

行政院國家科學委員會專題研究計畫成果報告  
由共振腔結構決定的非線性雷射動力學研究  
Resonator configuration dependent nonlinear laser dynamics

計畫編號：NSC 90-2112-M009-050  
執行期限：90 年 8 月 1 日至 91 年 7 月 31 日  
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### 一、中文摘要

在本報告中我們提出在軸向幫浦的固態雷射中，當共振腔操作在簡併結構下且幫浦光斑小於空腔光斑時，雷射的本徵模是由多回徑高斯模之個別回徑之電場之線性組合構成。這個新發現的本徵模存在多焦點、低閾值和低功率轉化效率等特性。上述之結果我們不但在實驗上得到證實，而且也發現雷射如操作在單回徑模和多回徑交界之不連續區將發展成不穩定之渾沌現象。

**關鍵詞：**雷射、共振腔、光渾沌現象、固態雷射。

### Abstract

We propose a laser eigenmode must be represented as the superposition of consecutive multipass Gaussian modes and new universal instabilities when pumping beam size is less than bare cavity beam size in specific degenerate configurations. It exhibits multiple focii when it images by a lens in free-space. This laser eigenmode has lower threshold and lower slope efficiency. The laser develops into instabilities that correspond to discontinuity in the transverse-mode profile is subject to transition between single-pass transverse mode and multi-pass transverse mode. The multipass transverse modes and instabilities are experimentally observed in a tightly focused pumped Nd-YVO<sub>4</sub> laser.

**Keywords:** Laser, Resonator, Optical Chaos, Solid-State Laser.

### 二、緣由與目的

The analysis of laser beams and design of resonators are interesting subjects in laser physics after invention of laser.[1] The numerical mode-calculation procedure like the “Fox and Li” approach [2] was used to find the eigenmode of an empty resonator in the earliest days of laser studies. An arbitrary initial field will eventually converge a steady state that self-reproduce in amplitude and phase pattern each round trip. The question is raised: Does an optical resonator have a “multipass transverse eigenmode” that must reproduce itself after several round trips? In the Chapter 14 of the “Lasers” text book, [1] it states that “...such a multipass transverse (MPT) eigenmode would really consist of a mixture of the true single-pass transverse (SPT) eigenmodes”.

When the gain of a laser has a transverse variation, due to gain aperture and gain saturation, the

nonlinear gain becomes part of the propagation kernel that determines the eigenmode of laser. One may ask: Does the MPT eigenmode exist in the real system? What are its propagation behavior and spatial distribution? On the basis of nonlinear dynamics, [3,4] periodic solutions exist under persistent nonlinear effect at low-order resonance that correspond to specific transverse-mode degenerate cavity configurations. An eigenmode may be represented by the superposition of the period-n electric field solutions that belong to the consecutive n round trips of the low order resonant cavities, [4] but it cannot be expanded by the linear superposition of orthogonal period-1 bases. This reveals a new MPT eigenmode may differ from that stated in the text book [1] and implies it may be observable in a continuous wave (cw) laser at degenerations.

The diode-pumped solid-state lasers are generously used mainly because they favor to operate with the fundamental Gaussian mode with low lasing threshold and large slope efficiency under small pump size. [5] However, at the special configurations such as the self-imaging confocal cavity, [6] it had been demonstrated a Nd-YAG laser can self-adjust its mode pattern to fit optimal pump volume because the cavity can support arbitrary beam patterns. Recently, we obtained a relatively low threshold but low slope efficiency in the proximity of the transverse-mode degenerations in an axially pumped Nd-YVO<sub>4</sub> laser. [7] In particular, we observed the laser operated with a mode that display beam waist shrinkage[7] and multiple minimal beam waists<sup>8</sup> in the path of beam propagation through a focal lens.

