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Robustness Design of Fuzzy Control for Nonlinear Multiple Time-Delay Large-Scale Systems via Neural-Network-Based Approach

Feng-Hsiag Hsiao, Sheng-Dong Xu, Chia-Yen Lin, and Zhi-Ren Tsai

***Abstract*—The stabilization problem is considered in this correspondence for a nonlinear multiple time-delay large-scale system. First, the neural-network (NN) model is employed to approximate each subsystem. Then, a linear differential inclusion (LDI) state-space representation is established for the dynamics of each NN model. According to the LDI state-space representation, a robustness design of fuzzy control is proposed to overcome the effect of modeling errors between subsystems and NN models. Next, in terms of Lyapunov's direct method, a delay-dependent stability criterion is derived to guarantee the asymptotic stability of nonlinear multiple time-delay large-scale systems. Finally, based on this criterion and the decentralized control scheme, a set of fuzzy controllers is synthesized to stabilize the nonlinear multiple time-delay large-scale system.**

***Index Terms*—Delay-dependent stability criterion, large-scale systems, modeling error, neural network (NN).**

I. Introduction

In the past few years, neural-network (NN)-based modeling has become an active research field because of its unique merits in solving complex nonlinear system identification and control problems (see [3]–[5] and the references therein). NNs are composed of simple elements operating in parallel. These elements are inspired by biological nervous systems. As a result, we can train an NN to represent a particular function by adjusting the weights between elements. Hence, the nonlinear system is approximated as close as desired by the NN models via repetitive training. Recently, many reports on the success of NN applications in control systems have appeared in the literature (see [6]–[11]). For instance, Limanond *et al.* [7] applied NNs to optimal etch time control design for a reactive ion etching process. Enns and Si [10] advanced an NN-based approximate dynamic programming control mechanism to helicopter flight control. Despite several promising empirical results and the nonlinear mapping approximation property, the rigorous closed-loop stability results for systems using NN-based controllers are still difficult to establish. Therefore, an LDI state-space representation was also introduced to deal with the stability analysis of NN models (for example, see [4] and [5]).

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During the last decade, fuzzy control has been successfully applied to the control design of nonlinear systems (see [1], [2], [12]–[25]). In these papers, a so-called Takagi–Sugeno (T–S) fuzzy model was employed to approximate a nonlinear plant; then, a model-based fuzzy controller was designed to stabilize the T–S fuzzy model. All of them, however, neglect the modeling error between the nonlinear system and the fuzzy model. In fact, existence of modeling error may be a potential source of instability for control designs that have been based on the assumption that the fuzzy model exactly matches the nonlinear plant [26]. Recently, Kiriakidis [26], Chen *et al.* [27], Cao and Frank [28], and Cao and Lin [29] highlighted the importance of modeling error for the stability analysis of nonlinear systems. However, a literature search indicates that the effect of modeling errors between nonlinear multiple time-delay large-scale systems and NN models has not been discussed yet. A robustness design of T–S fuzzy control for nonlinear multiple time-delay large-scale systems is hence proposed in this correspondence to overcome the influence of modeling error via NN-based approach.

This correspondence is organized as follows. The system description is presented in Section II. In Section III, a robustness design of T–S fuzzy control and stability analysis of the nonlinear multiple time-delay large-scale systems are proposed. Finally, the conclusions are drawn in Section IV.

II. System Description

Consider a nonlinear multiple time-delay large-scale system $\boldsymbol{N}$ composed of J interconnected subsystems N_j, $j = 1, 2, \ldots, J$. The jth subsystem N_j is described as follows:

$$\dot{X}_j(t) = f_j\left(X_j(t), U_j(t)\right) + \sum_{k=1}^{L_j}\sum_{n=1}^{J} \rho_{knj}\left(X_n(t-\tau_{knj})\right) + \sum_{\substack{n=1\\ n\neq j}}^{J} b_{nj}\left(X_n(t)\right) \tag{2.1}$$

where $f_j(\cdot)$ and $\rho_{knj}(\cdot)$ are the nonlinear vector-valued functions, $X_j(t)$ denotes the state vector, $U_j(t)$ is the input vector, $b_{nj}(\cdot)$ is the nonlinear interconnections between the nth and jth subsystems, $\tau_{kjj}(k = 1, 2, \ldots, L_j)$ are the time delays in the jth subsystems, and $\tau_{knj}(n = 1, 2, \ldots, J,\ n \neq j)$ are the time delays in the interconnections.

