Analytical Solution for Tidal Propagation in a Leaky Aquifer Extending Finite Distance under the Sea

Mo-Hsuing Chuang[1] and Hund-Der Yeh[2]

Abstract: This paper focuses on groundwater dynamics in response to the tidal fluctuation in a coastal aquifer system. An analytical solution is derived to describe groundwater level fluctuation in a leaky aquifer extending finite distance under the sea. Based on this solution, the joint effects of various parameters, such as the dynamic effect of water table fluctuation and the leakages of the inland and offshore, on the behavior of the groundwater level fluctuations in the inland part of the leaky confined aquifer can be thoroughly analyzed. When the roof length is increased, the dynamic effect of the water table fluctuation on the dimensionless groundwater amplitude, intrusion distance, and fixed phase shift in the unconfined aquifer become more important and the water table fluctuation approaches constant values when the roof length is greater than a threshold value. However, given the same values of dimensionless leakage and roof length, the dimensionless groundwater amplitude, intrusion distance, and fixed phase shift in the leaky aquifer with considering the dynamic effects are always larger than those of neglecting such effects.

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Introduction

The subject of dynamic relation between groundwater and seawater has received a great deal of attention in the recent years (e.g., Jacob 1950; Gregg 1966; Carr and Van der Kamp 1969; Van der Kamp 1972; Taigbenu et al. 1984; Pandit et al. 1991; Farrell 1994; Svitil 1996; Sun 1997; Oki et al. 1998; Uchiyama et al. 2000). These studies included aquifer parameter estimation, beach dewatering, marine environment, marine retaining structures, and/or seawater intrusion. These papers mentioned that the evaluation of water-table fluctuation in a coastal aquifer is important for various hydrogeological, engineering, ecological, and environmental problems. Some previous studies showed that dynamic effects of the phreatic aquifer on the tidal head fluctuations in the confined aquifer plays an active role in solving coupled leaky-confined/phreatic coastal aquifer problems. For example, based on the assumption that the water-table fluctuation in the shallow unconfined aquifer was negligible, Jiao and Tang (1999) presented an analytical solution to study the groundwater head fluctuations in the confined aquifer of a coastal aquifer system by ignoring the elastic storage of the leaky layer. They found that the leakage has a significant damping effect on the groundwater fluctuation amplitude in the confined aquifer. Li et al. (2001) used perturbation approach to derive an approximate solution in examining dynamic effects of the overlying aquifer. Volker and Zhang (2001) used the finite element program 2DFEMFAT to assess the errors induced by neglecting water level changes in the unconfined aquifer of a leaky aquifer system subjected to tidal sea boundary condition. However, Jiao and Tang (2001) mentioned there is no significant error to neglect dynamic effects of the overlying aquifer, because it is inappropriate to use the leakage value which is as great as 1 per day. Jeng et al. (2002) presented an analytical solution for the tidal response in a fully coupled leaky confined aquifer system considering the effects of the water table fluctuations in the unconfined aquifer. They concluded that the dynamic effects are important under a relatively large leakage and phreatic aquifer transmissivity. Ignoring these effects could lead to errors in estimating aquifer properties based on the tidal signals. Li and Jiao (2001a,b) presented complete analytical solutions to describe tidal groundwater wave propagation in coastal two aquifer systems with considering both the leakage and the storativity of the leaky layer. They found that the assumption of neglecting the effects of the leakage and storativity of the leaky layer is valid only when the storage ratio of the semipermeable to the confined aquifers is less than 0.5 and the storage of the semipermeable layer is small.

The other important topic involved in this study is that the roof of a coastal aquifer may extend for a certain distance under the tidal water. Van der Kamp (1972) derived a solution to describe the groundwater fluctuation in the aquifer with considering an extreme assumption that the roof length is infinite. Chuang and Yeh (2007) developed an analytical solution to investigate the effects of tidal fluctuations and leakage on the groundwater head of leaky confined aquifer extending an infinite distance under the sea. They found the effects of the storativity and transmissivity of the unconfined aquifer on the head fluctuation of the leaky confined aquifer are obvious when the leakage of the inland aquitard is larger than 0.001 per day. In addition, those effects are comparatively noticeable when the leakage of the inland aquitard is large and that of the offshore aquitard is small. In contrast, Li and Chen (1991a,b) considered the situation where the roof length is

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J. Hydraul. Eng. 2008.134:447-454.

