行政院國家科學委員會補助專題研究計畫成果報告

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☐國際合作研究計畫國外研究報告書一份

執行單位：交通大學電子物理系

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成果報告

1. 英文摘要：

We show that the degenerate positive-norm physical propagating fields of the open bosonic string are mere gauge artifacts of the higher spin fields at the same mass level. This is demonstrated by using the lowest order gauge transformation of Witten's string field theory (WSFT) up to the fourth massive level (spin-five), and is found to be consistent with previous conformal field theory calculation based on the first quantized generalized sigma-model approach.

2. 中文摘要：

我們証明玻色開弦的粒子譜中大量態簡併之物理態，其運動方程式散射振幅乃由同質量之高自旋態決定。我們用Witten的弦場論証明此現象到自旋5的質量態。此結果亦用保角場論之計算結果相符。

I. INTRODUCTION

It was pointed out more than ten years ago by Gross [1] that, in addition to the strong coupling regime, the most important nonperturbative regime of string theory is the high-energy stringy ($\alpha' \to \infty$) behavior of the theory. It is in this regime that the theory becomes very different from point particle field theory. Among many interesting stringy behaviors, it was believed that an infinite broken gauge symmetry get restored at energy much higher than the Planck energy. Moreover, this symmetry is powerful enough to link different string scattering amplitudes and, in principle, can be used to express all string amplitudes in terms of say dilaton amplitude.

Instead of studying stringy scattering amplitudes [2], one altenative to explicitely derive stringy symmetry is to use the generalized worldsheet sigma-model approach. In this approach, one uses conformal field theory to calculate the equations of motion for massive string background fields in the lowest order weak field approximation but valid to all orders in α'. Weak field approximation is thus the appropriate approximation scheme to study high energy symmetry of the string. An infinite set of on-shell stringy gauge symmetry is then derived by requiring the decoupling of both types of zero-norm physical states in the OCFQ spectrum [3]. In particular, all physical propagating states at each fixed mass level are found to form a large gauge multiplet. This begins to show up at the second massive level (spin-three). Moreover, it was remarkable to discover that [4], in addition to zero-norm states, the degenerate positive-norm physical propagating fields of the open bosonic string are mere gauge artifacts of the higher spin fields at the same mass level. Instead of being gauged away as the usual zero-norm states, these positive-norm states can be gauged to the higher spin fields by the existence of zero-norm states with the same Young representations. It was also shown [5] that the scattering amplitudes of these degenerate positive-norm states can be expressed in terms of those of higher spin states at the same mass level through massive Ward identities. This begins to show up at the third massive level (spin-four), and was argued to be a sigma-model n+1 loop result for the nth massive level. These stringy phenomena seem to be closely related to the results in Ref. [1]. In fact, an infinite number of linear relations between the string tree-level scattering amplitudes of different string states, similar to those claimed in [1], were derived through an infinite number of zero-norm states [5]. To claim that the decoupling phenomenon persist for general higher levels, it would be very important a prior to see whether one can rederive it from the second quantized off-shell WSFT [6].

Recently there is a revived interest in WSFT, mainly due to Sen's conjecture in tachyon condensation on D-brane [7]. And it becomes more and more clear that a second quantized field theory of string is unavoidable especially when one wants to study higher string modes. On the other hand, a cross check by both first and second quantized approaches of any reliable string theory result would be of great importance. Unfortunately most of the recent researches on string field theory were confined to the scalar modes on identification of nonperturbative string vacuum [8]. Our aim in this paper is to consider the gauge transformation of all string modes with any spin and in arbitrary gauge [9]. We will first prove the decoupling phenomenon of the third massive level of open bosonic string claimed in Ref. [4] by WSFT. The result is then generalized to the fourth massive level by both first and second quantized approaches. This paper is organized as following. In section II we first summarize the previous first quantized approach. In section III we explicitly calculate the lowest order gauge transformation level by level up to the third massive level in WSFT, and compare them with those of the first quantized approach. Some important observations will be made for ghost fields in WSFT and zero-norm states in OCFQ spectrum. The transformation will be seperated into the matter and ghost fields parts in WSFT. The matter part is found to be consistent with previous calculation [5] based on the old covariant string field gauge transformation of Banks and Perskin [10]. The ghost part is argued to correspond to the lift of on-shell (including on-mass-shell, gauge and traceless) conditions of zero-norm states in the OCFQ calculation. Section IV is devoted to the fourth massive level. Both first and second quantized calculations there are new and will be presented. A brief conclusion is made in section V. The lengthy gauge transformation of ghost fields for level four will be collected in the appendix.

II. OLD COVARIANT FIRST QUANTIZED APPROACH

The old covariant quantization is one of the three standard quantization schemes of string. In addition to the physical positive-norm propagating modes, there exist two types of physical zero-norm states in the bosonic open string spectrum [11]. They are

$$\text{Type I}: \quad L_{-1}|\chi\rangle, \quad \text{where } L_m|\chi\rangle = 0, m \geq 1, L_0|\chi\rangle = 0; \tag{1}$$

$$\text{Type II}: \quad \left(L_{-2} + \frac{3}{2}L_{-1}^2\right)|\tilde{\chi}\rangle, \quad \text{where } L_m|\tilde{\chi}\rangle = 0, m \geq 1, (L_0+1)|\tilde{\chi}\rangle = 0. \tag{2}$$

Type I states have zero-norm at any spacetime dimension, while type II states have zero-norm only at $D = 26$. Their existence turns out to be important in the following disscusion. The explicit forms of these zero-norm states have been calculated and their Young tabulation, together with positive-norm states, up to the third massive level are listed in the following table. Note that zero-norm states are not included in the light-cone quantization.

mass level	positive-norm states	zero-norm states
$m^2 = -2$	$\bullet$	
$m^2 = 0$	$\square$	$\bullet$(singlet)
$m^2 = 2$	$\square\square$	$\square, \bullet$
$m^2 = 4$	$\square\square\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square\square, 2\times\square, \bullet$
$m^2 = 6$	$\square\square\square\square, \begin{smallmatrix}\square&\square\\\square&\end{smallmatrix}, \square\square, \bullet$	$\square\square\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}, 2\times\square\square, 3\times\square, 2\times\bullet$

Table.1 OCFQ spectrum of open bosonic string.

It was demonstrated in the first order weak field approximation that for each zero-norm state there corresponds a on-shell gauge transformation for the positive-norm background field ($\alpha' \equiv \frac{1}{2}$) [3]:

$$m^2 = 0: \quad \delta A_\mu = \partial_\mu \theta; \tag{3a}$$
$$\partial^2 \theta = 0. \tag{3b}$$

$$m^2 = 2: \quad \delta B_{\mu\nu} = \partial_{(\mu}\theta_{\nu)}; \tag{4a}$$
$$\partial^\mu \theta_\mu = 0, (\partial^2 - 2)\theta_\mu = 0. \tag{4b}$$
$$\delta B_{\mu\nu} = \frac{3}{2}\partial_\mu\partial_\nu\theta - \frac{1}{2}\eta_{\mu\nu}\theta; \tag{5a}$$
$$(\partial^2 - 2)\theta = 0. \tag{5b}$$