In this report, we numerically verify our previous observations [7,8] that the new MPT eigenmodes do exist in the axial pump cw lasers when the pump radius,  $w_p$ , is less than the waist radius,  $w_c$ , of the bare cavity fundamental mode in the proximity of degenerate configurations. The steady-state laser mode can be decomposed into the superposition of the optical fields belonging to the multipass fundamental Gaussian mode in successive round trips. The laser develops into instabilities in the region corresponds to discontinuity in the transverse-mode profile is subject to transition between SPT mode and MPT mode.

### 三、研究方法與步驟

To compare with the previous experimental results [7,8], we simulate an end-pumped Nd:YVO<sub>4</sub>

laser with a plano-concave cavity having the  $g_1g_2$ -parameter around 1/4. The flat mirror as the reference plane is formed by a facet of the 1 mm thick crystal coated with 100% reflectivity and the concave mirror as the output coupler with radius of curvature  $R = 8$  cm has 90% reflectivity at 1.064  $\mu\text{m}$ . Propagation of electric field through laser crystal is described by an amplitude transmission function due to gain effect of thin slice active medium with the stimulated emission cross section of the laser transition  $25 \times 10^{-19} \text{ cm}^2$ , the upper laser level lifetime 50  $\mu\text{s}$  and the saturation intensity  $3.2 \times 10^7 \text{ W/m}^2$ .

Propagation of electric field in the free space of the cavity is according to the generalized Huygen' integral for a round trip according to the round-trip ray matrix  $M$ . Because  $w_p$  is less than 108  $\mu\text{m}$  of  $w_c$ , we chose 1mm aperture such that the Fresnel number is large enough. We input an initial electric field under specified pump or gain then calculate the internal field each round trip until it converges to a steady state.

#### 四、結果與討論

The intensity distributions of output fields at ( $L = 6$  cm) and far from ( $L = 6.15$  cm) the degeneracy are shown in Fig. 1(a) with  $w_p = 60\mu\text{m}$ . When the cavity configuration is tuned toward degeneracy, the peak intensity increases with decreasing waist radius defined by  $1/e^2$  of the peak intensity. The waist radius nearly equals the pump radius at degeneracy that means the gain-aperture effect dominates the mode pattern. This waist radius shrinkage near degeneracy shown in Fig. 1(b) with solid circles agrees well with the experiments. [7,8] However, if the spot radius is defined by the second moment, it increases with cavity tuning toward the degeneracy as shown in Fig. 1(b) with solid squares. Also shown with the open circles is the waist radius of the bare cavity ordinary fundamental Gaussian mode. We can see far from the degeneracy the laser tends to operate with the ordinary fundamental Gaussian mode; whereas the second moment waist radius becomes larger close to degeneracy, contrary to the inclination of the central waist radius. This intensity distribution looks like a centralized profile bases on a plateau. It may result from superposition of either high-order Laguerre-Gaussian modes or multipass fundamental modes. We will show shortly that the new MPT mode is indeed the latter case when  $w_p < w_c$  and is even more obvious for smaller pump size.

Gain-aperture effect offers the nonlinearity to support the periodic dynamics in the system such that the stable period-3 solutions exist [4] for  $g_1g_2 = 1/4$ , that is a field with any  $q$ -parameter or electric field will reproduce itself after three round trips inside the bare cavity due to  $M^3 = I$ . This self-consistent eigenmode will not be separated for individual round trip in a cw steady-state laser because three different electric fields for different round trips simultaneously

exist at any plane in the cavity. Thus, the mode distribution at degeneracy should be dominated by the gain-loss mechanism and this is confirmed by the fact that the waist radius at degeneracy is equal to the pump radius. [8]