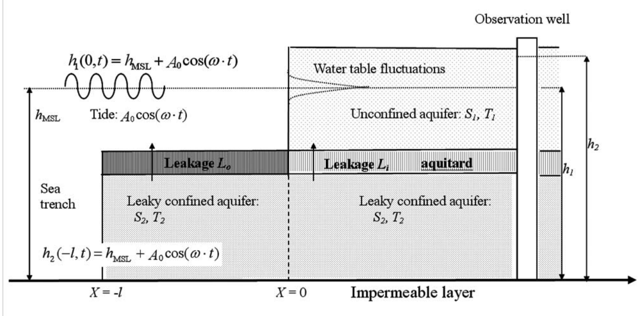

Fig. 1. Schematic diagram of a leaky aquifer

finite. They assumed that there is no leakage from the confining layer. Li and Jiao (2001a) presented an analytical solution for tidal-induced groundwater fluctuation in a coastal leaky confined aquifer extending under the sea to investigate the influences of tidal efficiency, roof length, and leakage of the semipermeable layer on tide-induced groundwater fluctuations. They relaxed the assumption that there is no leakage from the confining layer but the water table fluctuation in the unconfined aquifer is assumed negligible and considered that the leakage of the offshore aquitard is the same as that of the inland aquitard. They showed that there exists a finite threshold value (l_u) of roof length (l), and when $l \geq l_u$, the tidal propagation in the inland aquifer will behave as if the roof length were infinite. They found that the impacts of leakage from the offshore and inland portions of the confining unit are different and the fluctuation increases with the tidal efficiency when the roof length is large and the leakage is small.

The objectives of this paper are to derive a new analytical solution for describing groundwater level fluctuation in a leaky aquifer extending finite distance under the sea and to investigate the dynamic response of the aquifer system to the tidal fluctuation. The leakages of the offshore and inland aquitards are considered different. The leakage effects of both inland and offshore aquitards on the head distribution of the tidal leaky confined aquifer are therefore analyzed. This new solution differs form the solutions of Li and Jiao (2001a) with following two situations: (1) the offshore and inland parts of the aquifer have different hydraulic properties and (2) the water table in the unconfined aquifer fluctuates with tide. The consideration of fluctuation in the unconfined aquifer makes the solution closer to the physical reality of the real world problem. An attempt is made to investigate the influence of those two situations on the behavior of the groundwater level fluctuations in the inland part of the leaky aquifer. In addition, the solution of Chuang and Yeh (2007) can be considered as a special case of this newly derived solution when the roof length extends to infinity under the sea. The joint effects of various parameters, such as dynamic effect of water table fluctuation, roof length, and the leakages of the inland and offshore, on the behavior of the groundwater level fluctuations in the inland part of the leaky confined aquifer can be thoroughly examined.

Problem Setup and Boundary Conditions

Fig. 1 presents a coastal aquifer system with an unconfined aquifer, a leaky aquifer, and an aquitard between them. The origin of the x axis is at the intersection of the mean sea surface and both the coastal line and the x axis are horizontal, positive landward. Consider that both the unconfined and the leaky aquifers, interacting with each other through leakage, have dynamic responses to the tidal fluctuation. The unconfined aquifer terminates at the coast, whereas the aquitard and the leaky aquifer extend finite distance (l) under the sea. Assume that there is a sea trench located beyond the distance l as indicated in Fig. 1. The leakages of the offshore and inland aquitards are different and the bottom of the leaky aquifer is impermeable.

Assume that the aquifer is homogeneous and isotropic and the thickness of the unconfined aquifer is very large in comparison with the magnitude of the tidal fluctuation, therefore allowing the application of confined-aquifer theory to the unconfined one. The flow velocity in the leaky aquifer is assumed horizontal, and a vertical leakage through the aquitard exists. The initial hydraulic head in the whole system is uniform and equals h_{MSL}, which is the distance from the groundwater level to a convenient reference datum. In addition, the aquitard storage is negligible and leakage is linearly proportional to the head difference of the unconfined aquifer and leaky confined aquifer (Bear and Verruijt 1987; Li and Jiao 2001a). Accordingly, the governing equations for the head fluctuation of the inland unconfined and the leaky confined aquifers ($x>0$) can be written, respectively, as (Bear and Verruijt 1987; Li and Jiao 2001a)

$$S_1\frac{\partial h_1}{\partial t}=T_1\frac{\partial^2 h_1}{\partial x^2}+L_i(h_2-h_1) \tag{1}$$

and

$$S_2\frac{\partial h_2}{\partial t}=T_2\frac{\partial^2 h_2}{\partial x^2}+L_i(h_1-h_2) \tag{2}$$

and for the offshore aquifer ($-l<x<0$) is

$$S_2\frac{\partial h_2}{\partial t}=T_2\frac{\partial^2 h_2}{\partial x^2}+S_2T_e\frac{dh_s}{dt}+L_o(h_s-h_2) \tag{3}$$

where h_1 and h_2=hydraulic heads in the unconfined and the leaky aquifers, respectively; h_s=hydraulic head of the sea tide; S_1 and S_2, as well as T_1 and T_2=storativities and transmissivities of these two aquifers, respectively; L_o and L_i=leakages of the offshore and inland aquitards, respectively; and T_e=tidal efficiency, which reflects the fluctuation of groundwater level caused by compression of both the aquifer skeleton and groundwater due to the tidal loading above the offshore aquitard (Jacob 1950). The leakage is defined as the hydraulic conductivity of the aquitard over the thickness of the aquitard. Note that the hydraulic conductivity and/or thickness of the inland aquitard are distinct from those of the offshore aquitard due to the difference of depositional sediment.