$$m^2 = 4: \quad \delta C_{\mu\nu\lambda} = \partial_{(\mu}\theta_{\nu\lambda)}; \tag{6a}$$
$$\partial^\mu \theta_{\mu\nu} = \theta_\mu^{\ \mu} = 0, \ (\partial^2 - 4)\theta_{\mu\nu} = 0. \tag{6b}$$
$$\delta C_{\mu\nu\lambda} = \frac{5}{2}\partial_{(\mu}\partial_\nu\theta^1_{\lambda)} - \eta_{(\mu\nu}\theta^1_{\lambda)}; \tag{7a}$$
$$\partial^\mu \theta^1_\mu = 0, \ (\partial^2 - 4)\theta^1_\mu = 0. \tag{7b}$$
$$\delta C_{\mu\nu\lambda} = \frac{1}{2}\partial_{(\mu}\partial_\nu\theta^2_{\lambda)} - 2\eta_{(\mu\nu}\theta^2_{\lambda)}, \ \delta C_{[\mu\nu]} = 9\partial_{[\mu}\theta^2_{\nu]}; \tag{8a}$$
$$\partial^\mu \theta^2_\mu = 0, \ (\partial^2 - 4)\theta^2_\mu = 0. \tag{8b}$$
$$\delta C_{\mu\nu\lambda} = \frac{3}{5}\partial_\mu\partial_\nu\partial_\lambda\theta - \frac{1}{5}\eta_{(\mu\nu}\partial_{\lambda)}\theta; \tag{9a}$$
$$(\partial^2 - 4)\theta = 0. \tag{9b}$$

In the above equations, A, B, C are positive-norm background fields, θ's represent zero-norm background fields, and $\partial^2 \equiv \partial^\mu\partial_\mu$. There are on-mass-shell, gauge and traceless conditions on the transformaion parameters θ's, which will correspond to BRST ghost fields in a one-to-one manner in WSFT as will be discussed in the next section. Eq (3) is of course the usual on-shell gauge transformation, and eq (5) is the first residual stringy gauge symmetry. Note that θ^1_μ and θ^2_μ in eqs (7) and (8) are some linear combination of the oringinal type I and type II vector zero-norm states calculated by eqs (1) and (2). It is interesting to see that eq (8) implies that the two second massive level modes $C_{\mu\nu\lambda}$ and $C_{[\mu\nu]}$ form a larger gauge multiplet [3]. This is a generic feature for higher massive level and had also been justified from S-matrix point of view [12]. One might want to generalize the calculation to the second order weak field to see the inter-mass level symmetry. This however suffers from the so-called non-perturbative non-renormalizability of 2-d σ-model and one is forced to introduce infinite number of counter-terms to perserve the worldsheet conformal invariance [13].

Instead of calculating the stringy gauge symmetry at level $m^2 = 6$, it was discovered that an even more interesting phenomenon begins to show up at this mass level. Take the energy-momentum tensor in the first order weak field approximation to be of the following form.

$$\begin{aligned} T_{++} = & -\frac{1}{2}\eta_{\mu\nu}\partial X^\mu \partial X^\nu + D_{\mu\nu\alpha\beta}\partial X^\mu \partial X^\nu \partial X^\alpha \partial X^\beta + D_{\mu\nu\alpha}\partial X^\mu \partial X^\nu \partial^2 X^\alpha \\ & + D^0_{\mu\nu}\partial^2 X^\mu \partial^2 X^\nu + D^1_{\mu\nu}\partial X^\mu \partial^3 X^\nu + D_\mu \partial^4 X^\mu, \qquad (10) \\ T_{--} = & \; T_{++}(\partial X^\mu \to \bar{\partial} X^\mu). \qquad (11) \end{aligned}$$

This is the most general worldsheet coupling in the generalized σ-model apporoach consistent with vertex operator consideration [14]. The Virasoro operators are defined to be

$$L_n = \frac{1}{\pi}\int_0^\pi d\sigma \left(e^{in\pi}T_{++} + e^{-in\pi}T_{--}\right). \qquad (12)$$

The conditions to cancel all q-number worldsheet conformal anomalous terms correspond to cancelling all kinds of loop divergences up to the four loop order in the 2-d conformal field theory. It is easier to use operator-product calculation and the conditions read [4]

$$2\partial^\mu D_{\mu\nu\alpha\beta} - D_{(\nu\alpha\beta)} = 0, \qquad (13a)$$

$$\partial^\mu D_{\mu\nu\alpha} - 2D^0_{\nu\alpha} - 3D^1_{\nu\alpha} = 0, \qquad (13b)$$

$$\partial^\mu D^1_{\mu\nu} - 12D_\nu = 0, \qquad (13c)$$

$$3D^\mu{}_{\mu\nu\alpha} + \partial^\mu D_{\nu\alpha\mu} - 3D^1_{(\nu\alpha)} = 0, \qquad (13d)$$

$$D^\mu{}_{\mu\nu} + 4\partial^\mu D^0_{\mu\nu} - 24D_\nu = 0, \qquad (13e)$$

$$2D_{\mu\nu}{}^\nu + 3\partial^\nu D^1_{\mu\nu} - 12D_\mu = 0, \qquad (13f)$$

$$2D^{0\,\mu}_\mu + 3D^{1\,\mu}_\mu + 12\partial^\mu D_\mu = 0, \qquad (13g)$$

$$(\partial^2 - 6)\phi = 0. \qquad (13h)$$

Here, ϕ represents all background fields introduced in eq (10). It is now clear through (13b) and (13d) that both $D^0_{\mu\nu}$ and $D^1_{(\mu\nu)}$ can be expressed in terms of $D_{\mu\nu\alpha\beta}$ and $D_{\mu\nu\alpha}$ and are mere gauge artifacts. $D^1_{[\mu\nu]}$ can be expressed in terms of $D_{\mu\nu\alpha\beta}$ and $D_{\mu\nu\alpha}$ by (13b). Equations (13a) and (13c) imply that $D_{(\mu\nu\alpha)}$ and D_μ can also be expressed in terms of $D_{\mu\nu\alpha\beta}$ and $D_{\mu\nu\alpha}$. Finally eqs (13e)-(13g) are the gauge conditions for $D_{\mu\nu\alpha\beta}$ and $D_{\mu\nu\alpha}$ after substuting $D^0_{\mu\nu}$, $D^1_{\mu\nu}$ and D_μ in terms of $D_{\mu\nu\alpha\beta}$ and $D_{\mu\nu\alpha}$. The remaining scalar particle is automatically a gauge artifact since eq (10) is already the most general form of background-field coupling. This means that the degenerate spin two and scalar positive-norm states are mere gauge artifacts and can be gauged to the higher spin fields $D_{\mu\nu\alpha\beta}$ and $D_{\mu\nu\alpha}$. This is very different from the analysis of lower massive levels where only zero-norm states are gauge artifacts and can be gauged away. Presumably, this decoupling phenomenon comes from the ambiguity in defining positive-norm states due to the existence of zero-norm states in the same Young representations. We will justify this decoupling by WSFT in the next section. Finally one expects this decoupling persists if one includes the higher order corrections in weak field approximation, as there will be even stronger relations between background fields order by order through iteration.

III. WITTEN STRING FIELD THEORY APPROACH

It would be much more convincing if one can rederive the stringy phenomena discussed in the previous section from WSFT. One can not only compare the first quantized string with the second quantized string, but also compare the old covariant quantized string with the BRST quantized string. Although the calculation is lengthy, the result, as we shall see, are still controllable by comparing them with our results from first quantized approach in section II . There existed important consistency checks of first quantized string results from WSFT in the literature, e.g. the rederivation of Veneziano and Kubo-Nielson amplitutes from WSFT [15]. In some stringy cases, the calculations can only be done in string field theory approach. The recently developed pp-wave string amplitudes can only be calculated in the light-cone string field theory[16]. Sen's recent conjectures of tachyon condensation on D-brane were mostly justified by string field theory. It is by now clear that a consistent check by both first and second quantized approaches of any reliable string results would be of great importance.

