi(k)}{\delta k}. \qquad (4)$$

The dynamic system tends to stay at the lower stability factor, so $\gamma(k) < 0$ represents more stable with increment of k.

The iterative map for the resonator configuration is derived from propagating the complex beam parameter q in the cavity. By adopting the transfer matrix of the q-parameter propagating across the Kerr medium [11], we can obtain all matrices for Gaussian beam across all optical components. Then the iterative map is easily derived from ABCD law [14]. Assuming that $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the round-trip transfer matrix and the reference plane chosen to be just after the beam leaving the end mirrors M_1. We can relate the q-parameter of the (n+1)-th round-trip to the n-th one as

$$R_{n+1} = \left\{ \mathrm{Re}\left[\frac{C + D\left(\frac{1}{R_n} - i\frac{\lambda}{\pi w_n^2}\right)}{A + B\left(\frac{1}{R_n} - i\frac{\lambda}{\pi w_n^2}\right)} \right] \right\}^{-1} \qquad (5)$$

and

$$w_{n+1} = \left\{ -\frac{\pi}{\lambda}\mathrm{Im}\left[\frac{C + D\left(\frac{1}{R_n} - i\frac{\lambda}{\pi w_n^2}\right)}{A + B\left(\frac{1}{R_n} - i\frac{\lambda}{\pi w_n^2}\right)} \right] \right\}^{-1/2},$$

(6)

Then we obtained the two-dimensional iterative map, which the time interval is equal to round-trip time of the resonator. The fixed point, (R_0, w_0), of the map is the self-consistent solution of the geometrical resonator, i.e., the steady-state solution [9,10]

Solving the eigenvalues of Jacobian matrix at the fixed point on the map, we get the stability factor

$$\chi = \left(\left(A_r + \frac{B_r}{R_0} + \frac{B_i \lambda}{\pi w_0^2} \right)^2 + \left(A_i + \frac{B_i}{R_0} - \frac{B_r \lambda}{\pi w_0^2} \right)^2 \right)^{-1} \quad (7)$$

where the subscript r and i represent the real and imaginary parts of the elements in the round-trip transfer matrix.

When all optical components are represented as the first-order real transfer matrices, we have proved that the dynamical behavior of the map is equivalent to the behavior of the simple harmonic oscillation. The iterative map belongs to a Hamiltonian system. Furthermore, the loss optical component is usually represented as the transfer matrix having the complex elements and the dynamical behavior becomes a damping oscillation and the imaginary part of the matrix element corresponds to the damping parameter, it is still governed by its Hamiltonian [2]. The real system always has loss such as the mirror having finite extend. A Gaussian function is usually used to taper the mirror with finite extend as a loss component [16] and the taper constant will correspond to the damping parameter. When the loss is included, the stability factor stands for the converge rate of the system against perturbation. Under the same damping parameter, the faster converge rate with the smaller stability factor implies the more stable of the system between neighbor dynamical parameter.

The system with the variation of the Kerr effect is still equivalent to the simple harmonic oscillation and just has different focusing strength for different K. Whether this "oscillator" prefers to operate at the cw operation (K=0) or KLM operation (K>0) is determined by the relative stability

$$\gamma_0 = \left. \frac{\delta\chi}{\delta K} \right|_{K=0} . \quad (8)$$

If $\gamma_0<0$, KLM operation have faster converge rate against perturbation than cw one under the same damping parameter. In other word, the KLM operation is more stable than cw one in such resonator structure. We will use the criterion, $\gamma_0<0$, to determine the resonator for preferable KLM operation. It is worth to note that the map has only one fixed point associated with the steady-state solution for a fixed K. This fixed point stands for cw operation as K=0 and KLM operation as K>0. Discussing the stability neighbor K=0 is capable to determine the tendency about the resonator preferring toward KLM or cw operation.