The tidal boundary conditions may be written as

$$h_1(0,t)=h_s(t)=h_{\mathrm{MSL}}+A_0\cos(\omega\cdot t) \tag{4}$$

$$h_2(-l,t)=h_s(t)=h_{\mathrm{MSL}}+A_0\cos(\omega\cdot t) \tag{5}$$

where $h_1(0,t)$=hydraulic head at $x=0$; $-l$=distance extending under the sea; A_0=amplitude of the tidal change; and ω=tidal speed and is equal to $2\pi/t_0$, where t_0=tidal is the tidal period. The continuity conditions of the hydraulic head and flux at $x=0$ require, respectively

$$\lim_{x\downarrow 0} h_2(x,t)=\lim_{x\uparrow 0} h_2(x,t) \tag{6}$$

and

J. Hydraul. Eng. 2008.134:447-454.

$$\lim_{x\downarrow 0}\frac{\partial h_2(x,t)}{\partial x}=\lim_{x\uparrow 0}\frac{\partial h_2(x,t)}{\partial x} \tag{7}$$

The boundary conditions for Eqs. (1) and (2) on the inland side may be expressed, respectively, as

$$\lim_{x\to\infty}\frac{\partial h_1}{\partial x}=0 \tag{8}$$

and

$$\lim_{x\to\infty}\frac{\partial h_2}{\partial x}=0 \tag{9}$$

Analytical Solution

Some normalized parameters used in Li and Jiao (2001a) are also adopted hereinafter for the convenience of comparison. The tidal propagation parameter is defined as $a_1=\sqrt{\omega S_1/2T_1}=\sqrt{\pi S_1/T_1 t_0}$ for the unconfined aquifer and $a_2=\sqrt{\omega S_2/2T_2}=\sqrt{\pi S_2/T_2 t_0}$ for the confined aquifer. The dimensionless leakage is $u_i=L_i/\omega S_2$ for the inland aquitard and $u_o=L_o/\omega S_2$ for the offshore aquitard. The dimensionless storativity is defined as $n=S_2/S_1$. The hydraulic heads for the unconfined and the inland leaky aquifers ($x>0$) can be assumed, respectively, as

$$h_1(x,t)=h_{\mathrm{MSL}}+\mathrm{Re}[A_0 X_1(x)e^{-i\omega t}] \tag{10a}$$

and

$$h_2(x,t)=h_{\mathrm{MSL}}+\mathrm{Re}[A_0 X_2(x)e^{-i\omega t}] \tag{10b}$$

and that for offshore aquifer ($-l<x<0$) is

$$h_2(x,t)=h_{\mathrm{MSL}}+\mathrm{Re}[A_0 X_2(x)e^{-i\omega t}] \tag{10c}$$

where Re denotes the real part of the complex expression and $i=\sqrt{-1}$. The variables $X_1(x)$ and $X_2(x)$ are unknown and functions of x.

The solutions for the $X_1(x)$ and $X_2(x)$ with the conditions (4)–(9) can be obtained by substituting Eqs. (10*a*)–(10*c*) into Eqs. (1)–(3) (Chuang and Yeh 2007). Note that a no-flow boundary at $x\to-\infty$ is specified as the remote boundary condition in Chuang and Yeh [2007, Eq. (9)], whereas a free boundary at $x=-l$ is chosen as the tidal boundary condition, Eq. (5), in this paper. The results of $X_1(x)$ and $X_2(x)$ for the inland aquifer ($x>0$) are:

$$X_1(x)=\alpha_1 e^{-\lambda_1 x}+\alpha_2 e^{-\lambda_2 x} \tag{10d}$$

$$X_2(x)=\alpha_1\beta_1 e^{-\lambda_1 x}+\alpha_2\beta_2 e^{-\lambda_2 x} \tag{10e}$$

and that for offshore aquifer ($-l<x<0$) is

$$X_2(x)=\alpha_3 e^{\lambda_3 x}+\alpha_4 e^{-\lambda_4 x}+\beta_3 \tag{10f}$$

The variables α_1, α_2, α_3, α_4, β_1, β_2, β_3, λ_1, λ_2, and λ_3 are defined as

$$\alpha_1=\frac{D_1}{D} \tag{11a}$$

$$\alpha_2=\frac{D_2}{D} \tag{11b}$$

$$\alpha_3=\frac{D_3}{D} \tag{11c}$$

$$\alpha_4=\frac{D_4}{D} \tag{11d}$$

$$\begin{aligned}D={}&e^{-\lambda_3 l}(\beta_1\lambda_3-\beta_1\lambda_1-\beta_2\lambda_3+\beta_2\lambda_2)\\&+e^{\lambda_3 l}(\beta_1\lambda_3+\beta_1\lambda_1-\beta_2\lambda_3-\beta_2\lambda_2)\end{aligned} \tag{11e}$$

$$\begin{aligned}D_1={}&e^{-\lambda_3 l}(\beta_3\lambda_3+\beta_2\lambda_2-\beta_2\lambda_3)+e^{\lambda_3 l}(\beta_3\lambda_3-\beta_2\lambda_2-\beta_2\lambda_3)\\&-2(\lambda_3-\beta_3\lambda_3)\end{aligned} \tag{11f}$$