In the following, each subsystem is approximated by an NN model. Then, the dynamics of the NN models are converted into linear differential inclusion (LDI) state-space representations. Subsequently, a set of model-based fuzzy controllers is designed to stabilize the nonlinear multiple time-delay large-scale system $\boldsymbol{N}$.

A. NN Model

The jth subsystem of N is approximated by an NN model, which, as shown in Fig. 1, has $S_j(j = 1, 2, \ldots, J)$ layers with $R_j^{\sigma}(\sigma = 1, 2, \ldots, S_j)$[1] neurons for each layer, in which $x_{1n}(t) \sim x_{\delta_n n}(t)$ $(n \neq j)$ are the interconnected state variables and $u_{1j}(t) \sim u_{m_j j}(t)$

[1]For simplicity of notation, we use S instead of S_j in the remainder of this correspondence.

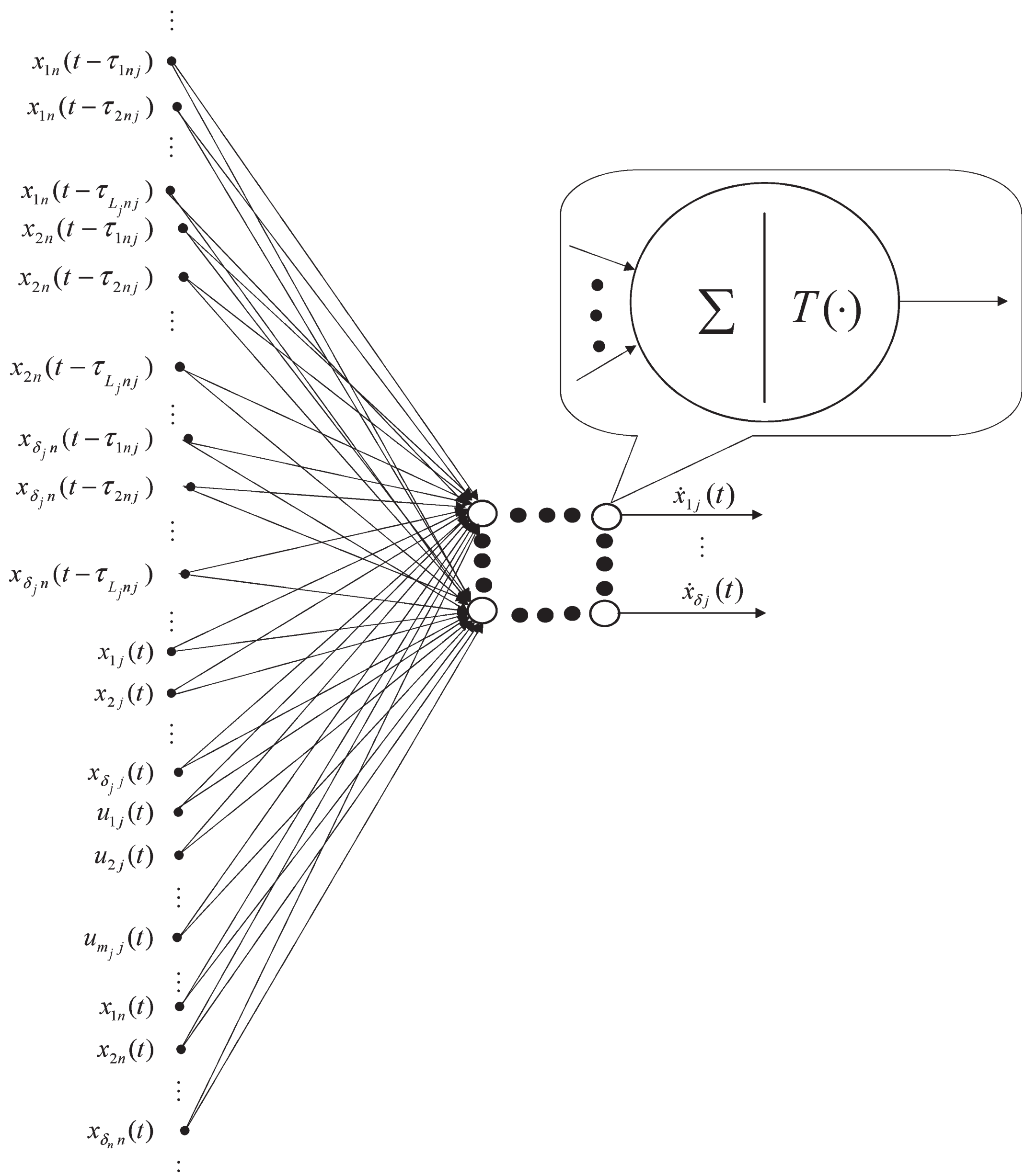

Fig. 1. The jth NN model.

are the input variables. In order to distinguish among these layers, the superscripts are used for identifying the layers. Specifically, we append the number of the layer as a superscript to the names for each of these variables. Thus, the weight matrix for the $\boldsymbol{\sigma}$th layer is written as $\mathbf{W}_j^\sigma$. Moreover, it is assumed that $v(t)$ is the net input and $T(v(t))$ is the transfer function of the neuron. Subsequently, the transfer function vector of the σth ($\sigma = 1, 2, \ldots, S$) layer is defined as

$$\Psi_j^\sigma\left(v(t)\right) \equiv \left[T\left(v_1^\sigma(t)\right) T\left(v_2^\sigma(t)\right) \cdots T\left(v_{R_j^\sigma}^\sigma(t)\right)\right]^T \tag{2.2}$$