四、結果與討論

For comparing the theoretical results with the experimental data [6], our studies focused on the symmetrically hard aperturing KLM resonator. The resonator's parameters, shown in Fig.1, are the same as in Ref. [6]. The equal arms d_1 and d_2 are 850 mm, the radii of curvature on the curved mirrors M_2 and M_3 are both 100 mm, and the Brewster-cut Ti:sapphire rod is L=20 mm. Considering the astigmatism compensation of Brewster-cut about the rod, the curved mirrors are tilted by $\theta=14.5^0$. The separation of the curved mirrors, z, and the distance, x, between the curved mirror M_2 and the rod endface I are the adjustable variables. Moreover, we considered the resonator as two orthogonal astigmatic optical systems corresponding to the tangential and sagittal planes, then we will construct the iterative maps for corresponding planes. In a hard aperturing KLM laser, one normally inserts a slit near M_1 to constrain the tangential spot size. Thus, the map of tangential plane determines the stability of the resonator from calculating the eigenvalues of the map's Jacobian matrix at the fixed point. The hard aperturing with $\delta<0$ [6], δ denoting as the small signal relative spot size variation, represents that the system has the capability to sustain the KLM operation. Of course, the system also has the capability to sustain the cw operation due to the resonator satisfying with geometrically stable condition. Whether the resonator prefers the KLM operation is further determined by γ_0 of tangential plane.

The dynamical behavior of the system is governed by its Hamiltonian and the loss associates with the damping effect. The preferable operation will not change as varying the tapering constant. The numerical verification is shown in Fig. 2. Fig. 2 shows the relation between γ_0 and the tapering constant with z=116.5 mm. Owing to the tapering constant just corresponding to the damping parameter, we simplified to add a Gaussian tapering at M_4. From Fig. 2, the tendency is classified into two cases. One is that γ_0 is always greater than zero and a monotonically decreasing function of the tapering constant such as at x=45 mm in Fig. 2. The other has contrary change with γ_0 being always less than zero and increasing γ_0 against the tapering constant such as at x=50 mm in Fig. 2. Although γ_0 depends on the tapering constant for a resonator, the sign of γ_0 is unchanged, i.e., the preferable operation ($\gamma_0<0$ for KLM or $\gamma_0>0$ for cw) of the resonator is independent of the tapering constant. The result agrees with the previous discussion. Thus the tapering constant is set as 10 cm in the following simulations.

For example, at z=116.5 mm and x=50 mm, the stability factor is a monotonically decreasing function of K and the relative stability $\gamma_0<0$. The laser system is more stable by appending power to mode-locking rather than to cw and the resonator prefers the KLM operation. In fact, the stability factor is not always a monotonically decreasing function of K for some z and x with the relative stability $\gamma_0<0$. They have one minimum stability factor under the reasonable range of K value in experiments (K<0.4). Owing to the K standing for the beam power of the KLM laser, the above cases represent that the higher power operation is unstable than the lower power one and these resonators are not easy to obtain higher KLM power. This phenomenon had been observed in various experiments, e.g., Ref. [18].

The contour figure of γ_0 as a function of z and x in the tangential plane is shown in Fig. 3 where the dot marks are the duplicated self-starting results in Ref. [6] for comparison. The resonator configuration with $\gamma_0<0$ prefers to KLM operation when a mechanism, such as the hard aperturing in this case, has capability to sustain the KLM and cw operations. We find that the regions with $\gamma_0<0$ agree with the self-starting regions of Ref. [6] when the Kerr medium is placed around the center of the resonator. However, when the Kerr medium is placed far away the center of the resonator, the beam waist may be located far away the center of Kerr medium or outside the material. Then the effects of beam focalization [11] and the efficiencies of extracting power from gain medium must be considered in practice. We think that this is main reason for unpredicted results of our method. Owing to the strength of mechanical tapping may be beyond that of the intrinsic perturbation discussed above to cause large cavity structure change, our simple approach is not suitable for the KLM initiated by mechanical tapping.

On the other hand, another approach borrowed from the classical mechanics can also be used to verify the previous results. We can obtain the Hamiltonian for a resonator without considering the loss, which is the function of K and donated as H(K). Because the Hamiltonian represents the energy of the harmonic oscillation system and the system prefers to stay at the lower energy, $\left.\frac{\delta H(K)}{\delta K}\right|_{K=0} < 0$ stands for the system having lower energy with larger K and it preferring to operate at the K>0 (KLM operation). From our numerical experiment, the same regions of the preferable resonators for Kerr-lens mode-locking are obtained from these two approaches. This result also verifies that the dynamical behavior about the preferable operation is governed by the Hamiltonian whatever the system is loss or lossless.