The infinitesimal gauge transformation of WSFT is

$$\delta\Phi = Q_B\Lambda + g_0(\Phi * \Lambda - \Lambda * \Phi). \tag{14}$$

To compare with our first quantized results in section II, we will only calculate the first term on the right hand side of eq (14). Up to the second massive level, Φ and Λ can be expressed as

$$\begin{aligned}\Phi = \Big\{ & \phi(x) + iA_\mu(x)\alpha^\mu_{-1} + \alpha(x)b_{-1}c_0 - B_{\mu\nu}(x)\alpha^\mu_{-1}\alpha^\nu_{-1} + iB_\mu(x)\alpha^\mu_{-2} \\ & + i\beta_\mu(x)\alpha^\mu_{-1}b_{-1}c_0 + \beta^0(x)b_{-2}c_0 + \beta^1(x)b_{-1}c_{-1} \\ & - iC_{\mu\nu\lambda}(x)\alpha^\mu_{-1}\alpha^\nu_{-1}\alpha^\lambda_{-1} - C_{\mu\nu}(x)\alpha^\mu_{-2}\alpha^\nu_{-1} + iC_\mu(x)\alpha^\mu_{-3} \\ & - \gamma_{\mu\nu}(x)\alpha^\mu_{-1}\alpha^\nu_{-1}b_{-1}c_0 + i\gamma^0_\mu(x)\alpha^\mu_{-1}b_{-2}c_0 + i\gamma^1_\mu(x)\alpha^\mu_{-1}b_{-1}c_{-1} + i\gamma^2_\mu(x)\alpha^\mu_{-2}b_{-1}c_0 \\ & + \gamma^0(x)b_{-3}c_0 + \gamma^1(x)b_{-2}c_{-1} + \gamma^2(x)b_{-1}c_{-2}\Big\} c_1|k\rangle, \end{aligned} \tag{15}$$

$$\begin{aligned}\Lambda = \Big\{ & \epsilon^0(x)b_{-1} - \epsilon^0_{\mu\nu}(x)\alpha^\mu_{-1}\alpha^\nu_{-1}b_{-1} + i\epsilon^0_\mu(x)\alpha^\mu_{-1}b_{-1} + i\epsilon^1_\mu(x)\alpha^\mu_{-2}b_{-1} + i\epsilon^2_\mu(x)\alpha^\mu_{-1}b_{-2} \\ & + \epsilon^1(x)b_{-2} + \epsilon^2(x)b_{-3} + \epsilon^3(x)b_{-1}b_{-2}c_0\Big\}|\Omega\rangle. \end{aligned} \tag{16}$$

where Φ and Λ are restricted to ghost number 1 and 0 respectively, and the BRST charge is

$$Q_B = \sum_{n=-\infty}^{\infty} L^{\text{matt}}_{-n} c_n + \sum_{m,n=-\infty}^{\infty} \frac{m-n}{2} : c_m c_n b_{-m-n} : - c_0. \tag{17}$$

The transformation one gets for each mass level are

$$m^2 = 0, \quad \delta A_\mu = \partial_\mu \epsilon^0, \tag{18a}$$

$$\delta\,\alpha = \frac{1}{2}\partial^2\epsilon^0; \tag{18b}$$

$$m^2 = 2, \quad \delta B_{\mu\nu} = -\partial_{(\mu}\epsilon^0_{\nu)} - \frac{1}{2}\epsilon^1\eta_{\mu\nu}, \tag{19a}$$

$$\delta B_\mu = -\partial_\mu\epsilon^1 + \epsilon^0_\mu, \tag{19b}$$

$$\delta\beta_\mu = \frac{1}{2}(\partial^2 - 2)\epsilon^0_\mu, \tag{19c}$$

$$\delta\beta^0 = \frac{1}{2}(\partial^2 - 2)\epsilon^1, \tag{19d}$$

$$\delta\beta^1 = -\partial^\mu\epsilon^0_\mu - 3\epsilon^1; \tag{19e}$$

$$m^2 = 4, \quad \delta C_{\mu\nu\lambda} = -\partial_{(\mu}\epsilon^0_{\nu\lambda)} - \frac{1}{2}\epsilon^2_{(\mu}\eta_{\nu\lambda)}, \tag{20a}$$

$$\delta C_{[\mu\nu]} = -\partial_{[\nu}\epsilon^1_{\mu]} - \partial_{[\mu}\epsilon^2_{\nu]}, \tag{20b}$$

$$\delta C_{(\mu\nu)} = -\partial_{(\nu}\epsilon^1_{\mu)} - \partial_{(\mu}\epsilon^2_{\nu)} + 2\epsilon^0_{\mu\nu} - \epsilon^2\eta_{\mu\nu}, \tag{20c}$$

$$\delta C_\mu = -\partial_\mu\epsilon^2 + 2\epsilon^1_\mu + \epsilon^2_\mu, \tag{20d}$$

$$\delta\gamma_{\mu\nu} = \frac{1}{2}(\partial^2 - 4)\epsilon^0_{\mu\nu} - \frac{1}{2}\epsilon^3\eta_{\mu\nu}, \tag{20e}$$

$$\delta\gamma^0_\mu = \frac{1}{2}(\partial^2 - 4)\epsilon^2_\mu + \partial_\mu\epsilon^3, \tag{20f}$$

$$\delta\gamma^1_\mu = -2\partial^\nu\epsilon^0_{\nu\mu} - 2\epsilon^1_\mu - 3\epsilon^2_\mu, \tag{20g}$$

$$\delta\gamma^2_\mu = \frac{1}{2}(\partial^2 - 4)\epsilon^1_\mu - \partial_\mu\epsilon^3, \tag{20h}$$

$$\delta\gamma^0 = \frac{1}{2}(\partial^2 - 4)\epsilon^2 - \epsilon^3, \tag{20i}$$

$$\delta\gamma^1 = -\partial^\mu\epsilon^2_\mu - 4\epsilon^2 - 2\epsilon^3, \tag{20j}$$

$$\delta\gamma^2 = -2\partial^\mu\epsilon^1_\mu - 5\epsilon^2 + 4\epsilon^3 + \epsilon^{0\ \mu}_\mu. \tag{20k}$$

It is interesting to note that eq (18b) corresponds to the lift of on-mass-shell condition in eqs (3b), (19c) and (19d) correspond to on-mass-shell condition in (5b) and (4b) and eq (19e) corresponds to the gauge condition in (4b). Similar correspondence applies to level $m^2 = 4$. Eqs (20e), (20f), (20h) and (20i) correspond to on-mass-shell conditions in eqs (6b), (7b), (8b) and (9b). Eqs (20g), (20j) and (20k) correspond to gauge conditions in eqs (6b), (7b) and (8b). The traceless condition in (6b) corresponds to the trace part of eq (20e). Also, only zero-norm state transformation parameters appear on the r.h.s. of matter transformation A,B,C, and all ghost transformations correspond, in a one-to one manner, to the lift of on-shell conditions (including on-mass-shell, gauge and traceless conditions) in the OCFQ approach. These important observations simplify the demonstration of decouplng of degenerate positive-norm states for complicated higher mass levels, $m^2 = 6$ and $m^2 = 8$ in WSFT as will be discussed in the rest of this paper.

For $m^2 = 4$, it can be checked that only $C_{\mu\nu\lambda}$ and $C_{[\mu\nu]}$ are dynamical fields (not gauge artifacts) and they form a gauge multiplet, which is consistent with result of first quantized calculation presented in section II .

