

The generalized synchronization of a Quantum-CNN chaotic oscillator with different order systems

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Abstract

This paper presents a special kind of the generalized synchronization of different order systems, proved by Lyapunov asymptotical stability theorem. A sufficient condition is given for the asymptotical stability of the null solution of an error dynamics. The generalized synchronization developed may be applied to the design of secure communication. Finally, numerical results are studied for a Quantum-CNN oscillator synchronized with three different order systems respectively to show the effectiveness of the proposed synchronization strategy.

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1. Introduction

In recent years, the synchronization of chaotic systems has been studied in various fields [1–4]. For a particular chaotic system, a master, together with an identical or a different system, a slave system, our goal is to synchronize them via coupling or other methods.

Among many kinds of synchronizations [5], the generalized synchronization is investigated [6–10]. It means there exists a functional relationship between the states of the master and those of the slave. In this paper, a special kind of generalized synchronizations

$$y = x + F(t) \quad (1)$$

is studied, where x , y are the state vectors of the master and the slave respectively, $F(t)$ is a given vector function of time, which may take various forms, either regular or chaotic functions of time. The generalized synchronization developed may be applied to the design of secure communication. When $F(t) = 0$, it reduces to a complete synchronization [12–21].

As numerical examples, recently developed quantum cellular neural network (Quantum-CNN) chaotic oscillator is used to synchronize with three different order systems respectively. Quantum-CNN oscillator equations are derived from a Schrödinger equation taking into account quantum dots cellular automata structures to which in the last decade a wide interest has been devoted, with particular attention towards quantum computing [11].

This paper is organized as follows. In Section 2, by the Lyapunov asymptotical stability theorem, a generalized synchronization scheme is given. In Section 3, various feedback controllers are designed for the synchronization of the

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Quantum-CNN oscillator with a Lorenz system and with a Chen system respectively. Numerical simulations are also given in Section 3. Finally, some concluding remarks are given in Section 4.

2. Generalized synchronization scheme

There are two identical nonlinear dynamical systems, and the master system controls the slave system. The master system is given here

$$\dot{x} = Ax + f(x) \tag{2}$$

where $x = (x_1, x_2, \dots, x_n)^T \in R^n$ denotes a state vector, A is a $n \times n$ coefficient matrix, and f is a nonlinear vector function.

The slave system is given here

$$\dot{y} = By + h(y) + u(t) \tag{3}$$

where $y = (y_1, y_2, \dots, y_{n-\alpha})^T \in R^{n-\alpha}$ denotes a state vector, α is a positive integer, $1 \leq \alpha \leq n$, B is a $(n - \alpha) \times (n - \alpha)$ coefficient matrix, h is a nonlinear vector function, and $u(t) = (u_1(t), u_2(t), \dots, u_{n-\alpha}(t))^T \in R^{n-\alpha}$ is a control input vector.

Our goal is to design a controller $u(t)$ so that the state vector of the slave system (3) asymptotically approaches the state vector of the master system (2) plus a given vector function $F(t) = (F_1(t), F_2(t), \dots, F_{n-\alpha}(t))^T$, and finally the synchronization will be accomplished in the sense that the limit of the error vector $e(t) = (e_1, e_2, \dots, e_{n-\alpha})^T$ approaches zero:

$$\lim_{t \rightarrow \infty} e = 0 \tag{4}$$

where

$$e = x - y + F(t) \tag{5}$$

From Eq. (5) we have

$$\dot{e} = \dot{x} - \dot{y} + \dot{F}(t) \tag{6}$$

$$\dot{e} = (A_{n-\alpha} - B)e + f(x) - h(y) + \dot{F}(t) - u(t) \tag{7}$$

A Lyapunov function $V(e)$ is chosen as a positive definite function

$$V(e) = \frac{1}{2} e^T e \tag{8}$$

Its derivative along any solution of Eq. (7) is

$$\dot{V}(e) = e^T \{ (A_{n-\alpha} - B)[x_i - y_i] + f(x) - h(y) + \dot{F}(t) - u(t) \}, \quad i = 1, 2, \dots, n - \alpha \tag{9}$$

where $[x_i - y_i]$ is a $(n - \alpha) \times 1$ column matrix, $u(t)$ is chosen so that, $\dot{V} = e^T C_{n-\alpha} e$, $C_{n-\alpha}$ is a diagonal negative definite matrix, and \dot{V} is a negative definite function of e . By the Lyapunov theorem of asymptotical stability, we have

$$\lim_{t \rightarrow \infty} e = 0 \tag{10}$$

The generalized synchronization is obtained [22–31].