where $T(v_\varsigma^\sigma(t))(\varsigma = 1, 2, \ldots, R_j^\sigma)$ is the transfer function of the ςth neuron. Then, the final output of the jth NN model can be inferred as follows:

$$\dot{X}_j(t) = \Psi_j^S\left(\mathbf{W}_j^S \Psi_j^{S-1}\left(\mathbf{W}_j^{S-1}\Psi_j^{S-2}\right.\right.$$
$$\left.\left.\left(\cdots \Psi_j^2\left(\mathbf{W}_j^2 \Psi_j^1\left(\mathbf{W}_j^1 \Lambda_j(t)\right)\right)\cdots\right)\right)\right) \tag{2.3}$$

where

$$\begin{aligned}\Lambda_j^T(t) = \Big[&\cdots X_n^T(t-\tau_{1nj}) X_n^T(t-\tau_{2nj}) \cdots X_n^T(t-\tau_{knj}) \\ &\cdots X_n^T(t-\tau_{L_jnj}) \cdots X_j^T(t) U_j^T(t)\ X_1^T(t) X_2^T(t) \\ &\cdots X_n^T(t) \cdots X_J^T(t)\Big]\end{aligned}$$

with

$$X_n^T(t-\tau_{knj}) = \left[x_{1n}(t-\tau_{knj})\, x_{2n}(t-\tau_{knj}) \cdots x_{\delta_n n}(t-\tau_{knj})\right]$$

for $n = 1, 2, \ldots, J; \quad k = 1, 2, \ldots, L_j$

$$\begin{aligned} X_j^T(t) &= \left[x_{1j}(t) \quad x_{2j}(t) \quad \cdots \quad x_{\delta_j j}(t)\right] \\ U_j^T(t) &= \left[u_{1j}(t) \quad u_{2j}(t) \quad \cdots \quad u_{m_j j}(t)\right] \\ X_n^T(t) &= \left[x_{1n}(t) \quad x_{2n}(t) \quad \cdots \quad x_{\delta_n n}(t)\right], \qquad n \neq j. \end{aligned}$$

Remark 1: In NNs, neurons may use whatever differentiable transfer functions to generate their output, so we have many choices about choosing or determining the transfer function, such as the sigmoid function and the hyperbolic tangent function. The selection of transfer functions $T(v(t))$ in this correspondence is just one of the feasible choices.