We now calculate the decoupling phenomenon for the third massive level $m^2 = 6$, in which Φ and Λ can be expanded as

$$\Phi_4 = \Big\{ D_{\mu\nu\alpha\beta}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}a^{\beta}_{-1} - iD_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-2} - D^0_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-2} - D^1_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-3}$$
$$+iD_{\mu}(x)\alpha^{\mu}_{-4} - i\xi_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-1}c_0 - \xi^0_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}b_{-1}c_0 - \xi^1_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-2}c_0$$
$$-\xi^2_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-1}c_{-1} + i\xi^0_{\mu}(x)\alpha^{\mu}_{-3}b_{-1}c_0 + i\xi^1_{\mu}(x)\alpha^{\mu}_{-2}b_{-2}c_0 + i\xi^2_{\mu}(x)\alpha^{\mu}_{-1}b_{-3}c_0$$
$$+i\xi^3_{\mu}(x)\alpha^{\mu}_{-2}b_{-1}c_{-1} + i\xi^4_{\mu}(x)\alpha^{\mu}_{-1}b_{-2}c_{-1} + i\xi^5_{\mu}(x)\alpha^{\mu}_{-1}b_{-1}c_{-2} + \xi^0(x)b_{-4}c_0 + \xi^1(x)b_{-3}c_{-1}$$
$$+\xi^2(x)b_{-2}c_{-2} + \xi^3(x)b_{-1}c_{-3} + \xi^4(x)b_{-2}b_{-1}c_{-1}c_{-0}\Big\}c_1|k\rangle, \tag{21}$$

$$\Lambda_4 = \Big\{ -i\epsilon^0_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-1} - \epsilon^1_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}b_{-1} - \epsilon^2_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-2} + i\epsilon^3_{\mu}(x)\alpha^{\mu}_{-3}b_{-1}$$
$$+i\epsilon^4_{\mu}(x)\alpha^{\mu}_{-2}b_{-2} + i\epsilon^5_{\mu}(x)\alpha^{\mu}_{-1}b_{-3} + i\epsilon^6_{\mu}(x)\alpha^{\mu}_{-1}b_{-2}b_{-1}c_0 + \epsilon^4(x)b_{-4}$$
$$+\epsilon^5(x)b_{-3}b_{-1}c_0 + \epsilon^6(x)b_{-2}b_{-1}c_{-1}\Big\}|\Omega\rangle. \tag{22}$$

The transformations for the matter part are

$$\delta D_{\mu\nu\alpha\beta} = -\partial_{(\beta}\epsilon^0_{\mu\nu\alpha)} - \frac{1}{2}\epsilon^2_{(\mu\nu}\eta_{\alpha\beta)}, \tag{23a}$$
$$\delta D_{\mu\nu\alpha} = -\partial_{(\mu}\epsilon^1_{|\alpha|\nu)} - \partial_{\alpha}\epsilon^2_{\nu\mu} + 3\epsilon^0_{\mu\nu\alpha} - \frac{1}{2}\epsilon^4_{\alpha}\eta_{\nu\mu} - \epsilon^5_{(\mu}\eta_{\nu)\alpha}, \tag{23b}$$
$$\delta D^1_{[\mu\nu]} = -\partial_{[\mu}\epsilon^3_{\nu]} - \partial_{[\nu}\epsilon^5_{\mu]} + 2\epsilon^1_{[\nu\mu]}, \tag{23c}$$
$$\delta D^1_{(\mu\nu)} = -\partial_{(\mu}\epsilon^3_{\nu)} - \partial_{(\nu}\epsilon^5_{\mu)} + 2\epsilon^1_{(\nu\mu)} + 2\epsilon^2_{\mu\nu} - \epsilon^4\eta_{\mu\nu}, \tag{23d}$$
$$\delta D^0_{\mu\nu} = -\partial_{(\mu}\epsilon^4_{\nu)} + \epsilon^1_{(\mu\nu)} - \frac{1}{2}\epsilon^4\eta_{\mu\nu}, \tag{23e}$$
$$\delta D_{\mu} = -\partial_{\mu}\epsilon^4 + 3\epsilon^3_{\mu} + 2\epsilon^4_{\mu} + \epsilon^5_{\mu}. \tag{23f}$$

It can be checked from eqs (23) that only $D_{\mu\nu\alpha\beta}$ and mixed-symmetric $D_{\mu\nu\alpha}$ cannot be gauged away, which is consistent with the result of the first quantized approach in sec. II. That is , the spin-two and scalar positive-norm physical propagating modes are mere gauge artifacts and have been gauged to $D_{\mu\nu\alpha\beta}$ and mixed symmetric $D_{\mu\nu\alpha}$. In fact, $D_{\mu\nu\alpha}, D^1_{[\mu\nu]}, D^1_{(\mu\nu)}, D^0_{\mu\nu}$ and D_{μ} can be gauged away by $\epsilon^0_{\mu\mu\lambda}, \epsilon^1_{[\mu\nu]}, \epsilon^1_{(\mu\nu)}, \epsilon^2_{\mu\nu}$ and one of the vector parameters, say ϵ^3_{μ}. The rest, $\epsilon^4_{\mu}, \epsilon^5_{\mu}$ and ϵ^4 are gauge artifacts of $D_{\mu\nu\alpha\beta}$ and mixed symmetric $D_{\mu\nu\alpha}$.

The transformation for the ghost part are

$$\delta\xi_{\mu\nu\alpha} = \frac{1}{2}(\partial^2 - 6)\epsilon^0_{\mu\nu\alpha} - \frac{1}{2}\epsilon^6_{(\mu}\eta_{\nu\alpha)}, \tag{24a}$$

$$\delta\xi^0_{[\mu\nu]} = \frac{1}{2}(\partial^2 - 6)\epsilon^1_{[\mu\nu]} - \partial_{[\mu}\epsilon^6_{\nu]}, \tag{24b}$$

$$\delta\xi^0_{(\mu\nu)} = \frac{1}{2}(\partial^2 - 6)\epsilon^1_{(\mu\nu)} - \partial_{(\mu}\epsilon^6_{\nu)} + \epsilon^5\eta_{\mu\nu}, \tag{24c}$$

$$\delta\xi^1_{\mu\nu} = \frac{1}{2}(\partial^2 - 6)\epsilon^2_{\mu\nu} + \partial_{(\nu}\epsilon^6_{\mu)}, \tag{24d}$$

$$\delta\xi^2_{\mu\nu} = -3\partial^\alpha\epsilon^0_{\mu\nu\alpha} - 2\epsilon^1_{(\mu\nu)} - 3\epsilon^2_{\mu\nu} - \frac{1}{2}\epsilon^6\eta_{\mu\nu}, \tag{24e}$$

$$\delta\xi^0_\mu = \frac{1}{2}(\partial^2 - 6)\epsilon^3_\mu - \partial_\mu\epsilon^5 + \epsilon^6_\mu, \tag{24f}$$

$$\delta\xi^1_\mu = \frac{1}{2}(\partial^2 - 6)\epsilon^4_\mu - \epsilon^6_\mu, \tag{24g}$$

$$\delta\xi^2_\mu = \frac{1}{2}(\partial^2 - 6)\epsilon^5_\mu + \partial_\mu\epsilon^5 - \epsilon^6_\mu, \tag{24h}$$

$$\delta\xi^3_\mu = -\partial^\nu\epsilon^1_{\mu\nu} - \partial_\mu\epsilon^6 - 3\epsilon^3_\mu - 3\epsilon^4_\mu, \tag{24i}$$