3. Numerical results of the generalized chaos synchronization of the Quantum-CNN oscillator with different order systems

Case I. A complete synchronization as a special case of the generalized synchronization

For a two-cell Quantum-CNN, the following differential equations are obtained [11]:

$$\begin{cases} \frac{d}{dt}x_1 = -2a_1\sqrt{1-x_1^2}\sin x_2 \\ \frac{d}{dt}x_2 = -\omega_1(x_1-x_2) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 \\ \frac{d}{dt}x_3 = -2a_2\sqrt{1-x_3^2}\sin x_4 \\ \frac{d}{dt}x_4 = -\omega_2(x_3-x_1) + 2a_2\frac{x_3}{\sqrt{1-x_3^2}}\cos x_4 \end{cases} \tag{11}$$

where x_1, x_3 are polarizations, x_2, x_4 are quantum phase displacements, a_1 and a_2 are proportional to the inter-dot energy inside each cell and ω_1 and ω_2 are parameters that weigh effects on the cell of the polarization of neighboring cells, like the cloning templates in traditional CNNs. Let $a_1 = 19.4, a_2 = 13.1, \omega_1 = 9.529, \omega_2 = 7.94$.

A Lorenz system is described by

$$\begin{cases} \frac{d}{dt}y_1 = \sigma(y_2 - y_1) \\ \frac{d}{dt}y_2 = \gamma y_1 - y_2 - y_1 y_3 \\ \frac{d}{dt}y_3 = y_1 y_2 - \beta y_3 \end{cases} \tag{12}$$

where $\sigma = 10, \gamma = 28, \beta = \frac{3}{8}$.

Take $\alpha = 1$. In order to lead (y_1, y_2, y_3) to $(x_1 + F_1(t), x_2 + F_2(t), x_3 + F_3(t))$, we add u_1, u_2 , and u_3 to the first, the second, and the third equations of Eq. (12) respectively.

$$\begin{cases} \frac{d}{dt}y_1 = \sigma(y_2 - y_1) + u_1 \\ \frac{d}{dt}y_2 = \gamma y_1 - y_2 - y_1 y_3 + u_2 \\ \frac{d}{dt}y_3 = y_1 y_2 - \beta y_3 + u_3 \end{cases} \tag{13}$$

Subtracting Eq. (13) from the first three equations of Eq. (11), we obtain an error dynamics. The initial values of the Quantum-CNN system and the Lorenz system are taken as $x_1(0) = 0.8, x_2(0) = -0.77, x_3(0) = -0.72, x_4(0) = 0.57, y_1(0) = -0.2, y_2(0) = -0.42$, and $y_3(0) = -0.11$.

$$\lim_{t \rightarrow \infty} e_i = \lim_{t \rightarrow \infty} (x_i - y_i) = 0, \quad i = 1, 2, 3 \tag{14}$$

$$\begin{aligned} \dot{e}_1 &= -2a_1 \sqrt{1 - x_1^2} \sin x_2 - \sigma(y_2 - y_1) - u_1 \\ \dot{e}_2 &= -\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1 - x_1^2}} \cos x_2 - \gamma y_1 + y_2 + y_1 y_3 - u_2 \\ \dot{e}_3 &= -2a_2 \sqrt{1 - x_3^2} \sin x_4 - y_1 y_2 + \beta y_3 - u_3 \end{aligned} \tag{15}$$

where $e_1 = x_1 - y_1, e_2 = x_2 - y_2, e_3 = x_3 - y_3$.

Choose a Lyapunov function in the form of the positive definite function:

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{16}$$

Its time derivative along any solution of Eq. (15) is

$$\begin{aligned} \dot{V} &= e_1 \left(-2a_1 \sqrt{1 - x_1^2} \sin x_2 - \sigma(y_2 - y_1) - u_1 \right) \\ &\quad + e_2 \left(-\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1 - x_1^2}} \cos x_2 - \gamma y_1 + y_2 + y_1 y_3 - u_2 \right) \\ &\quad + e_3 \left(-2a_2 \sqrt{1 - x_3^2} \sin x_4 - y_1 y_2 + \beta y_3 - u_3 \right) \end{aligned} \tag{17}$$

Choose

$$\begin{aligned} u_1 &= -2a_1 \sqrt{1 - x_1^2} \sin x_2 - \sigma y_2 - \sigma x_1 \\ u_2 &= -\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1 - x_1^2}} \cos x_2 - \gamma y_1 + y_1 y_3 - x_2 \\ u_3 &= -2a_2 \sqrt{1 - x_3^2} \sin x_4 - y_1 y_2 + \beta x_3 \end{aligned}$$

Eq. (17) can be rewritten as

$$\dot{V} = -\sigma e_1^2 - e_2^2 - \gamma e_3^2 < 0 \tag{18}$$

which is negative definite. The Lyapunov asymptotical stability theorem is satisfied. This means that the complete chaos synchronization of the different order systems, the Quantum-CNN system and the Lorenz system, can be achieved. The numerical results are shown in Fig. 1. After 10 s, the motion trajectories enter a chaotic attractor.