Next, in order to deal with the stability problem of the nonlinear multiple time-delay large-scale system N, an LDI state-space representation is established for [30]

$$\dot{Y}(t) = A(a(t))\, Y(t)$$
$$A(a(t)) = \sum_{i=1}^{\phi} h_i(a(t))\, \overline{A}_i \tag{2.4}$$

where ϕ is a positive integer, $a(t)$ is a vector signifying the dependence of $h_i(\cdot)$ on its elements, $\overline{A_i}(i = 1, 2, \ldots, \phi)$ are constant matrices, and $Y(t) = [y_1(t)\ y_2(t)\ \cdots\ y_\Xi(t)]^T$. Furthermore, it is assumed that $h_i(a(t)) \geq 0$, $\sum_{i=1}^{\phi} h_i(a(t)) = 1$.

From the properties of LDI, without loss of generality, we can use $h_i(t)$ instead of $h_i(a(t))$. In the following, we present a procedure to represent the dynamics of the jth NN model (2.3) by LDI state-space representation [5].

To begin with, notice that the output $T(v_\varsigma^\sigma(t))$ satisfies

$$g_{\varsigma 1}^\sigma v_\varsigma^\sigma(t) \leq T\left(v_\varsigma^\sigma(t)\right) \leq g_{\varsigma 2}^\sigma v_\varsigma^\sigma(t), \qquad v_\varsigma^\sigma(t) \geq 0$$
$$g_{\varsigma 2}^\sigma v_\varsigma^\sigma(t) \leq T\left(v_\varsigma^\sigma(t)\right) \leq g_{\varsigma 1}^\sigma v_\varsigma^\sigma(t), \qquad v_\varsigma^\sigma(t) < 0$$

where $g_{\varsigma 1}^\sigma$ and $g_{\varsigma 2}^\sigma$ are the minimum and the maximum of the derivative of $T(v_\varsigma^\sigma(t))$, respectively, and they are given in the following:

$$g_{\varsigma\varphi}^\sigma = \begin{cases} \min\limits_{v} \frac{dT\left(v_\varsigma^\sigma(t)\right)}{dv_\varsigma^\sigma(t)}, & \text{when } \varphi = 1 \\ \max\limits_{v} \frac{dT\left(v_\varsigma^\sigma(t)\right)}{dv_\varsigma^\sigma(t)}, & \text{when } \varphi = 2. \end{cases} \tag{2.5}$$

Subsequently, the min–max matrix G^σ of the σth layer is defined as follows:

$$G^\sigma \equiv \operatorname{diag}\left[g_{\varsigma\varphi}^\sigma\right] = \begin{bmatrix} g_{1\varphi_1}^\sigma & 0 & 0 & \cdots & 0 \\ 0 & g_{2\varphi_2}^\sigma & 0 & \ddots & 0 \\ 0 & 0 & g_{3\varphi_3}^\sigma & 0 & \vdots \\ \vdots & \ddots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 0 & g_{R_j^\sigma \varphi_R}^\sigma \end{bmatrix}. \tag{2.6a}$$