$$\delta\xi^4_\mu = 2\partial^\nu\epsilon^2_{\mu\nu} + \partial_\mu\epsilon^6 - 2\epsilon^4_\mu - 4\epsilon^5_\mu - 2\epsilon^6_\mu, \tag{24j}$$

$$\delta\xi^5_\mu = -2\partial^\nu\epsilon^1_{\nu\mu} - 3\epsilon^3_\mu - 5\epsilon^5_\mu + 4\epsilon^6_\mu + 3\epsilon^{0\ \nu}_{\mu\nu}, \tag{24k}$$

$$\delta\xi^0 = \frac{1}{2}(\partial^2 - 6)\epsilon^4 - 2\epsilon^5, \tag{24l}$$

$$\delta\xi^1 = -\partial^\mu\epsilon^5_\mu - 5\epsilon^4 - 2\epsilon^5 - \epsilon^6, \tag{24m}$$

$$\delta\xi^2 = -2\partial^\mu\epsilon^4_\mu - 6\epsilon^4 - 3\epsilon^6 + \epsilon^{2\ \mu}_\mu, \tag{24n}$$

$$\delta\xi^3 = -3\partial^\mu\epsilon^3_\mu - 7\epsilon^4 + 6\epsilon^5 + 5\epsilon^6 + 2\epsilon^{1\ \mu}_\mu, \tag{24o}$$

$$\delta\xi^4 = \frac{1}{2}(\partial^2 - 6)\epsilon^6 + \partial^\mu\epsilon^6_\mu + 4\epsilon^5. \tag{24p}$$

There are nine on-mass-shell conditions, which contains a symmetric spin three,an antisymmetric spin two, two symmetric spin two, three vector and two scalar fields, and seven gauge conditions which amounts to sixteen equations in (24). This is consistent with counting from zero-norm states listed in the table. Three traceless conditions read from zero-norm states corresponds to the three equations involving $\delta\xi_{\mu\nu}^{\ \ \nu}, \delta\xi^{0\ \mu}_\mu, \delta\xi^{1\ \mu}_\mu$ which are contained in eqs (24a), (24c), and (24d).

It is important to note that the transformation for the matter parts, eqs (20a)-(20d) and eqs (23a)-(23f), are the same as calculation [5] based on the chordal gauge transformation of free covariant string field theory constructed by Banks and Peskin [10]. The Chordal gauge transformation can be written in the following form

$$\delta\Phi[X(\sigma)] = \sum_{n>0} L_{-n}\Phi_n[X(\sigma)], \tag{25}$$

where $\Phi[X(\sigma)]$ is the string field and $\Phi_n[X(\sigma)]$ are gauge parameters which are functions of $X[\sigma]$ only without ghost fields. This is because the pure ghost part of Q_B in eq (17) does not contribute to the transformation of matter background fields. It is interesting to note that the r.h.s. of eq (25) is in the form of spurious states in the OCFQ approach. It will become zero-norm states after imposing the physical (on-shell) state condition. Presumably, imposing the string field equation of motion, or eqs (23) and (24) will give us the on-shell stringy gauge transformation in the OCFQ approach. Finally, it can be shown that the number of scalar zero-norm states at n-th massive level ($n \geq 3$) is at least the sum (say K) of those at (n-2)-th and (n-1)-th massive levels. So at least K positive-norm scalar modes at n-th level, if they exist, will be decoupled according to our decoupling conjecture. The decoupling of these scalars has important implication on Sen's conjectures on the decay of open string tachyon. Since all scalars on D-brane including tachyon get non-zero vev in the false vacuum, they will decay toghther with tachyon and disappear eventually to the true closed string vacuum. Our result on scalar states as gauge artifacts of tensor fields in this paper implies that tensor fields of open string (D25-brane) will accompany the decay process, which means that the whole D-brane could disappear to the true closed string vacuum! This mechanism could provide a hint to solve the so-called $U(1)$-problem [17] in Sen's conjectures. A further study is in progress.

IV. THE FOURTH MASSIVE LEVEL

We will use both the first and second quantized approaches to test the decoupling conjecture for the fourth massive level $m^2 = 8$.

(A) The first quantized calculation

The positive-norm physical propagating fields can be found in Ref. [18]. Their Young tabulations are

$$\yng(5), \yng(3,1), \yng(2,1), \yng(3), \yng(1,1), \yng(1). \tag{26}$$

The Young tabulations of zero-norm states can then shown to be

$$\yng(4), \yng(2,1)', 2\times\yng(3), 2\times\yng(1,1), 4\times\yng(2), 5\times\yng(1), 3\times\bullet. \tag{27}$$

Note that the two representations $\yng(2,1)$ in (26)and $\yng(2,1)'$ in (27) are different. One corresponds to $\alpha^{\mu}_{-1}\alpha^{\nu}_{-2}\alpha^{\lambda}_{-2}$ and the other $\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\lambda}_{-3}$. So one expects the last three states in (26) are mere gauge artifacts and can be gauged to the higher spin fields. The most general worldsheet coupling consistent with vertex operator consideration is

$$
\begin{aligned}
T_{++} = \quad & -\frac{1}{2}\eta_{\mu\nu}\partial X^\mu \partial X^\nu + E_{\mu\nu\lambda\alpha\beta}\partial X^\mu \partial X^\nu \partial X^\lambda \partial X^\alpha \partial X^\beta + E_{\mu\nu\lambda\alpha}\partial X^\mu \partial X^\nu \partial X^\lambda \partial^2 X^\alpha \\
& + E^0_{\mu\nu\lambda}\partial X^\mu \partial X^\nu \partial^3 X^\lambda + E^1_{\mu\nu\lambda}\partial X^\mu \partial^2 X^\nu \partial^2 X^\lambda + E^0_{\mu\nu}\partial X^\mu \partial^4 X^\nu + E^1_{\mu\nu}\partial^2 X^\mu \partial^3 X^\nu \\
& + E_\mu \partial^5 X^\mu, \qquad (28) \\
T_{--} = \quad & T_{++}(\partial X^\mu \to \bar{\partial} X^\mu). \qquad (29)
\end{aligned}
$$

After a lengthy calculation, the condition to cancel all worldsheet q-number anomalies are

$$
\begin{aligned}
& 5\partial^\mu E_{\mu\nu\lambda\alpha\beta} - 2E_{(\nu\lambda\alpha\beta)} = 0, && (30a) \\
& \partial^\mu E^0_{\mu\nu} - 20E_\nu = 0, && (30b) \\
& \partial^\mu E_{\mu\nu\lambda\alpha} - 12E^0_{\nu\lambda\alpha} - 8E^1_{(\nu\lambda)\alpha} = 0, && (30c) \\
& \partial^\mu E^0_{\mu\nu\lambda} - 6E^0_{\nu\lambda} - E^1_{\nu\lambda} = 0, && (30d) \\
& \partial^\mu E^1_{\mu\nu\lambda} - 6E^1_{(\nu\lambda)} = 0, && (30e) \\
& 20E^\mu{}_{\mu\nu\lambda\alpha} + \partial^\mu E_{\nu\lambda\alpha\mu} - 12E^0_{(\nu\lambda\alpha)} = 0, && (31a) \\
& E^{0\mu}{}_{\mu\nu} + 4\partial^\mu E^1_{\mu\nu} - 120E_\nu = 0, && (31b) \\
& E^\mu{}_{\mu\nu\lambda} + 8\partial^\mu E^1_{\nu\lambda\mu} - 48E^0_{\nu\lambda} - 12E^1_{\lambda\nu} = 0, && (31c) \\
& E^\mu{}_{\nu\lambda\mu} + \partial^\mu E^0_{\nu\lambda\mu} - 4E^0_{(\nu\lambda)} = 0, && (32a) \\
& E^{1\mu}{}_{\mu\nu} + 12\partial^\mu E^1_{\nu\mu} - 240E_\nu = 0, && (32b) \\
& 3E^{0\mu}{}_{\nu\mu} + E^1_{\nu\mu}{}^\mu + 6\partial^\mu E^0_{\nu\mu} - 30E_\nu = 0, && (33) \\
& 2E^{0\mu}{}_\mu + E^1_\mu{}^\mu + 10\partial^\mu E_\mu = 0, && (34) \\
& (\partial^2 - 6)\phi = 0. && (35)
\end{aligned}
$$