$$\begin{aligned}D_2={}&e^{-\lambda_3 l}(\beta_1\lambda_3-\beta_1\lambda_1-\beta_3\lambda_3)+e^{\lambda_3 l}(\beta_1\lambda_3+\beta_1\lambda_1-\beta_3\lambda_3)\\&+2(\beta_3\lambda_3-\lambda_3)\end{aligned} \tag{11g}$$

$$\begin{aligned}D_3={}&\beta_1\lambda_3-\beta_1\lambda_1-\beta_2\lambda_3+\beta_2\lambda_2-\beta_1\beta_3\lambda_3+\beta_1\beta_3\lambda_1+\beta_2\beta_3\lambda_3\\&-\beta_2\beta_3\lambda_2+e^{\lambda_3 l}(\beta_2\beta_3\lambda_2+\beta_1\beta_2\lambda_1-\beta_1\beta_2\lambda_2-\beta_1\beta_3\lambda_1)\end{aligned} \tag{11h}$$

$$\begin{aligned}D_4={}&\beta_1\lambda_3+\beta_1\lambda_1-\beta_2\lambda_3-\beta_2\lambda_2-\beta_1\beta_3\lambda_3-\beta_1\beta_3\lambda_1+\beta_2\beta_3\lambda_3\\&+\beta_2\beta_3\lambda_2+e^{-\lambda_3 l}(\beta_1\beta_3\lambda_1-\beta_2\beta_3\lambda_2+\beta_1\beta_2\lambda_2-\beta_1\beta_2\lambda_1)\end{aligned} \tag{11i}$$

$$\beta_1=1-\frac{B_1}{2a_1^2 n u_i}-\frac{i}{n u_i} \tag{11j}$$

$$\beta_2=1-\frac{B_2}{2a_1^2 n u_i}-\frac{i}{n u_i} \tag{11k}$$

$$\beta_3=\frac{T_e i-u_o}{i-u_o} \tag{11l}$$

$$\lambda_1=\sqrt{B_1} \tag{11m}$$

$$\lambda_2=\sqrt{B_2} \tag{11n}$$

and

$$\lambda_3=2a_2\left(\frac{u_o-i}{2}\right)^{0.5} \tag{11o}$$

with the variables B_1 and B_2, respectively, defined as

$$B_1=-c_1-\sqrt{c_1^2-c_2} \tag{11p}$$

and

$$B_2=-c_1+\sqrt{c_1^2-c_2} \tag{11q}$$

and the variables c_1 and c_2 defined, respectively, as

$$c_1=-(na_1^2+a_2^2)u_i+(a_1^2+a_2^2)i \tag{11r}$$

and

$$c_2=-4a_1^2a_2^2(1+nu_i i+u_i i) \tag{11s}$$

Special Cases

If the roof length extends to infinity, the new solutions, Eqs. (10*a*)–(10*f*), will reduce to the solutions for tidal responses in a coupled coastal aquifer system consisting of a semipermeable

J. Hydraul. Eng. 2008.134:447-454.

layer and a leaky aquifer extending infinite distance under the sea. When $l \to \infty$, the variables α_1, α_2, α_3, and α_4 of Eqs. (11*a*)–(11*d*) become α_{1a}, α_{2a}, α_{3a}, and α_{4a}, respectively, and

$$\alpha_{1a} = \frac{\beta_3\lambda_3 - \beta_2\lambda_2 - \beta_2\lambda_3}{\beta_1\lambda_3 + \beta_1\lambda_1 - \beta_2\lambda_3 - \beta_2\lambda_2} \tag{12a}$$

$$\alpha_{2a} = \frac{\beta_1\lambda_3 + \beta_1\lambda_1 - \beta_3\lambda_3}{\beta_1\lambda_3 + \beta_1\lambda_1 - \beta_2\lambda_3 - \beta_2\lambda_2} \tag{12b}$$

$$\alpha_{3a} = \frac{\beta_2\beta_3\lambda_2 + \beta_1\beta_2\lambda_1 - \beta_1\beta_2\lambda_2 - \beta_1\beta_3\lambda_1}{\beta_1\lambda_3 + \beta_1\lambda_1 - \beta_2\lambda_3 - \beta_2\lambda_2} \tag{12c}$$

and

$$\alpha_{4a} = 0 \tag{12d}$$

Those equations, Eqs. (12*a*)–(12*d*), are equal to the solution of groundwater response to tidal fluctuation in a leaky aquifer presented in Chuang and Yeh (2007). Eqs. (11*j*)–(11*s*) are exactly the same as those defined in Chuang and Yeh (2007) except that the variables of β_1, β_2, β_3, λ_3, c_1, and c_2 are in terms of dimensionless parameters.