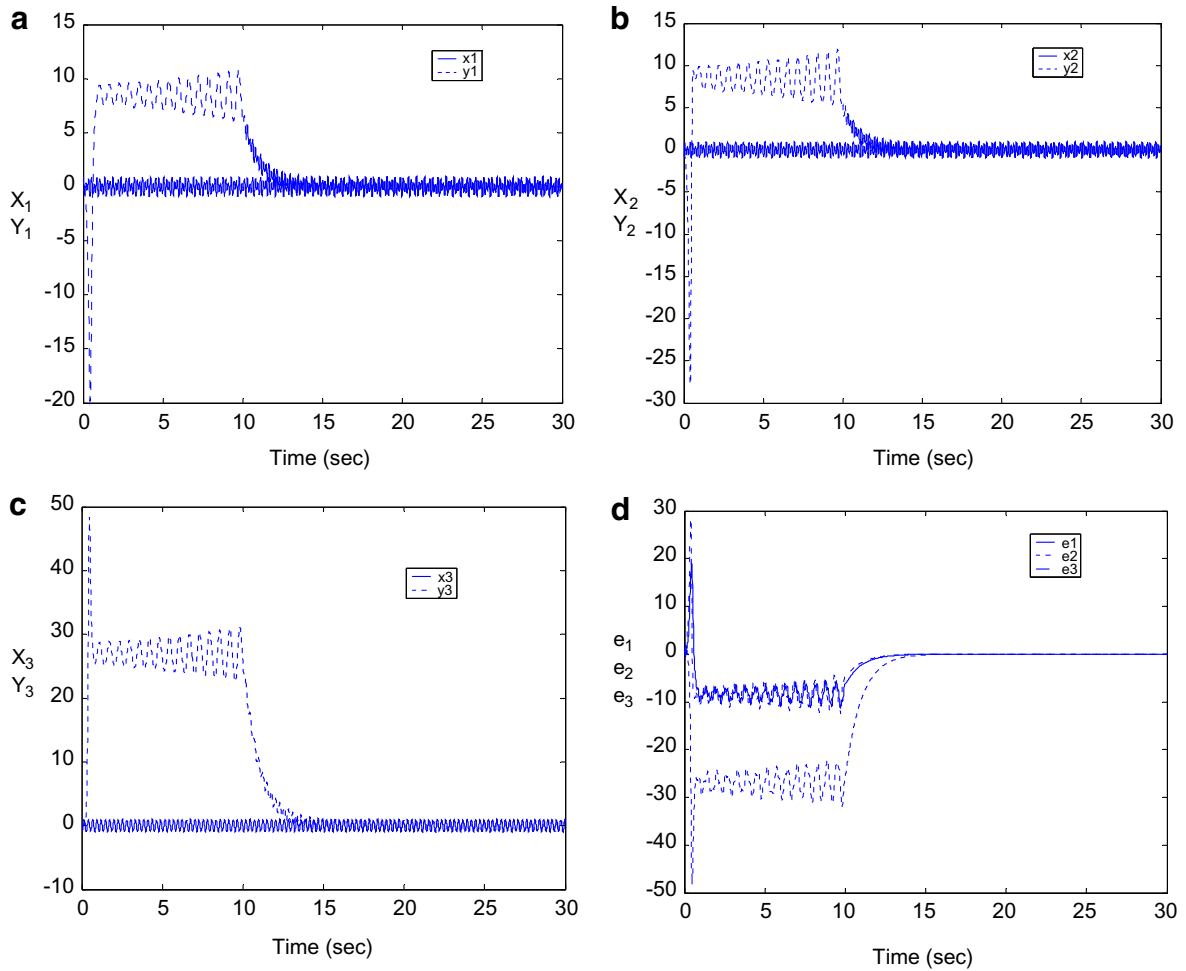


Fig. 1. Time histories of the master states, of the slave states, and of the synchronization errors for the Quantum-CNN system and the Lorenz system.