Here, ϕ again represents all background fields introduced in eq (28). Eqs (30a)-(30e) are extracted from $\frac{1}{(z-w)^3}$ anomalous terms in the operator product calculation, similarly (31a)-(31c), (32a)-(32b), (33) and (34) are extracted form $\frac{1}{(z-w)^4}$, $\frac{1}{(z-w)^5}$, $\frac{1}{(z-w)^6}$ and $\frac{1}{(z-w)^7}$ anomalous terms respectively. It can be carefully checked, as one did for the third massive level, that only $E_{\mu\nu\lambda\alpha\beta}$ and mixed-symmetric $E_{\mu\nu\lambda\alpha}$ and $E^1_{\mu\nu\lambda}$ corresponding to the first three Young representations in eq (26) are independent dynamical fields as the conjecture has claimed. The last three states in eq (26)have been gauged to $E_{\mu\nu\lambda\alpha\beta}$, mixed-symmetric $E_{\mu\nu\lambda\alpha}$ and $E^1_{\mu\nu\lambda}$ due to the existence of zero-norm states with the same Young representations in eq (27).

(B) WSFT calculation

Φ and Λ can be expanded at this massive level as

$$\Phi_5 = \Big\{ iE_{\mu\nu\lambda\alpha\beta}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}a^{\lambda}_{-1}a^{\beta}_{-1} + E_{\mu\nu\alpha\beta}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}a^{\beta}_{-2} - iE^{0}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-3}$$
$$-iE^{1}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-2}\alpha^{\alpha}_{-2} - E^{0}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-4} - E^{1}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-3} + iE_{\mu}(x)\alpha^{\mu}_{-5}$$
$$+\zeta_{\mu\nu\alpha\beta}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}\alpha^{\beta}_{-1}b_{-1}c_0 - i\zeta^{0}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-1}c_0 - i\zeta^{1}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-2}c_0$$
$$-i\zeta^{2}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-1}c_{-1} - \zeta^{0}_{\mu\nu}(x)\alpha^{\mu}_{-3}\alpha^{\nu}_{-1}b_{-1}c_0 - \zeta^{1}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-2}b_{-1}c_0$$
$$-\zeta^{2}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}b_{-2}c_0 - \zeta^{3}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-3}c_0 - \zeta^{4}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}b_{-1}c_{-1}$$
$$-\zeta^{5}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-2}c_{-1} - \zeta^{6}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-1}c_{-2} + i\zeta^{0}_{\mu}(x)\alpha^{\mu}_{-4}b_{-1}c_0 + i\zeta^{1}_{\mu}(x)\alpha^{\mu}_{-3}b_{-2}c_0$$
$$+i\zeta^{2}_{\mu}(x)\alpha^{\mu}_{-2}b_{-3}c_0 + i\zeta^{3}_{\mu}(x)\alpha^{\mu}_{-1}b_{-4}c_0 + i\zeta^{4}_{\mu}(x)\alpha^{\mu}_{-3}b_{-1}c_{-1} + i\zeta^{5}_{\mu}(x)\alpha^{\mu}_{-2}b_{-2}c_{-1}$$
$$+i\zeta^{6}_{\mu}(x)\alpha^{\mu}_{-1}b_{-3}c_{-1} + i\zeta^{7}_{\mu}(x)\alpha^{\mu}_{-2}b_{-1}c_{-2} + i\zeta^{8}_{\mu}(x)\alpha^{\mu}_{-1}b_{-2}c_{-2} + i\zeta^{9}_{\mu}(x)\alpha^{\mu}_{-1}b_{-1}c_{-3}$$
$$+i\zeta^{10}_{\mu}(x)\alpha^{\mu}_{-1}b_{-2}b_{-1}c_{-1}c_0 + \zeta^{0}(x)b_{-5}c_0 + \zeta^{1}(x)b_{-4}c_{-1} + \zeta^{2}(x)b_{-3}c_{-2} + \zeta^{3}(x)b_{-2}c_{-3}$$
$$+\zeta^{4}(x)b_{-1}c_{-4} + \zeta^{5}(x)b_{-3}b_{-1}c_{-1}c_{-0} + \zeta^{6}(x)b_{-2}b_{-1}c_{-2}c_{-0}\Big\} c_1|k\rangle \tag{36}$$

$$\Lambda_5 = \Big\{ \epsilon^{0}_{\mu\nu\alpha\beta}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}\alpha^{\beta}_{-1}b_{-1} - i\epsilon^{1}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-1} - i\epsilon^{2}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-2}$$
$$-\epsilon^{3}_{\mu\nu}(x)\alpha^{\mu}_{-3}\alpha^{\nu}_{-1}b_{-1} - \epsilon^{4}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-2}b_{-1} - \epsilon^{5}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}b_{-2} - \epsilon^{6}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-3}$$
$$-\epsilon^{7}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-2}b_{-1}c_0 + i\epsilon^{7}_{\mu}(x)\alpha^{\mu}_{-4}b_{-1} + i\epsilon^{8}_{\mu}(x)\alpha^{\mu}_{-3}b_{-2} + i\epsilon^{9}_{\mu}(x)\alpha^{\mu}_{-2}b_{-3} + i\epsilon^{10}_{\mu}(x)\alpha^{\mu}_{-1}b_{-4}$$
$$+i\epsilon^{11}_{\mu}(x)\alpha^{\mu}_{-2}b_{-2}b_{-1}c_0 + i\epsilon^{12}_{\mu}(x)\alpha^{\mu}_{-1}b_{-3}b_{-1}c_0 + i\epsilon^{13}_{\mu}(x)\alpha^{\mu}_{-1}b_{-2}b_{-1}c_{-1} + \epsilon^{7}(x)b_{-5}$$
$$+\epsilon^{8}(x)b_{-4}b_{-1}c_0 + \epsilon^{9}(x)b_{-3}b_{-2}c_0 + \epsilon^{10}(x)b_{-3}b_{-1}c_{-1} + \epsilon^{11}(x)b_{-2}b_{-1}c_{-2}\Big\}|\Omega\rangle. \tag{37}$$

