行政院國家科學委員會專題研究計畫成果報告

形態小波轉換理論與應用之研究

A Study on Morphological Wavelet Transform: Theory and Applications

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中文摘要

本計畫探討形態小波轉換之理論與應用。在理論方面，形態小波轉換結合了多重解析理論，離散小波轉換與形態骨架法。在應用上，可引進提升法已獲得更佳之資料壓縮效果。

關鍵詞：多重解析理論、離散小波轉換、形態骨架法、提升法

Abstract

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Keywords: Multiresolution Analysis, Discrete Wavelet Transform, Morphological Skeletonization, Lifting Scheme

1. INTRODUCTION

Multiresolution decomposition [1, 2, 3] based on discrete wavelet transform [4, 5] has significant applications in signal representation and compression. Since discrete wavelet transform is linear and the construction of a scaling function required Fourier transform, nonlinear approaches to wavelet-like decompositions without using Fourier transform have received many attentions [6-12]. The well-known lifting scheme [13, 14, 15] is one of the most successful nonlinear approaches.

Recently, Goutsias and Heijmans [16, 17] have proposed an axiomatic framework to unify the linear and nonlinear multiresolution analyses. Under this framework, wavelet decomposition based on mathematical morphology [18] can be constructed.

Mathematical morphology is known as an efficient tool for image analysis and processing. Thus, it is the purpose of this study to investigate the theory and applications of morphological wavelet decomposition.

2. WAVELET DECOMPOSITION

In a multi-resolution analysis, there are nested vector spaces

$$\Lambda \subset V_1 \subset V_0 \subset V_{-1} \subset \Lambda$$

such that

(1) the closure of the union of all V_j is the space $L^2(\Re)$ and the intersection of all V_j is {0};

(2) $f(t) \in V_j$ if and only if $f(2t) \in V_{j-1}$;

(3) there exists a **scaling function** $\phi(t)$ such that $\{\phi(t-k): k$ is an integer$\}$ form a basis for V_0.

Suppose W_j is the orthogonal complement of V_j in V_{j-1}. Then the vector space V_j can be written as

$$V_{j-1} = W_j \oplus W_{j+1} \oplus \Lambda \oplus W_{j+k} \oplus V_{j+k}$$

for any nonnegative *k*. It is known that there exists a function $\psi(t)$, the **wavelet**, such that $\{\psi(t-k): k$ is an integer$\}$ is a basis for W_1. Then any function $f \in L^2(\Re)$ can be represented as

$$f(t) = \sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} c_{k,l}\psi(2^{-k}t - l)$$

where

$$c_{k,l} = \int_{\Re} f(t) 2^{-\frac{k}{2}} \psi(2^{-\frac{k}{2}} t - l) dt$$

is the **discrete wavelet transform** of $f(t)$. Note that the discrete wavelet transform is linear and the sets V_j and W_j are required to be vector spaces.

Recently, Goutsias and Heijmans proposed an axiomatic framework to unify linear and nonlinear approaches. Let V_j and W_j be, respectively, the signal space and the detail space at level *j*. Consider the following analysis and synthesis operators:

(1) signal analysis operator
$G_j^{\downarrow}: V_j \to V_{j+1}$
(2) detail analysis operator
$H_j^{\downarrow}: V_j \to W_{j+1}$
(3) signal synthesis operator
$S_j: V_{j+1} \times W_{j+1} \to V_j$

If the following conditions are satisfied:

(1) perfect reconstruction condition
$S_j(G_j^{\downarrow}(x), H_j^{\downarrow}(x)) = x$
for all $x \in V_j$;
(2) nonredundant decomposition conditions
$G_j^{\downarrow} S_j(x, y) = x$ and $H_j^{\downarrow} S_j(x, y) = x$
for all $x \in V_{j+1}$ and $y \in W_{j+1}$,

then the recursive analysis scheme

$$x_0 \to \{x_1, y_1\} \to \{x_2, y_2, y_1\} \to \Lambda$$

where

$$x_{j+1} = G_j^{\downarrow}(x_j) \in V_{j+1}$$

and

$$y_{j+1} = H_j^{\downarrow}(x_j) \in W_{j+1}$$

is called a **wavelet decomposition scheme**. Note that x_0 can be recursively reconstructed from x_k and y_1, y_2, Λ, y_k by

$$x_j = S_j(x_{j+1}, y_{j+1}), \quad j = k-1, k-2, \Lambda, 0$$

Moreover, if there are operations $*_j$ on V_j and operators $G_j^{\uparrow}: V_{j+1} \to V_j$ and $H_j^{\uparrow}: W_{j+1} \to V_j$ such that

$$S_j(x, y) = G_j^{\uparrow}(x) *_{j+1} H_j^{\uparrow}(y)$$

for all $x \in V_{j+1}$ and $y \in W_{j+1}$, then the wavelet decomposition is called **uncoupled**.