By using  $w_p = 30 \mu\text{m}$  which was used in the experiments [7,8], we numerically obtained the one-round-trip self-consistent intensity profile at degeneracy as the solid squares in Fig. 2 with logarithmic vertical scale. To compare with the self-consistent intensity profile, we superpose the fields of consecutive three round trips with an initial Gaussian field having spot radius  $w_0$  and curvature of radius  $R_0$  on the flat mirror end and labeled as  $E_0(w_0, R_0 = \infty)$ . Then the electric field and  $q$ -parameter evolution in bare cavity repeatedly after each round trip follows  $E_1(w_1, R_1)$ ,  $E_2(w_1, -R_1)$  and  $E_0(w_0, \infty)$ . Since the mode distribution will be determined by pump beam due to gain aperture effect, we choose  $w_0 = w_p = 30\mu\text{m}$ . The intensity profile shown as open circles in Fig. 2 is the superposition mode of  $E_0$ ,  $E_1$ , and  $E_2$ . It consists of a centralized profile due to the contribution of  $E_0$  bases on a plateau that comes from the defocused fields  $E_1$  and  $E_2$ .

If we add a convergent lens of focal length 5.2 cm after the output coupler at a distance of 16.5 cm as in the experiments. [8] Three beam waists will be found as shown in Fig. 3(b) for both self-consistent (solid circles) and superposition (open circles) modes. The positions of three waists are the same as those of experimental observation. [8] The multiple-waist results show that the mode is mainly assembled by the superposition of period-3 solutions and cannot be superposed by using space and time dependence of SPT eigenmodes<sup>1</sup>. Because beam waists of all the SPT eigenmodes are located at the flat mirror, any one or superposition of those modes will diverge or has one imaging waist after imaged by a focusing lens. The new MPT modes appear near  $g_1g_2 = 1/4$  for  $w_p \approx 60 \mu\text{m}$ , otherwise the ordinary SPT mode recovers (see, for example, star curve in Fig. 3(a) for  $w_p = 90 \mu\text{m}$ ). They exist as the cavity length tuned from 5.96 cm to 6.06 cm for  $w_p = 30 \mu\text{m}$ , whereas the laser prefers to operate with the ordinary SPT mode when the cavity configuration is tuned away from the degeneracy.

#### 五、自我評估

本年度計畫下我們添購一部倍頻 Nd:YVO<sub>4</sub>雷射作為太藍寶石雷射之幫浦源，產生之飛秒雷射不但有相當高之穩定性，輸出功率、功率轉換效率、和波長之可調性多提高許多。這不但對本計畫共振腔結構相關之雷射非線性動力學之研究有相當之助益，對使用此雷射作為其他非線性光學材料之研究主題，也已有明顯之成果。今年度如本報告所述，首次發現並證實在所謂簡併共振腔結構下，當幫浦光斑小於空腔光斑時，穩定雷射輸出模並非眾人熟知之高斯本徵模，而是新型之所謂多回徑本徵

模。此結果可能改寫現有之雷射教科書。而在簡併與正常態之間存在相變區內，會造成雷射極度不穩定，甚至形成光渾沌現象。這個發現可能發展出新的渾沌通訊的架構。

另外，我們在鐵電材料晶格動力學方面，也證實庫倫作用力對鐵電性之影響，及  $\text{PbTiO}_3$  和  $\text{BaTiO}_3$  中 Pb 與 Ba 離子性之大小決定了鐵電性質。這些結果將提供理論研究學者，修改理論模型以更擬合實驗結果之依據。

## 六、結論

We have verified the previous experimental observation that a new MPT eigenmode must be expressed as superposition of multipass Gaussian modes, but not as linear combination of single-pass high-order LG modes near degenerate cavity configuration. These modes, exhibiting multiple beam waists in free-space propagation or after a simple imaging lens, can occur close to specific degenerations when  $w_p < w_c$ , but the ordinary fundamental Gaussian modes recover far from these degenerations. Moreover, they are more obvious with smaller pump size due to stronger gain aperture effect. Lower threshold accompanied with lower slope efficiency are illustrated for the new MPT modes. The laser develops into instabilities that correspond to discontinuity in the transverse-mode profile is subject to transition between SPT mode and MPT mode. Therefore, axially pumped solid-state lasers should be cautiously used and can be used to study nonlinear dynamics since there are many degenerate positions that are sensitive to nonlinear effect within the geometrically stable regime.