The transformations for the matter part are

$$\delta E_{\mu\nu\lambda\alpha\beta} = -\partial_{(\beta}\epsilon^{0}_{\mu\nu\lambda\alpha)} + \frac{1}{2}\epsilon^{2}_{(\lambda\alpha\beta}\eta_{\mu\nu)}, \tag{38a}$$
$$\delta E_{\mu\nu\alpha\beta} = -\partial_{(\mu}\epsilon^{1}_{|\beta|\alpha\nu)} - \partial_{\beta}\epsilon^{2}_{\alpha\mu\nu} + 4\epsilon^{0}_{\mu\nu\alpha\beta} - \frac{1}{2}\epsilon^{5}_{\beta(\nu}\eta_{\alpha\mu)} - \epsilon^{6}_{(\alpha\mu}\eta_{\nu)\beta}, \tag{38b}$$
$$\delta E^{0}_{\mu\nu\alpha} = -\partial_{(\mu}\epsilon^{3}_{|\alpha|\nu)} - \partial_{\alpha}\epsilon^{6}_{\nu\mu} + 2\epsilon^{1}_{\alpha\nu\mu} + 3\epsilon^{2}_{\alpha\nu\mu} - \frac{1}{2}\epsilon^{7}_{\alpha}\eta_{\nu\mu} - \epsilon^{9}_{(\mu}\eta_{\nu)\alpha}, \tag{38c}$$
$$\delta E^{1}_{\mu\nu\alpha} = -\partial_{\mu}\epsilon^{4}_{\nu\alpha} - \partial_{(\alpha}\epsilon^{5}_{\nu)\mu} + 2\epsilon^{1}_{(\alpha\nu)\mu} - \epsilon^{8}_{(\alpha}\eta_{\nu)\mu} - \frac{1}{2}\epsilon^{9}_{\mu}\eta_{\nu\alpha}, \tag{38d}$$
$$\delta E^{0}_{[\mu\nu]} = -\partial_{[\mu}\epsilon^{7}_{\nu]} - \partial_{[\nu}\epsilon^{9}_{\mu]} + 3\epsilon^{3}_{[\nu\mu]} + 2\epsilon^{5}_{[\nu\mu]}, \tag{38e}$$
$$\delta E^{1}_{[\mu\nu]} = -\partial_{[\mu}\epsilon^{7}_{\nu]} - \partial_{[\nu}\epsilon^{8}_{\mu]} + \epsilon^{3}_{[\nu\mu]} + \epsilon^{5}_{[\mu\nu]}, \tag{38f}$$
$$\delta E^{0}_{(\mu\nu)} = -\partial_{(\mu}\epsilon^{7}_{\nu)} - \partial_{(\nu}\epsilon^{9}_{\mu)} + 3\epsilon^{3}_{(\nu\mu)} + 2\epsilon^{5}_{(\nu\mu)} + 2\epsilon^{6}_{\nu\mu} - \epsilon^{7}\eta_{\mu\nu}, \tag{38g}$$
$$\delta E^{1}_{(\mu\nu)} = -\partial_{(\mu}\epsilon^{7}_{\nu)} - \partial_{(\nu}\epsilon^{8}_{\mu)} + \epsilon^{3}_{(\nu\mu)} + 2\epsilon^{4}_{\nu\mu} + \epsilon^{5}_{(\mu\nu)} - \epsilon^{7}\eta_{\mu\nu}, \tag{38h}$$
$$\delta E_{\mu} = -\partial_{\mu}\epsilon^{7} + 7\epsilon^{7}_{\mu} + 2\epsilon^{8}_{\mu} + \epsilon^{9}_{\mu}. \tag{38i}$$

Again these are the same as calculation by eq (25). It can be carefully checked, as one did for the third massive level, that all background fields except $E_{\mu\nu\lambda\alpha\beta}$, mixed-symmetric $E_{\mu\nu\lambda\alpha}$ and $E^1_{\mu\nu\lambda}$ can be either gauged away or gauged to $E_{\mu\nu\lambda\alpha\beta}$, $E_{\mu\nu\lambda\alpha}$ and $E^1_{\mu\nu\lambda}$ by zero-norm states, which is consistent with the result of the first quantized approach presented in subsection (A). The transformation for the ghost part are very lengthy and can be found in the appendix. There are 18 on-mass-shell conditions, which contains a spin four, a mixed-symmetric spin three, two symmetric spin three, two antisymmetric spin two, four symmetric spin two, five vector and three scalar fields, and 15 gauge conditions which are consistent with counting from number of zero-norm states listed in eq (27).

V. CONCLUSION

We have explictly shown that the degenerate positive-norm states at the third and fourth massive levels of bosonic open string theory are mere gauge artifacts and can be gauged to the higher spin fields at the same mass level. This is demonstrated by using both OCFQ string and WSFT. We have compared the on-shell conditions of zero-norm states in OCFQ string to the background ghost fields of WSFT. This important observation makes the lengthy calculations in both the first and second quantized approaches controllable and more importantly form a double consistency check of our results. The interesting stringy behaviors discussed in this paper and those in Ref. [1, 2] seem to imply that there must exist enormous exotic high-energy properties of string theory which remained to be uncovered. One interesting application of the decoupling of higher scalar modes is the decay of tensor fields on D-brane into the true closed string vacuum in Sen's conjectures discussed in the end of section III.

It is straightforward to generalize our calculation to closed string theory for the first quantized approach. A reliable second quantized closed string field theory may help uncover more high energy stringy properties.

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Appendix

Gauge transformation for background ghost fields of the fourth massive level are:

$$\delta\zeta_{\mu\nu\alpha\beta} = \frac{1}{2}(\partial^2 - 8)\epsilon^0_{\mu\nu\alpha\beta} - \frac{1}{2}\epsilon^7_{(\mu\nu}\eta_{\alpha\beta)}, \tag{A.1}$$

$$\delta\zeta^0_{\mu\nu\alpha} = \frac{1}{2}(\partial^2 - 8)\epsilon^1_{\mu\nu\alpha} - \partial_\mu\epsilon^7_{\nu\alpha} - \frac{1}{2}\epsilon^{11}_\mu\eta_{\nu\alpha} - \epsilon^{12}_{(\nu}\eta_{\alpha)\mu}, \tag{A.2}$$

$$\delta\zeta^1_{\mu\nu\alpha} = \frac{1}{2}(\partial^2 - 8)\epsilon^2_{\mu\nu\alpha} + \partial_{(\mu}\epsilon^7_{\nu\alpha)}, \tag{A.3}$$

$$\delta\zeta^2_{\mu\nu\alpha} = -4\partial^\beta\epsilon^0_{\mu\nu\alpha\beta} - 2\epsilon^1_{(\mu\nu\alpha)} - 3\epsilon^2_{\mu\nu\alpha} - \frac{1}{2}\epsilon^{13}_{(\mu}\eta_{\nu\alpha)}, \tag{A.4}$$

$$\delta\zeta^0_{[\mu\nu]} = \frac{1}{2}(\partial^2 - 8)\epsilon^3_{[\mu\nu]} - \partial_{[\mu}\epsilon^{12}_{\nu]}, \tag{A.5}$$

$$\delta\zeta^2_{[\mu\nu]} = \frac{1}{2}(\partial^2 - 8)\epsilon^5_{[\mu\nu]} + \partial_{[\nu}\epsilon^{11}_{\mu]}, \tag{A.6}$$

$$\delta\zeta^4_{[\mu\nu]} = -2\partial^\alpha\epsilon^1_{[\mu\nu]\alpha} - \partial_{[\mu}\epsilon^{13}_{\nu]} - 3\epsilon^3_{[\mu\nu]} - 3\epsilon^5_{[\mu\nu]}, \tag{A.7}$$

$$\delta\zeta^0_{(\mu\nu)} = \frac{1}{2}(\partial^2 - 8)\epsilon^3_{(\mu\nu)} - \partial_{(\mu}\epsilon^{12}_{\nu)} + 2\epsilon^7_{\mu\nu} - \epsilon^8\eta_{\mu\nu}, \tag{A.8}$$