Case II. A sine function synchronization

We have

$$\lim_{t \rightarrow \infty} e_i = \lim_{t \rightarrow \infty} (x_i - y_i + F_i \sin \omega t) = 0, \quad i = 1, 2, 3 \tag{19}$$

where $\dot{e} = \dot{x} - \dot{y} + F\omega \cos \omega t$.

Let $F_1 = F_2 = F_3 = F$, Eq. (7) becomes

$$\begin{aligned} \dot{e}_1 &= -2a_1 \sqrt{1 - x_1^2} \sin x_2 - \sigma(y_2 - y_1) - u_1 + F\omega \cos \omega t \\ \dot{e}_2 &= -\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1 - x_1^2}} \cos x_2 - \gamma y_1 + y_2 + y_1 y_3 - u_2 + F\omega \cos \omega t \\ \dot{e}_3 &= -2a_2 \sqrt{1 - x_3^2} \sin x_4 - y_1 y_2 + \beta y_3 - u_3 + F\omega \cos \omega t \end{aligned} \tag{20}$$

where $e_1 = x_1 - y_1 + F \sin \omega t$, $e_2 = x_2 - y_2 + F \sin \omega t$, $e_3 = x_3 - y_3 + F \sin \omega t$. F and ω are taken as $F = 0.7$, $\omega = 1$.

Choose a Lyapunov function in the form of the positive definite function:

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{21}$$

Its time derivative along any solution of Eq. (20) is

$$\begin{aligned} \dot{V} = & e_1 \left(-2a_1 \sqrt{1-x_1^2} \sin x_2 - \sigma(y_2 - y_1) - u_1 + F\omega \cos \omega t \right) \\ & + e_2 \left(-\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1-x_1^2}} \cos x_2 - \gamma y_1 + y_2 + y_1 y_3 - u_2 + F\omega \cos \omega t \right) \\ & + e_3 \left(-2a_2 \sqrt{1-x_3^2} \sin x_4 - y_1 y_2 + \beta y_3 - u_3 + F\omega \cos \omega t \right) \end{aligned} \tag{22}$$

Choose

$$\begin{aligned} u_1 &= -2a_1 \sqrt{1-x_1^2} \sin x_2 - \sigma y_2 + \sigma x_1 + F(\omega \cos \omega t + \sin \omega t) \\ u_2 &= -\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1-x_1^2}} \cos x_2 - \gamma y_1 + y_1 y_3 + x_2 + F(\omega \cos \omega t + \sin \omega t) \\ u_3 &= -2a_2 \sqrt{1-x_3^2} \sin x_4 - y_1 y_2 + \beta x_3 + F(\omega \cos \omega t + \sin \omega t) \end{aligned}$$

Eq. (22) can be rewritten as

$$\dot{V} = -\sigma e_1^2 - e_2^2 - \gamma e_3^2 < 0 \tag{23}$$

which is negative definite. The Lyapunov asymptotical stability theorem is satisfied. This means that the sine function synchronization of the different order systems, the Quantum-CNN system and the Lorenz system, can be achieved. The numerical results are shown in Fig. 2. After 10 s, the motion trajectories enter a chaotic attractor.

Case III. A Chen system state synchronization

The goal system for synchronization is a Chen system

$$\begin{cases} \frac{d}{dt} z_1 = a(z_2 - z_1) \\ \frac{d}{dt} z_2 = (c - a)z_1 - z_1 z_3 - cz_2 \\ \frac{d}{dt} z_3 = z_1 z_2 - bz_3 \end{cases} \tag{24}$$

where $a = 10$, $b = \frac{3}{8}$, $c = 28$. The initial values of the states of the Chen system is taken as $z_1(0) = 0.5$, $z_2(0) = 0.5$, and $z_3(0) = 0.5$. The chaotic vector state $z = (z_1, z_1, z_1)$ is chosen as $F(t)$. Then

$$\lim_{t \rightarrow \infty} e_i = \lim_{t \rightarrow \infty} (x_i - y_i + z_1) = 0, \quad i = 1, 2, 3 \tag{25}$$

Eq. (7) becomes

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{y} + \dot{z} \\ \dot{e}_1 &= -2a_1 \sqrt{1-x_1^2} \sin x_2 - \sigma(y_2 - y_1) - u_1 + a(z_2 - z_1) \\ \dot{e}_2 &= -\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1-x_1^2}} \cos x_2 - \gamma y_1 + y_2 + y_1 y_3 - u_2 + a(z_2 - z_1) \\ \dot{e}_3 &= -2a_2 \sqrt{1-x_3^2} \sin x_4 - y_1 y_2 + \beta y_3 - u_3 + a(z_2 - z_1) \end{aligned} \tag{26}$$

where $e_1 = x_1 - y_1 + z_1$, $e_2 = x_2 - y_2 + z_1$, $e_3 = x_3 - y_3 + z_1$.