In addition, when both n and $T_1 \to 0$, the water level in the phreatic aquifer can be considered as a constant. Under this circumstance, the variables α_1, α_2, α_3, and α_4 become α_{1b}, α_{2b}, α_{3b}, and α_{4b}, respectively, if $u = u_i = u_o$. Based on Eqs. (11*a*)–(11*d*), one can obtain:

$$\lambda_1 = \lambda_2 = \lambda_3 = a_2(2u - 2i)^{0.5} = a(p - qi) \tag{13a}$$

$$\beta_3 = \frac{u - iT_e}{u - i} = \lambda - i\mu \tag{13b}$$

$$X_2(x) = \alpha_{1b}\beta_1 e^{-\lambda_1 x} + \alpha_{2b}\beta_2 e^{-\lambda_2 x} = (\alpha_{1b}\beta_1 + \alpha_{2b}\beta_2)e^{-\lambda_1 x} = \gamma e^{-\lambda_1 x} \tag{14a}$$

for inland aquifer ($x > 0$), and

$$X_2(x) = \alpha_{3b} e^{-\lambda_3 x} + \alpha_{4b} e^{\lambda_3 x} + \beta_3 \tag{14b}$$

for offshore aquifer ($-l < x < 0$) with the variables γ, α_{3b}, and α_{4b} defining, respectively, as

$$\gamma = (1 - \beta_3)e^{-\lambda_3 l} + \frac{\beta_3}{2} + \frac{\beta_3}{2}e^{-2\lambda_3 l} = C_1 - C_2 = C_3 \tag{15a}$$

$$\alpha_{3b} = -\frac{\beta_3}{2} = C_2 \tag{15b}$$

$$\alpha_{4b} = (1 - \beta_3)e^{-\lambda_3 l} + \frac{\beta_3}{2}e^{-2\lambda_3 l} = C_1 \tag{15c}$$

Note that the variables a, p, q, λ, μ, C_1, C_2, and C_3 have the same definition as those in Li and Jiao (2001a). The equations, Eqs. (14*a*) and (14*b*), are identical to the solution of groundwater response to tidal fluctuation in a leaky aquifer presented in Li and Jiao [2001a, Eqs. (A9b) and (A9a)]. However, the complex expression used in Li and Jiao (2001a) is $\mathrm{Re}(e^{i\omega t})$, whereas that used in this study is $\mathrm{Re}(e^{-i\omega t})$.

Results and Discussion

Eqs. (10*a*) and (10*b*) are the solutions for the groundwater heads in the inland part of unconfined and confined aquifers, respectively, and Eq. (10*c*) is the solution in the offshore part of aquifer. Most field studies on coastal aquifers focus mainly on the inland part of aquifer and the measurement of groundwater heads in the offshore area is not available. Thus, only the groundwater heads in the inland part of aquifers are discussed in this paper. The dimensionless fixed phase shift (ph) used in Li and Jiao (2001a) is defined as $\mathrm{ph} = \mathrm{Re}(\alpha_1 + \alpha_2)/\mathrm{Im}(\alpha_1 + \alpha_2)$ where Im denotes the imaginary part of complex expression. The amplitude of the tidal fluctuation, A_0, is assumed constant and the normalized groundwater amplitude, $|H_2|/A_0$ or simply HA, is defined as the groundwater fluctuation amplitude of the inland confined leaky aquifer over the tide amplitude. In addition, HA is denoted as HA_{x0} when $x = 0$. Consider that the tidal intrusion distance ($x_{\max}$) is the farthest landward distance from the coastline to the location where HA is less than 10^{-2}. In the following section, the influences of the water table fluctuation, the dimensionless roof length ($a_2 l$), and the leakages on the tidal fluctuations are analyzed through two case studies.

Joint Effects of Water Table Fluctuation and Roof Length

In the first case, the dimensionless inland and offshore leakages are set equal, therefore the dimensionless leakage u is $u = u_i = u_o$. Consider that u varies from 1 to 30 and $a_1 = 10a_2$ for the tidal propagation parameters and the tidal efficiency T_e is equal to 0. Figs. 2(a–c) show the curves of the normalized groundwater amplitude at $x = 0$, HA_{x0}, dimensionless fixed phase shift, ph, and dimensionless intrusion distance in the leaky aquifer, $a_2 x_{\max}$, versus the dimensionless roof length, $a_2 l$. The solid lines denote the present solution, the dash lines represent the present solution when n or $T_1 \to 0$, and the square symbol stands for the solution of Li and Jiao (2001a) without considering the effect of the fluctuation of groundwater level in the unconfined aquifer. The solid lines of Figs. 2(a and b) clearly show that both HA_{x0} and $a_2 x_{\max}$ decrease with increasing $a_2 l$ and their decreasing rates increase with u for small $a_2 l$ and decrease with u for large $a_2 l$. In contrast, the solid lines of Fig. 2(c) display that ph increases with $a_2 l$ and its increasing rate increases with u for small $a_2 l$ and decreases with u for large $a_2 l$. The solid lines in Figs. 2(a–c) demonstrate that the HA_{x0}, $a_2 x_{\max}$, and ph in the leaky aquifer will approach constant values when $a_2 l$ is greater than a threshold value. In other words, when $a_2 l$ increases, the HA_{x0}, $a_2 x_{\max}$ and ph become less and less sensitive to the $a_2 l$. In addition, the threshold value of $a_2 l$ decreases with increasing u.