$$\delta\zeta^1_{\mu\nu} = \frac{1}{2}(\partial^2 - 8)\epsilon^4_{\mu\nu} - \partial_{(\mu}\epsilon^{11}_{\nu)} - \frac{1}{2}\epsilon^8\eta_{\alpha\beta}, \tag{A.9}$$

$$\delta\zeta^2_{(\mu\nu)} = \frac{1}{2}(\partial^2 - 8)\epsilon^5_{(\mu\nu)} + \partial_{(\nu}\epsilon^{11}_{\mu)} - 2\epsilon^7_{\mu\nu} - \epsilon^9\eta_{\mu\nu}, \tag{A.10}$$

$$\delta\zeta^3_{\mu\nu} = \frac{1}{2}(\partial^2 - 8)\epsilon^6_{\mu\nu} + \partial_{(\mu}\epsilon^{12}_{\nu)} - \epsilon^7_{\mu\nu} + \frac{1}{2}\epsilon^9\eta_{\mu\nu}, \tag{A.11}$$

$$\delta\zeta^4_{(\mu\nu)} = -2\partial^\alpha\epsilon^1_{(\mu\nu)\alpha} - \partial_{(\mu}\epsilon^{13}_{\nu)} - 3\epsilon^3_{(\mu\nu)} - 4\epsilon^4_{\mu\nu} - 3\epsilon^5_{(\mu\nu)} - \epsilon^{10}\eta_{\mu\nu}, \tag{A.12}$$

$$\delta\zeta^5_{\mu\nu} = -3\partial^\alpha\epsilon^2_{\mu\nu\alpha} + \partial_{(\mu}\epsilon^{13}_{\nu)} - 2\epsilon^5_{(\mu\nu)} - 4\epsilon^6_{\mu\nu} - 2\epsilon^7_{\mu\nu}, \tag{A.13}$$

$$\delta\zeta^6_{\mu\nu} = -2\partial^\alpha\epsilon^1_{\alpha\mu\nu} + 6\epsilon^0_{\mu\nu\alpha\beta}\eta^{\alpha\beta} - 3\epsilon^3_{(\mu\nu)} - 5\epsilon^6_{\mu\nu} + 4\epsilon^7_{\mu\nu} - \frac{1}{2}\epsilon^{11}\eta_{\mu\nu}, \tag{A.14}$$

$$\delta\zeta^0_\mu = \frac{1}{2}(\partial^2 - 8)\epsilon^7_\mu - \partial_\mu\epsilon^8 + 2\epsilon^{11}_\mu + \epsilon^{12}_\mu, \tag{A.15}$$

$$\delta\zeta^1_\mu = \frac{1}{2}(\partial^2 - 8)\epsilon^8_\mu - \partial_\mu\epsilon^9 - 2\epsilon^{11}_\mu, \tag{A.16}$$

$$\delta\zeta^2_\mu = \frac{1}{2}(\partial^2 - 8)\epsilon^9_\mu + \partial_\mu\epsilon^9 - \epsilon^{12}_\mu, \tag{A.17}$$

$$\delta\zeta^3_\mu = \frac{1}{2}(\partial^2 - 8)\epsilon^{10}_\mu + \partial_\mu\epsilon^8 - 2\epsilon^{12}_\mu, \tag{A.18}$$

$$\delta\zeta^4_\mu = -\partial^\nu\epsilon^3_{\mu\nu} - \partial_\mu\epsilon^{10} - 4\epsilon^7_\mu - 3\epsilon^8_\mu + \epsilon^{13}_\mu, \tag{A.19}$$

$$\delta\zeta^5_\mu = -\partial^\nu\epsilon^5_{\mu\nu} - 3\epsilon^8_\mu - 4\epsilon^9_\mu - 2\epsilon^{11}_\mu - \epsilon^{13}_\mu, \tag{A.20}$$

$$\delta\zeta^6_\mu = -2\partial^\nu\epsilon^6_{\mu\nu} + \partial_\mu\epsilon^{10} - 2\epsilon^9_\mu - 5\epsilon^{10}_\mu - 2\epsilon^{12}_\mu - \epsilon^{13}_\mu, \tag{A.21}$$

$$\delta\zeta^7_\mu = -4\partial^\nu\epsilon^4_{\mu\nu} - \partial_\mu\epsilon^{11} - 4\epsilon^7_\mu - 5\epsilon^9_\mu + 4\epsilon^{11}_\mu + \epsilon^1_{\mu\nu\alpha}\eta^{\nu\alpha}, \tag{A.22}$$

$$\delta\zeta^8_\mu = -2\partial^\nu\epsilon^5_{\nu\mu} + \partial_\mu\epsilon^{11} - 3\epsilon^8_\mu - 6\epsilon^{10}_\mu - 3\epsilon^{13}_\mu + 3\epsilon^2_{\mu\nu\alpha}\eta^{\nu\alpha}, \tag{A.23}$$

$$\delta\zeta^9_\mu = -3\partial^\nu\epsilon^3_{\mu\nu} - 4\epsilon^7_\mu - 7\epsilon^{10}_\mu + 6\epsilon^{12}_\mu + 5\epsilon^{13}_\mu + 4\epsilon^1_{\nu\alpha\mu}\eta^{\nu\alpha}, \tag{A.24}$$

$$\delta\zeta^{10}_\mu = \frac{1}{2}(\partial^2 - 8)\epsilon^{13}_\mu + 2\partial^\nu\epsilon^7_{\mu\nu} + 2\epsilon^{11}_\mu + 4\epsilon^{12}_\mu, \tag{A.25}$$

$$\delta\zeta^0 = \frac{1}{2}(\partial^2 - 8)\epsilon^7 - 3\epsilon^8 - \epsilon^9, \tag{A.26}$$

$$\delta\zeta^1 = -\partial^\mu\epsilon^{10}_\mu - 6\epsilon^7 - 2\epsilon^8 - 2\epsilon^{10}, \tag{A.27}$$

$$\delta\zeta^2 = -\partial^\mu\epsilon^9_\mu - 7\epsilon^7 - 4\epsilon^9 - 3\epsilon^{10} - \epsilon^{11} + \epsilon^6_{\mu\nu}\eta^{\mu\nu}, \tag{A.28}$$

$$\delta\zeta^3 = -3\partial^\mu\epsilon^8_\mu - 8\epsilon^7 + 6\epsilon^9 - 4\epsilon^{11} + 2\epsilon^5_{\mu\nu}\eta^{\mu\nu}, \tag{A.29}$$

$$\delta\zeta^4 = -4\partial^\mu\epsilon^7_\mu - 9\epsilon^7 + 8\epsilon^8 + 7\epsilon^{10} + 6\epsilon^{11} + 3\epsilon^3_{\mu\nu}\eta^{\mu\nu} + 4\epsilon^4_{\mu\nu}\eta^{\mu\nu}, \tag{A.30}$$

$$\delta\zeta^5 = \frac{1}{2}(\partial^2 - 8)\epsilon^{10} + \partial^\mu\epsilon^{12}_\mu + 5\epsilon^8 + 3\epsilon^9, \tag{A.31}$$

$$\delta\zeta^6 = \frac{1}{2}(\partial^2 - 8)\epsilon^{11} + 2\partial^\mu\epsilon^{11}_\mu + 6\epsilon^8 - 5\epsilon^9 - \epsilon^7_{\mu\nu}\eta^{\mu\nu}. \tag{A.32}$$

There are 18 on-mass-shell conditions and 15 gauge conditions in eqs (A.1)-(A.32), which are consistent with counting from number of zero-norm states listed in eq (27). Note that there are two irreducible components in (A.2).

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