Choose a Lyapunov function in the form of the positive definite function:

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{27}$$

Its time derivative along any solution of Eq. (26) is

$$\begin{aligned} \dot{V} = & e_1(-2a_1 \sqrt{1-x_1^2} \sin x_2 - \sigma(y_2 - y_1) - u_1 + a(z_2 - z_1)) + e_2(-\omega_1(x_1 - x_3) + 2a_1 \frac{x_1}{\sqrt{1-x_1^2}} \cos x_2 - \gamma y_1 \\ & + y_2 + y_1 y_3 - u_2 + a(z_2 - z_1)) + e_3(-2a_2 \sqrt{1-x_3^2} \sin x_4 - y_1 y_2 + \beta y_3 - u_3 + a(z_2 - z_1)) \end{aligned} \tag{28}$$

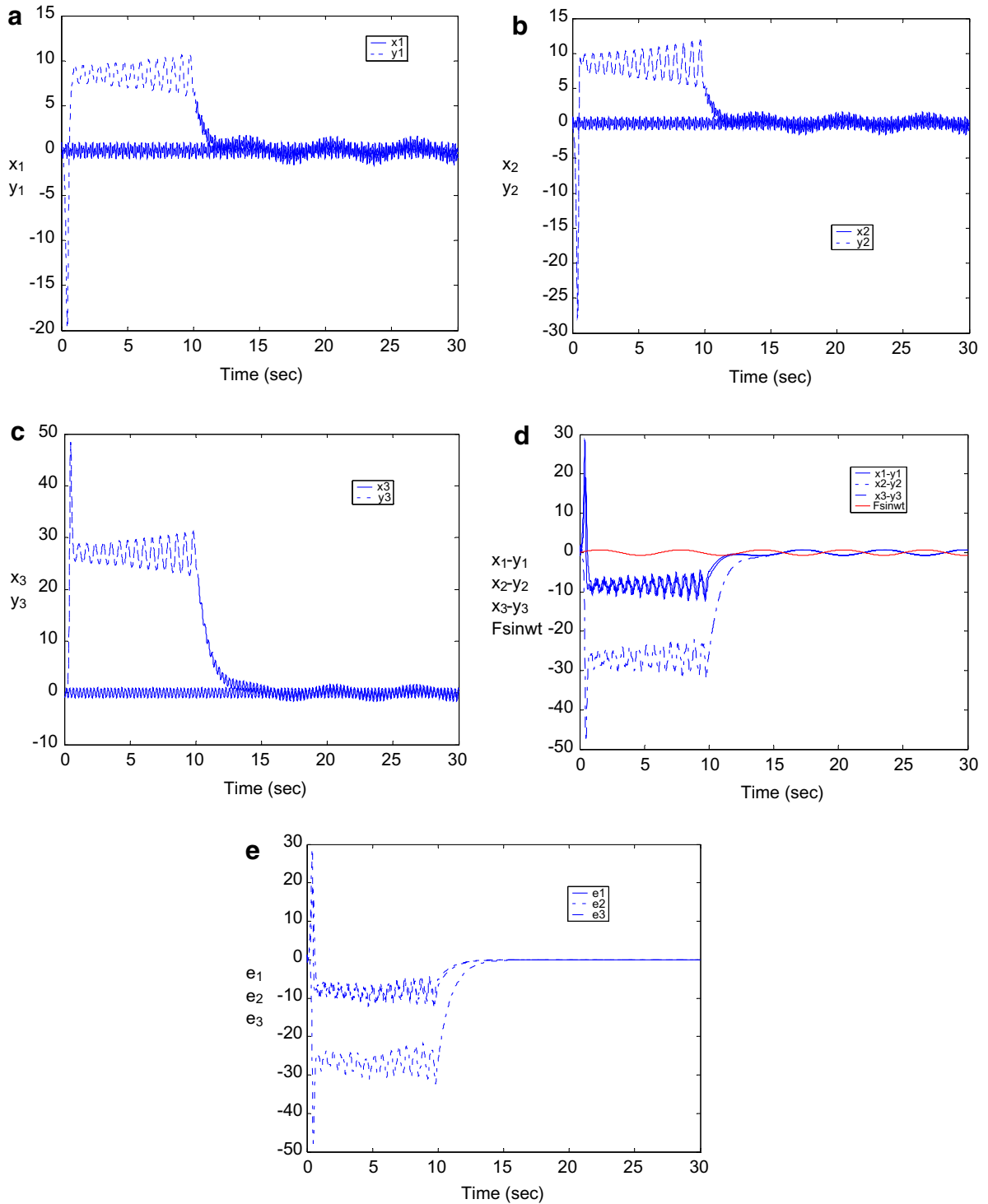


Fig. 2. Time histories of the master states, of the slave states, and of the sine function synchronization errors for the Quantum-CNN system and the Lorenz system, where $e_i = x_i - y_i + F \sin \omega t$, $i = 1, 2, 3$.

Choose

$$\begin{aligned} u_1 &= -2a_1\sqrt{1-x_1^2}\sin x_2 - \sigma y_2 + \sigma x_1 + a(z_2 - z_1) + z_1 \\ u_2 &= -\omega_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 - \gamma y_1 + y_1 y_3 - x_2 + a(z_2 - z_1) + z_1 \\ u_3 &= -2a_2\sqrt{1-x_3^2}\sin x_4 - y_1 y_2 + \beta x_3 + a(z_2 - z_1) + z_1 \end{aligned}$$

Eq. (28) can be rewritten as

$$\dot{V} = -\sigma e_1^2 - e_2^2 - \gamma e_3^2 < 0 \quad (29)$$

which is negative definite. The Lyapunov asymptotical stability theorem is satisfied. This means that the Chen system state synchronization of the different order systems, the Quantum-CNN system and the Lorenz system, can be achieved. The numerical results are shown in Fig. 3. After 10 s, the motion trajectories enter a chaotic attractor.

Case IV. A Chen system states synchronization

We have

$$\lim_{t \rightarrow \infty} e_i = \lim_{t \rightarrow \infty} (x_i - y_i + z_i) = 0, \quad i = 1, 2, 3 \quad (30)$$

and

$$\dot{e} = \dot{x} - \dot{y} + \dot{z}$$

where $\dot{z} = (\dot{z}_1, \dot{z}_2, \dot{z}_3)$. Eq. (7) becomes