For instances, the lazy wavelet is a uncoupled wavelet decomposition in which

$V_0 = V_j = W_j = \Re^Z$, j = 1, 2, …
$G^{\downarrow}(x)(n) = x(2n)$
$H^{\downarrow}(x)(n) = x(2n+1)$
$G^{\uparrow}(x)(2n) = x(n)$ and $G^{\uparrow}(x)(2n+1) = 0$
$H^{\uparrow}(y)(2n) = 0$ and $H^{\uparrow}(y)(2n+1) = y(n)$
$*$ = the standard addition in $\Re^Z$.

The one-dimensional morphological Haar wavelet is also a uncoupled wavelet decomposition in which

$V_0 = V_j = W_j = \Re^Z$, j = 1, 2, …
$G^{\downarrow}(x)(n) = x(2n) \wedge x(2n+1)$
$H^{\downarrow}(x)(n) = x(2n) - x(2n+1)$
$G^{\uparrow}(x)(2n) = G^{\uparrow}(x)(2n+1) = x(n)$
$H^{\uparrow}(y)(2n) = y(n) \vee 0$
$H^{\uparrow}(y)(2n+1) = -(y(n) \wedge 0)$

$* =$ the standard addition in $\Re^Z$.

3. MORPHOLOGICAL WAVELETS

Under the framework alluded above, wavelet decomposition based on morphological operators can be constructed. Indeed, the well-known morphological skeletonization [19] is a uncoupled wavelet decomposition in which

$$V_0 = V_j = W_j = \Re^Z, j = 1, 2, \ldots$$
$$G^{\downarrow}(x) = E_w(x)$$
$$H^{\downarrow}(x) = x - D_w E_w(x)$$
$$G^{\uparrow}(x) = D_w(x)$$
$$H^{\uparrow}(y) = y$$
$* =$ the standard addition in $\Re^Z$

where E_w and D_w are respectively the erosion and dilation with respect to the structuring element *w*.

In general, suppose there are operators $\phi_j^{\downarrow} : V_j \to V_{j+1}$ and $\phi_j^{\uparrow} : V_{j+1} \to V_j$ such that $(\phi_j^{\downarrow}, \phi_j^{\uparrow})$ forms an adjunction between V_j and V_{j+1}, i.e., $\phi_j^{\downarrow}(x) \le y \Leftrightarrow x \le \phi_j^{\uparrow}(y)$ for any $x \in V_j$ and $y \in V_{j+1}$, then the recursive analysis scheme

$$x_0 \to \{x_1, y_1\} \to \{x_2, y_2, y_1\} \to \Lambda$$

is a uncoupled wavelet decomposition in which

$$G_j^{\downarrow}(x_j) = \phi_j^{\downarrow}(x_j)$$
$$H_j^{\downarrow}(x_j) = x_j - \phi_j^{\uparrow}\phi_j^{\downarrow}(x_j)$$
$$G_j^{\uparrow}(x_{j+1}) = \phi_j^{\uparrow}(x_{j+1})$$
$$H_j^{\uparrow}(y_{j+1}) = y_{j+1}$$
$$S_j(x_{j+1}, y_{j+1}) = G_j^{\uparrow}(x_{j+1}) + H_j^{\uparrow}(x_{j+1})$$

4. APPLICATIONS

Wavelet decomposition has significant applications in signal representation and compression. Wavelets are building primitives for general signals on one hand and have the power to decorrelate data on the other hand. In order to quickly find a wavelet decomposition, the lifting scheme has been proposed by Sweldens to construct biorthogonal wavelets without employing the Fourier transform. The lifting scheme consists of three steps: split, predict, and update. It has been shown that any discrete wavelet transform can be factored into lifting steps [20].

In order to possess better compression properties, the lifting scheme can also be applied to morphological wavelets. A general lifting scheme would then consist the following steps: wavelet decomposition, predict, and update. That is, given the wavelet decomposition

$$x_0 \to \{x_1, y_1\} \to \{x_2, y_2, y_1\} \to \Lambda$$

if $P_j : V_j \to W_j$ and $U_j : W_j \to V_j$ are respectively the predict and update operators, then the lifting scheme is given by

$$y'_j = y_j - P_j(x_j)$$
$$x'_j = x_j + U_j(x_j)$$

5. CONCLUSIONS

In this study we investigate the morphological approach to wavelet transform. This approach can be viewed as a combination of multiresolution theory, morphological skeletonization, and the lifting scheme. It should have the potential applications in signal representation and compression.