Figs. 2(a–c) also indicate that when the $a_2 l$ is increased, the dynamic effect of the water table fluctuation in the unconfined aquifer becomes more and more import to $a_2 l$ and the water table fluctuation approaches a constant value when $a_2 l$ is greater than a threshold value. In addition, the dynamic effect becomes more and more important when the dimensionless leakage is increased. However, given the same values of u and $a_2 l$, the HA_{x0}, $a_2 x_{\max}$, and ph in the leaky aquifer with considering the dynamic effect are always larger than those of neglecting such effects.

Effects of the Inland and Offshore Leakages

The second case is to demonstrate how the inland and offshore leakages affect the tidal fluctuation when $a_2 l = 0.5$. Figs. 3(a–c)

J. Hydraul. Eng. 2008.134:447-454.

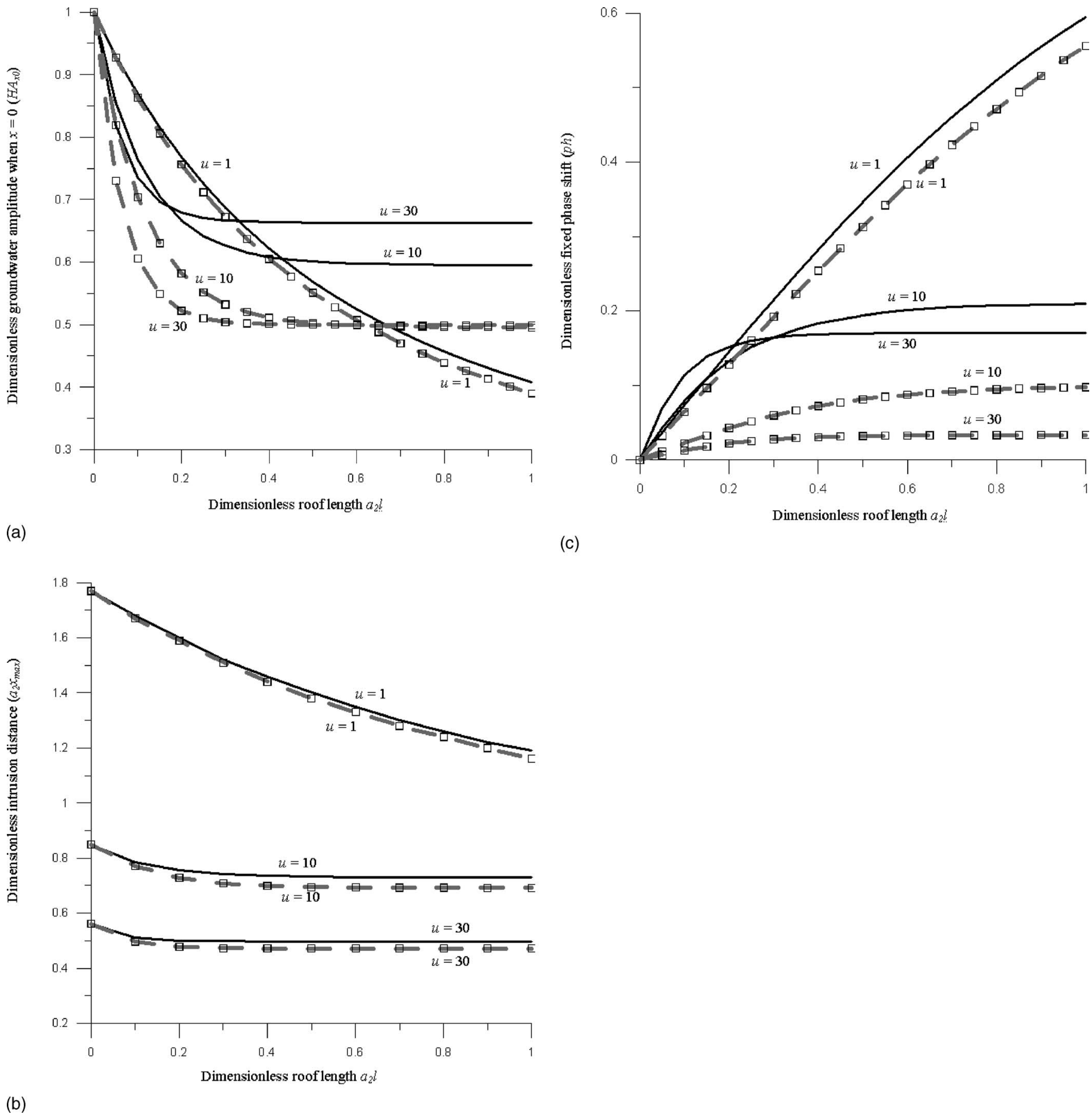

Fig. 2. The curves of (a) HA_{x0}; (b) ph, and (c) $a_2x_{\max}$ versus a_2l when the dimensionless leakage (u=u_i=u_o) varies from 1 to 30 with parameters a_1=$10a_2$ and T_e=0. The solid lines denote the present solution, the dash lines represent the present solution when n or $T_1 \rightarrow 0$, and the square symbol stands for solution of Li and Jiao (2001) without considering the fluctuation of groundwater level in the unconfined aquifer.