## 七、參考文獻

1. E. Siegmann, *Lasers*, University Science Book, Mill Valley, CA, (1986).
2. A. G. Fox, and T. Li, *Bell Sys. Tech. J.* 40, 453 (1961)
3. M.D. Wei, W.-F. Hsieh, and C. C. Sung, *Optics Commun.* 146, 201 (1998).
4. M.D. Wei, W.F. Hsieh, "Cavity configuration dependent nonlinear dynamics in Kerr lens mode-locked lasers," *J. Opt. Soc. Am. B* 17, 1335-1342 (2000).
5. P. Laporta and M. Brussard, "Design criteria for mode size optimization in diode-pumped solid-state lasers", *IEEE. J. Quantum Electron.* 27, 2319-2326 (1991).
6. V. Couderc, O. Guy, A. Barthelemy, C. Froehly, and F. Louradour, "Self-optimized resonator for optical pumping of solid-state lasers", *Opt. Lett.* 19, 1134 (1994)
7. H. H. Wu, C. C. Sheu, T. W. Chen, M. D. Wei and W.F. Hsieh, "Observation of power drop and low threshold due to beam waist shrinkage around critical configurations in an end-pumped Nd:YVO<sub>4</sub> laser," *Opt. Commu.* 165, 225 (1999).
8. H.H. Wu, W.F. Hsieh, "Observations of multi-pass transverse modes in an axially pumped solid state laser with different fractional degenerate resonator configurations," *J. Opt. Soc. Am. B* 18, 7-12 (2001).

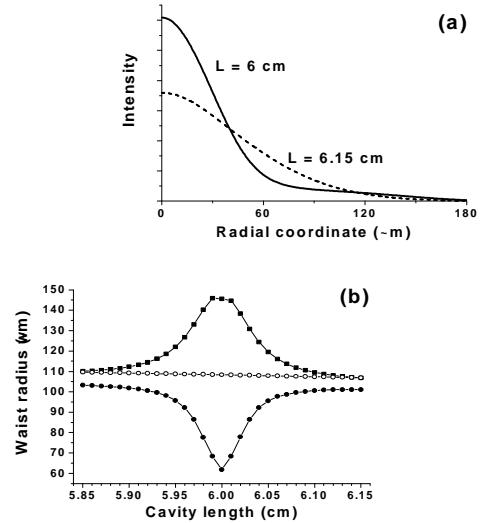


Fig. 1. The central part of intensity profiles (a) and waist radius (b) of the central part (solid circles), second moment (solid squares), and bare cavity Gaussian mode (open circles) near degeneracy as  $w_p = 60 \mu\text{m}$ .

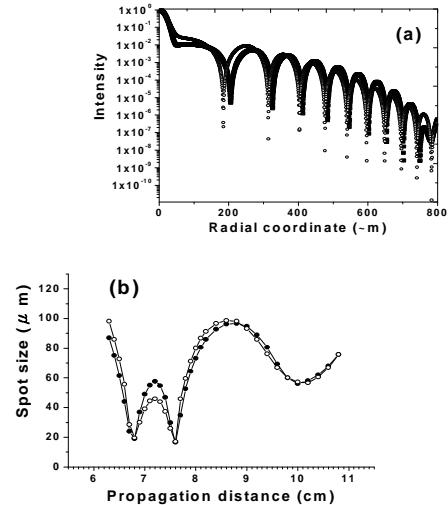


Fig. 2. (a) The normalized intensity profile of the self-consistent MPT mode (solid line) and the multipass superposition mode (open circle) (b) Propagating characteristic of the MPT mode through free-space through a focal lens for the self-consistent and the superposition modes, respectively.