However, in theoretical point of view, it is not clear that what structures should the signal spaces and the detail spaces have in order to satisfy the conditions required in morphological wavelets. We will work on the suitable structures in the future.

6. REFERENCES

[1] A. Akansu and R. Haddad, *Multiresolution Signal Decomposition*, Academic Press, San

Diego, 1992.
[2] P.P. Vaidyanathan, *Mutirate Systems and Filter Banks*, Prentice-Hall, Englewood Cliffs, New Jersey, 1993.
[3] O. Rioul, "A discrete-time multiresolution theory," *IEEE trans. Signal Processing*, vol. 41, pp. 2591-2606, Aug. 1993.
[4] M.T. Shensa, "The discrete wavelet transform: wedding the a trous and Mallat algorithms," *IEEE Trans. Signal Processing*, vol. 40, no. 10, pp. 2464-2482, Oct. 1992.
[5] S. Mallat, *A Wavelet Tour of Signal Processing*, Academic Press, San Diego, 1998.
[6] A.R. Calderbank, I. Daubechies, W. Sweldens, and B.-L. Yeo, "Wavelet transforms that map integers to integers," Technical report, Department of Mathematics, Princeton University, 1996.
[7] H. Cha and L.F. Chaparro, "A morphological wavelet transform," International Conference in Systems Engineering in the Service of Humans, Systems, Man and Cybernetics, pp. 501-506, 1993.
[8] F.J. Hampson and J.-C. Pesquet, "A nonlinear subband decomposition with perfect reconstruction," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing*, Atlanta, GA, pp. 1523-1526, 1996.
[9] A.R. Calderbank, I. Daubechies, W. Sweldens, and B.-L. Yeo, "Lossless image compression using integer to integer wavelet transform," *IEEE International Conference on Image Processing*, vol. 1, no. 385, pp. 596-599, 1997.
[10] H. Cha and L.F. Chaparro, "Adaptive morphological representation of signals: Polynomial and wavelet methods," *Multidimen. Syst. Signal Process.*, vol. 8, pp. 249-271, 1997.
[11] R. Claypoole, G. Davis, W. Sweldens, and R. Baraniuk, "Nonlinear wavelet transforms for image coding," *Proc. 31st Asilomar Conf. Signals, Systems, Computers*, vol. 1, pp. 662-667, 1997
[12] F.J. Hampson and J.-C. Pesquet, "M-band nonlinear subband decompositions with perfect reconstruction," *IEEE Trans. Image Processing*, vol. 7, pp. 1547-1560, Nov. 1998.
[13] W. Sweldens, "The lifting scheme: A new philosophy in biorthogonal wavelet constructions," in *Proc. SPIE Wavelet Applications Signal Image Processing III*, vol. 2569, A.F. Lain and M. Unser, Eds., pp. 68-79, 1995.
[14] W. Sweldens, "The lifting scheme: A custom-design construction of biorthogonal wavelets," in *Appl. Comput. Harmon. Anal.*, vol. 3, pp. 186-200, 1996.
[15] W. Sweldens, "The lifting scheme: A construction of second generation wavelets," in *SIAM J. Math.. Anal.*, vol. 29, pp. 511-546, 1998.
[16] J. Goutsias and H.J.A. Heijmans, "Nonlinear multiresolution signal decomposition schemes— Part I: Morphological pyramids," *IEEE Trans. Image Processing*, vol. 9, pp. 1862-1876, Nov. 2000.
[17] J. Goutsias and H.J.A. Heijmans, "Nonlinear multiresolution signal decomposition schemes— Part II: Morphological wavelets," *IEEE Trans. Image Processing*, vol. 9, pp. 1897-1913, Nov. 2000.
[18] J. Serra, Image Analysis and Mathematical Morphology, Academic Press, London, 1982.
[19] P. Maragos, "Morphological skeleton representation and coding of binary images," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 34, pp. 1228-1244, 1986.
[20] I. Daubechies and W. Sweldens, "Factoring wavelet transformations into lifting steps," *J. Fourier Anal. Appl.*, vol. 4, no. 3, pp. 245-267, 1998.

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