$$\begin{aligned} \dot{e}_1 &= -2a_1\sqrt{1-x_1^2}\sin x_2 - \sigma(y_2 - y_1) - u_1 + a(z_2 - z_1) \\ \dot{e}_2 &= -\omega_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 - \gamma y_1 + y_2 + y_1 y_3 - u_2 + (c - a)z_1 - z_1 z_2 - cz_2 \\ \dot{e}_3 &= -2a_2\sqrt{1-x_3^2}\sin x_4 - y_1 y_2 + \beta y_3 - u_3 + z_1 z_2 - bz_3 \end{aligned} \quad (31)$$

where $e_1 = x_1 - y_1 + z_1$, $e_2 = x_2 - y_2 + z_2$, $e_3 = x_3 - y_3 + z_3$.

Choose a Lyapunov function in the form of the positive definite function:

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \quad (32)$$

Its derivative along any solution of Eq. (31) is

$$\begin{aligned} \dot{V} &= e_1 \left(-2a_1\sqrt{1-x_1^2}\sin x_2 - \sigma(y_2 - y_1) - u_1 + a(z_2 - z_1) \right) \\ &\quad + e_2 \left(-\omega_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 - \gamma y_1 + y_2 + y_1 y_3 - u_2 + (c - a)z_1 - z_1 z_2 - cz_2 \right) \\ &\quad + e_3 \left(-2a_2\sqrt{1-x_3^2}\sin x_4 - y_1 y_2 + \beta y_3 - u_3 + z_1 z_2 - bz_3 \right) \end{aligned} \quad (33)$$

Choose

$$\begin{aligned} u_1 &= -2a_1\sqrt{1-x_1^2}\sin x_2 - \sigma y_2 + \sigma x_1 + a(z_2 - z_1) + z_1 \\ u_2 &= -\omega_1(x_1 - x_3) + 2a_1\frac{x_1}{\sqrt{1-x_1^2}}\cos x_2 - \gamma y_1 + y_1 y_3 + x_2 + (c - a)z_1 - z_1 z_2 + z_2(1 - c) \\ u_3 &= -2a_2\sqrt{1-x_3^2}\sin x_4 - y_1 y_2 + \beta x_3 + z_1 - z_2 + z_3(1 - b) \end{aligned}$$

Eq. (33) can be rewritten as

$$\dot{V} = -\sigma e_1^2 - e_2^2 - \gamma e_3^2 < 0 \quad (34)$$

which is negative definite. The Lyapunov asymptotical stability theorem is satisfied. This means that the Chen system states synchronization of the different order systems, the Quantum-CNN system and the Lorenz system, can be achieved. The numerical results are shown in Fig. 4. After 10 second, the motion trajectories enter a chaotic attractor.