show the distributions of HA_{x0}, $a_2x_{\max}$, and ph for various dimensionless leakage values of inland and offshore aquitards with a_1 =$10a_2$, a_2=0.00123/m, a_2l=0.5, and T_e=0. The dash line represents the dimensionless leakage, u, that u_i=u_o. Fig. 3(a) demonstrates that the HA_{x0} increases significantly with u_o when u_o ranges from 0 to 30. When u_o changes from 0 to 30, HA_{x0} varies from 0.54 to 0.87 when u_i=0, HA_{x0} varies from 0.44 to 0.69 when u_i=15, and HA_{x0} varies from 0.46 to 0.66 when u_i=30. Obviously, the effect of u_o on HA_{x0} is large when u_i is small. On the other hand, when u_i changes from 0 to 30, HA_{x0} varies from 0.54 to 0.46 when u_o=0, HA_{x0} varies from 0.81 to 0.61 when u_o=15, and HA_{x0} varies from 0.87 to 0.66 when u_o=30. Apparently, HA_{x0} decreases with increasing u_i and the effect of u_i on HA_{x0} is most important when u_o is large. The effect of u_i on HA_{x0} when u_o =0 is less than that of u_o when u_i=0. The dash line in Fig. 3(a) shows HA_{x0} varies from 0.54, 0.62, to 0.66 when u changes from 0, 15, to 30 indicating that the HA_{x0} increases with the dimensionless leakage.

Fig. 3(b) displays that $a_2x_{\max}$ decreases significantly with increasing u_i when u_i ranges from 0 to 30. On the other hand, $a_2x_{\max}$ is independent of u_o when u_i is small and slightly increases with u_o when u_i is relatively large. In addition, $a_2x_{\max}$ decreases