Due to the nature of KLM resonator is sensitive to geometrical configuration, the dynamics of Gaussian beam in bare resonator may govern the preferable condition for self-starting KLM. As a result, even though we do not consider the mechanism of the self-starting in KLM resonator, the regions with $\gamma_0<0$ agree with the self-starting regions of experiment. Moreover, not only the $\delta<0$ region contains $\gamma_0<0$ but also γ_0 is always greater than zero in the region with $\delta>0$. From this result, $\gamma_0<0$ seems to be more strict condition than the one with $\delta<0$. In addition, we can optimize the resonator design by the minimizing γ_0. The minimum γ_0 in the whole region is -8.06×10^{-4} at z=116.1 mm and x=50 mm under above mentioned resonator parameters. The optimal hard aperturing KLM laser is favor to operate near the confocal edge of the geometrically stable region. This result also agrees with the previous one that the KLM favors to operate at the borders of the stability region [6].

五、結論

By considering a two-dimensional iterative map derived from the propagation of q-parameter, we have generalized the stability factor as the modulus of eigenvalue at fixed point. The system tends to operate at lower stability factor as a successive variation on dynamical parameter k because the stability factor corresponds to converging rate against perturbation. We find that the variation of stability factor with respect to the Kerr parameter provides an available criterion for studying self-starting KLM lasers in the bare resonator with Kerr-lens effect only. As a result, the numerical simulation agrees with previous self-starting experimental data in the hard aperturing KLM lasers. In addition, this effective procedure can be used to study preferable resonator configuration for three-mirror KLM or the other mode competition systems. One can obtain optimal resonator designs based on simple mathematical calculations.

六、自我評估

在去年的研究中我們利用留數定理分析簡單雙端鏡共振腔保守系統映像，得到共振腔參數 g_1g_2=1/2, 1/4 和 3/4 時共振腔在非線性微擾下會呈不穩定現象。也以端幫平凹腔 Nd:YVO4 雷射得到驗證，此結果正撰文投稿中。本報告是考慮自聚焦為非線性效應，並分別考慮與不考慮損失的情況下利用映像的本徵值，定義穩定參數。

因為物理系統趨向於高穩定狀態，由克爾鎖模共振腔對鎖模和連續光操作的相對穩定性當選擇條件，我們決定的最佳自啟動克爾鎖模的範圍與實驗報導之結果相符。解釋了多年來無法正確預測此雷射自啟動的機制。最近實驗中我們已順利觀察並取得此雷射系統相關之渾沌現象，正進行數據分析與撰寫論文中，如預期之計劃進度。

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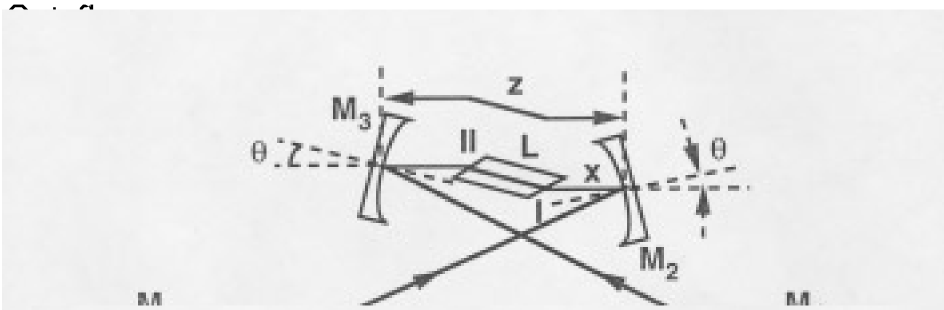