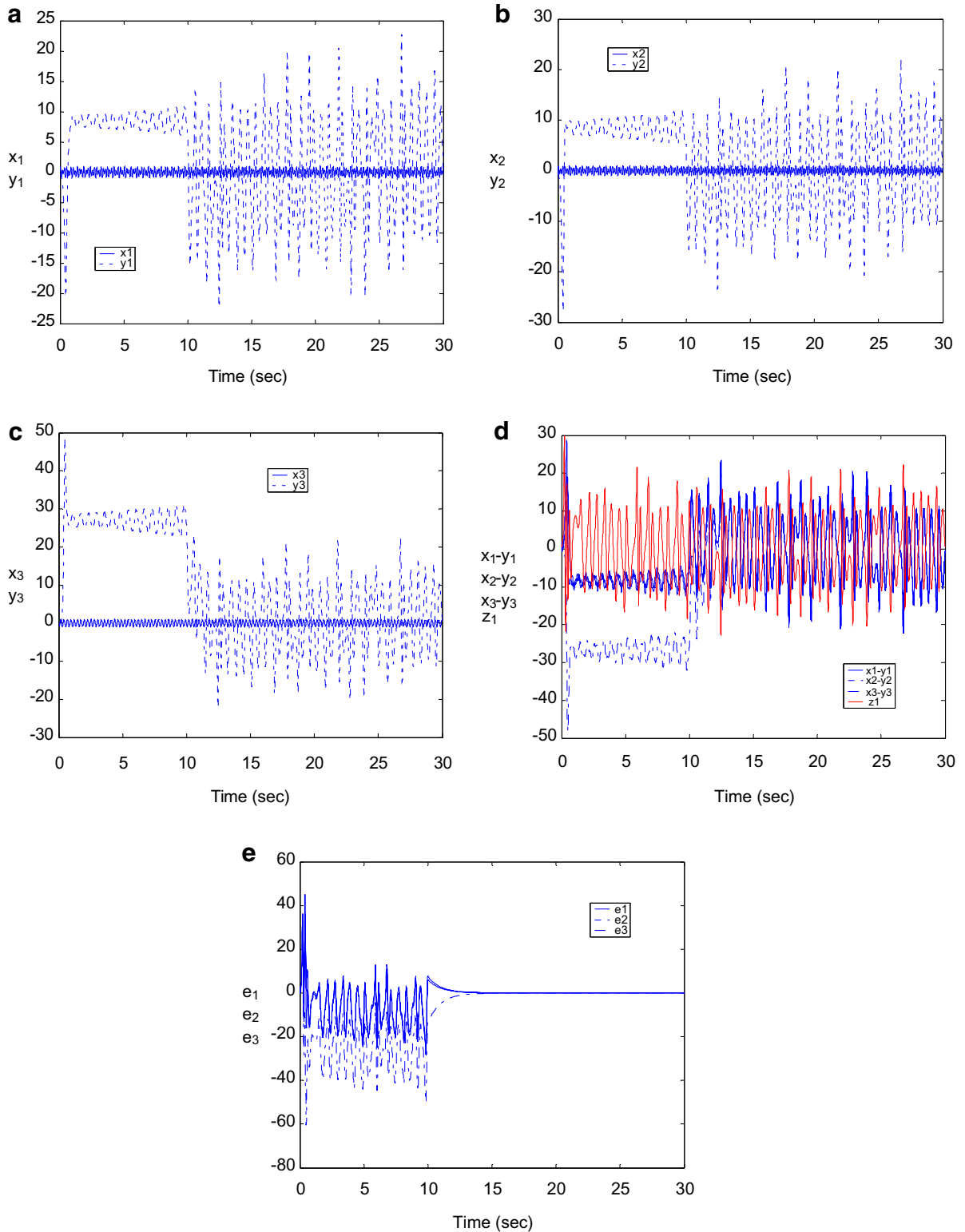


Fig. 3. Time histories of the master states, of the slave states, and of the Chen system state synchronization errors for the Quantum-CNN system and the Lorenz system, where $e_i = x_i - y_i + z_i$, $i = 1, 2, 3$.