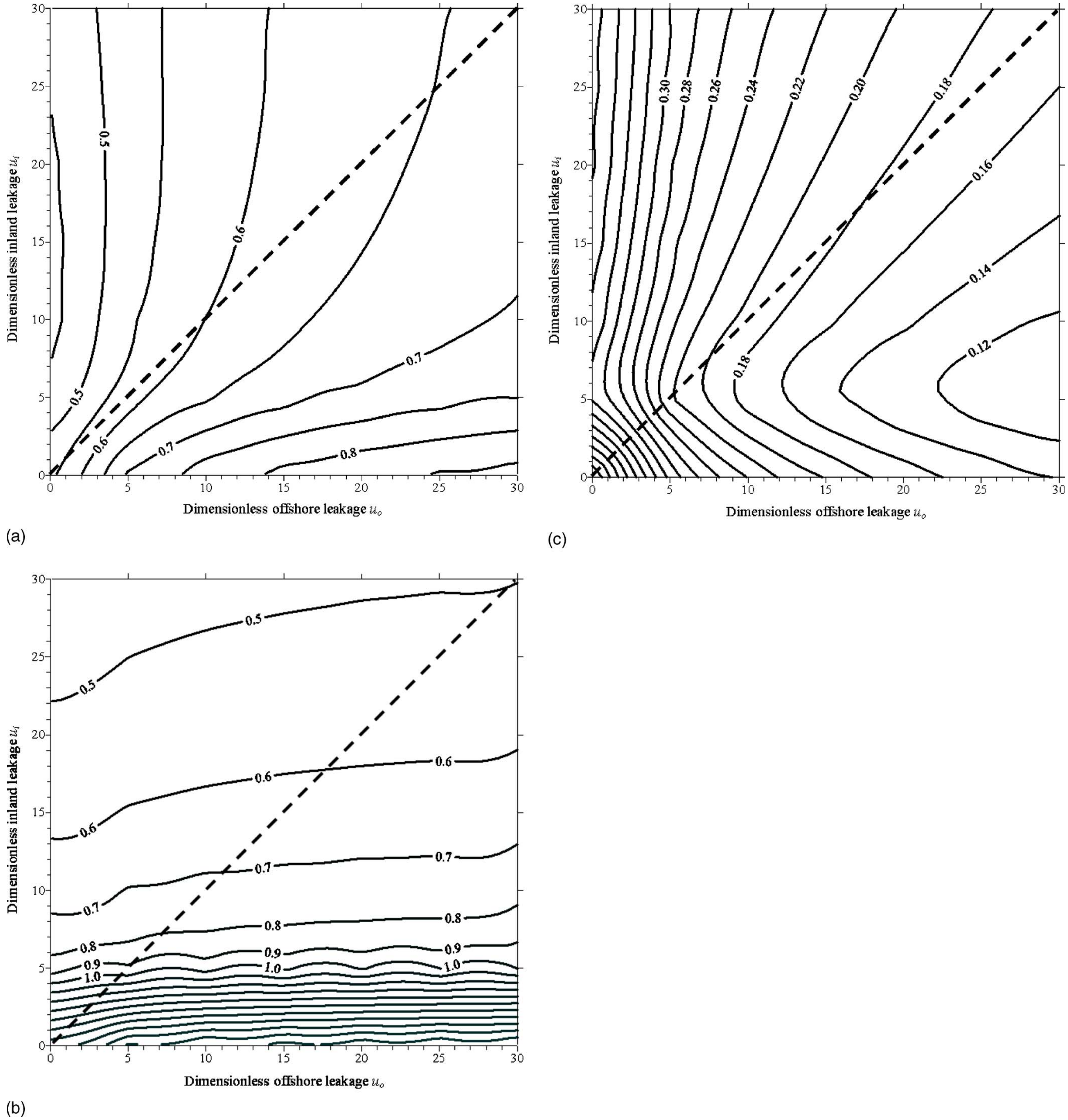

Fig. 3. The distributions of (a) HA_{x0}, (b) $a_2x_{\max}$, and (c) ph for various dimensionless leakage values of inland and offshore aquitards with $a_1 = 10\ a_2$, $a_2 = 0.00123/\mathrm{m}$, $a_2l = 0.5$, and $T_e = 0$. The dashed line represents the case that $u_i = u_o$.

quickly with increasing u_i when $u_i < 5$ and slowly when $u_i > 5$. The dash line of Fig. 3(b) shows that the $a_2x_{\max}$ increases conspicuously as u decreases. Fig. 3(c) shows that the ph decreases as u_i increases when $u_i < 4$ and then increases with u_i after $u_i > 6$. The effect of u_i on ph is obviously most significant when u_o is large. On the other hand, the ph decreases as u_o increases when u_i ranging from 0 to 30. The dash line in Fig. 3(c) shows that the ph decreases quickly with increasing u for small u and slowly for large u.

Conclusions

New analytical solutions had been derived to analyze the influences of the roof length, dynamic effect of water table fluctuation, and leakages of the inland and offshore on tidal responses in a coupled coastal aquifer system consisting of an unconfined aquifer, aquitard, and leaky aquifer. The unconfined aquifer ends at the coast, whereas the aquitard and leaky aquifer extend finite distance under the sea. These newly derived solutions can reduce to the solutions of Li and Jiao (2001a) with the assumption of neglecting water-table fluctuation in the unconfined aquifer. In addition, these solutions also reduce to the solutions of Chuang and Yeh (2007) when both the aquitard and leaky aquifer extend to an infinite distance. Both offshore and inland leakages are two dominant factors controlling the groundwater level fluctuations. The groundwater level fluctuation in the inland part of leaky aquifer increases significantly with the leakage of offshore aquitard. The dimensionless intrusion distance from the coast decreases significantly with the increasing leakage of inland aquitard. The dimensionless intrusion distance in the leaky aquifer decreases quickly with the increasing leakage of inland aquitard when the inland leakage is smaller than 5 and decreases slowly when the inland leakage is larger than 5. The dimensionless fixed phase shift decreases with the increasing dimensionless inland leakage (u_i) when $u_i < 4$ and then increases with u_i after $u_i > 6$. The dimensionless fixed phase shift also decreases with the increasing dimensionless offshore leakage. When the roof length is increased, the dynamic effect of the water table fluctuation on the dimensionless groundwater amplitude, intrusion distance and fixed phase shift in the unconfined aquifer become more and more important and the water table fluctuation approaches constant values when the roof length is greater than a threshold value. However, given the same values of dimensionless leakage and roof length, the dimensionless groundwater amplitude, intrusion distance and fixed phase shift in the leaky aquifer with considering the dynamic effects are always larger than those of neglecting such effects. In addition, the dynamic effect increases with the dimensionless leakage.

Notation

The following symbols are used in this paper:

A_0 = amplitude of the tidal change;
a_1 = unconfined aquifer's tidal propagation parameter;
a_2 = confined aquifer's tidal propagation parameter;
HA = normalized groundwater amplitude;
HA_{x0} = normalized groundwater amplitude when $x=0$;
h_s = hydraulic head of the sea tide;
h_1 = hydraulic head in the unconfined aquifer;
h_2 = hydraulic head in the leaky aquifer;
L_i = leakage of the inland aquitard;
L_o = leakage of the offshore aquitard;
l = roof length;
l_u = threshold value of roof length;
n = dimensionless storativity;
ph = dimensionless fixed phase shift;
S_1 = storativity of the unconfined aquifer;
S_2 = storativity of the leaky aquifer;
T_e = tidal efficiency;
T_1 = transmissivity of the unconfined aquifer;
T_2 = transmissivity of the leaky aquifer;
t_0 = tidal period;
u = dimensionless leakage *when* inland leakage is equal to offshore leakage;
u_i = dimensionless inland leakage;
u_o = dimensionless offshore leakage;
$x_{\max}$ = tidal intrusion distance is the farthest landward distance from the coastline to the location where HA is less than 10^{-2}; and
ω = tidal speed.

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