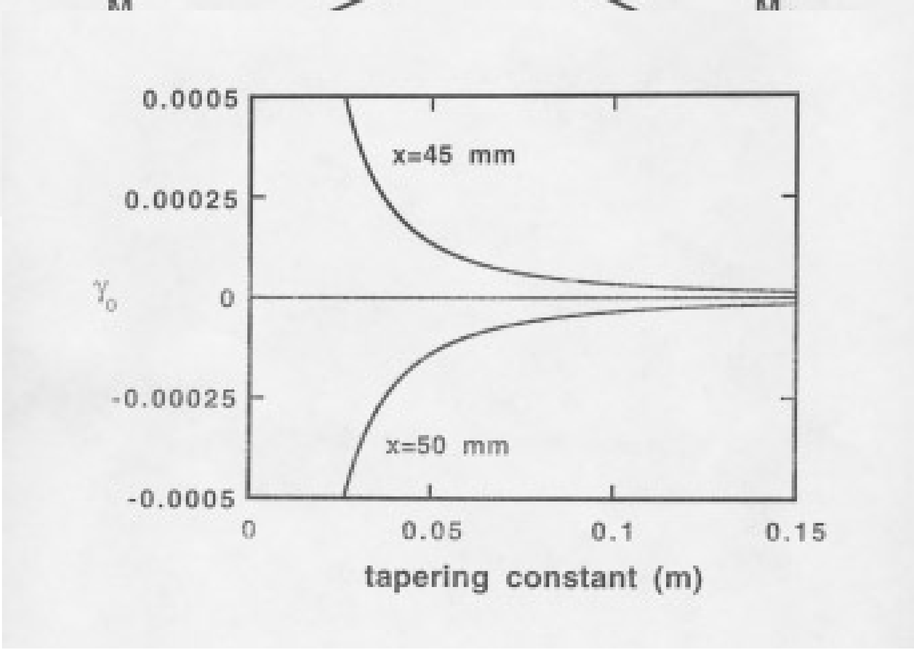

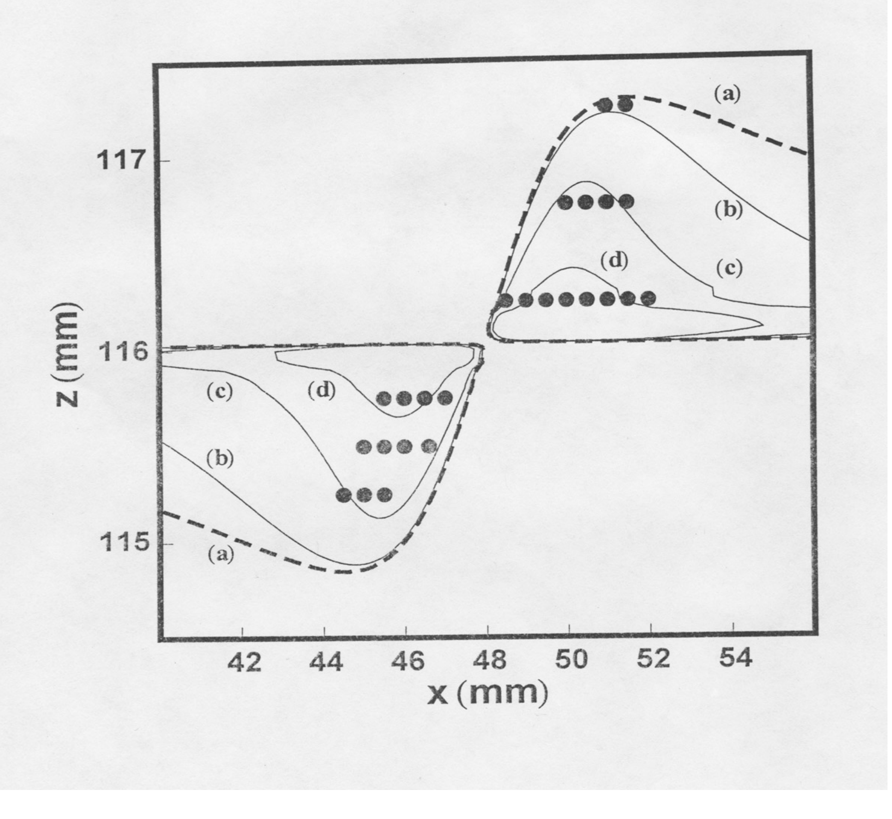

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