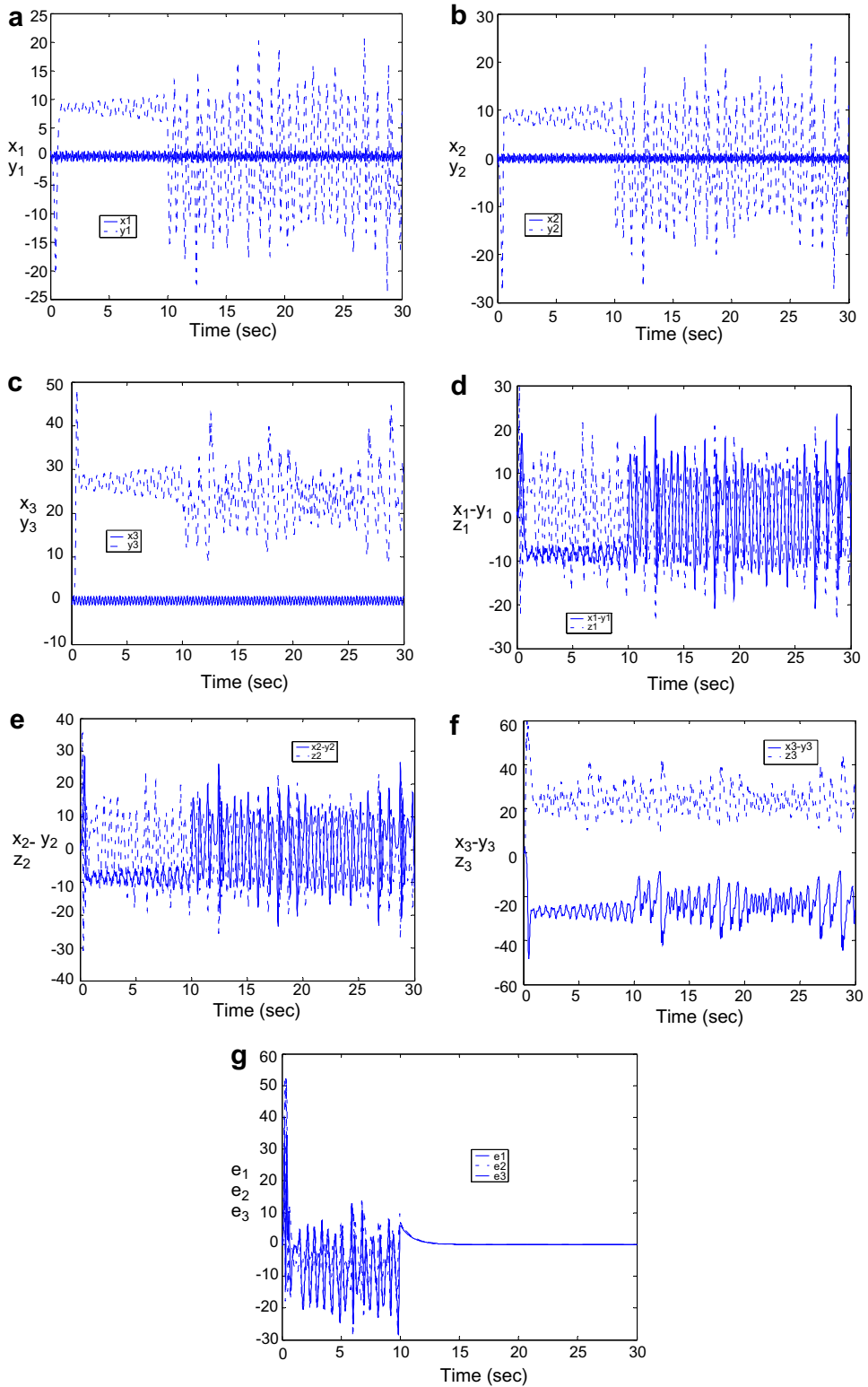


Fig. 4. Time histories of the master states, of the slave states, and of the Chen system states synchronization errors for the Quantum-CNN system and the Lorenz system, where $e_i = x_i - y_i + z_i$, $i = 1, 2, 3$.

4. Conclusions

The generalized chaos synchronization of the different order systems are investigated by using the Lyapunov asymptotical stability theorem. Two different chaotic dynamical systems, the Quantum-CNN system and the Lorenz system, are chosen for the complete synchronization as a special case and for the given regular time function synchronization. The Chen system is chosen for the given chaotic time function synchronizations. The generalized synchronization of chaos system can be used to increase the security of communication.

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