Moreover, based on the interpolation method, the transfer function $T(v_\varsigma^\sigma(t))$ can be represented as follows [5], [12]:

$$T\left(v_\varsigma^\sigma(t)\right) = \left(h_{\varsigma 1}^\sigma(t) g_{\varsigma 1}^\sigma + h_{\varsigma 2}^\sigma(t) g_{\varsigma 2}^\sigma\right) v_\varsigma^\sigma(t)$$
$$= \left(\sum_{\varphi=1}^{2} h_{\varsigma\varphi}^\sigma(t) g_{\varsigma\varphi}^\sigma\right) v_\varsigma^\sigma(t) \tag{2.6b}$$

where interpolation coefficients $h_{\varsigma\varphi}^\sigma(t) \in [0, 1]$ and $\sum_{\varphi=1}^{2} h_{\varsigma\varphi}^\sigma(t) = 1$. From (2.2) and (2.6b), we have

$$\begin{aligned} \Psi^\sigma(v(t)) &\equiv \left[T\left(v_1^\sigma(t)\right)\ T\left(v_2^\sigma(t)\right)\ \cdots\ T\left(v_{R^\sigma}^\sigma(t)\right)\right]^T \\ &= \left[\sum_{\varphi_1=1}^{2} h_{1\varphi_1}^\sigma(t) g_{1\varphi_1}^\sigma v_1^\sigma(t)\ \sum_{\varphi_2=1}^{2} h_{2\varphi_2}^\sigma(t) g_{2\varphi_2}^\sigma v_2^\sigma(t)\right. \\ &\qquad \left.\cdots \sum_{\varphi_R=1}^{2} h_{R^\sigma\varphi_R}^\sigma(t) g_{R^\sigma\varphi_R}^\sigma v_{R^\sigma}^\sigma(t)\right]^T. \end{aligned} \tag{2.6c}$$

Therefore, the final output of the NN model (2.3) can be reformulated as follows:

$$\begin{aligned} \dot{X}_j(t) &= \sum_{p=1}^{2} h_{\varsigma p}^S(t) G^S W_j^S \\ &\quad \times \left[\cdots \left[\sum_{m=1}^{2} h_{\varsigma m}^2(t) G^2 \left(W_j^2 \left[\sum_{r=1}^{2} h_{\varsigma r}^1(t) G^1 W_j^1 \Lambda_j(t)\right]\right)\right]\cdots\right] \\ &= \sum_{p=1}^{2} \cdots \sum_{m=1}^{2} \sum_{r=1}^{2} h_{\varsigma p}^S(t) \cdots h_{\varsigma m}^S(t) h_{\varsigma r}^S(t) G^S W_j^S \\ &\quad \cdots G^2 W_j^2 G^1 W_j^1 \Lambda_j(t) \\ &= \sum_{\Omega} h_{\varsigma\Omega}^\sigma(t) E_\Omega^\sigma \Lambda_j(t) \end{aligned} \tag{2.7}$$

where

$$\sum_{r=1}^{2} h_{\varsigma r}^1(t) \equiv \sum_{r_1=1}^{2} h_{1r_1}^1(t) \sum_{r_2=1}^{2} h_{2r_2}^1(t) \cdots \sum_{r_R=1}^{2} h_{R_j^1 r_R}^1(t)$$
$$\sum_{m=1}^{2} h_{\varsigma m}^2(t) \equiv \sum_{m_1=1}^{2} h_{1m_1}^2(t) \sum_{m_2=1}^{2} h_{2m_2}^2(t) \cdots \sum_{m_R=1}^{2} h_{R_j^2 m_R}^2(t)$$
$$\vdots$$
$$\sum_{p=1}^{2} h_{\varsigma p}^S(t) \equiv \sum_{p_1=1}^{2} h_{1p_1}^S(t) \sum_{p_2=1}^{2} h_{2p_2}^S(t) \cdots \sum_{p_R=1}^{2} h_{R_j^S p_R}^S(t)$$
$$\sum_{\Omega} h_{\varsigma\Omega}^\sigma(t) \equiv \sum_{p=1}^{2} \cdots \sum_{m=1}^{2} \sum_{r=1}^{2} h_{\varsigma p}^S(t) \cdots h_{\varsigma m}^2(t) h_{\varsigma r}^1(t),$$

$\varsigma = 1, 2 \ldots, R_\Omega^\sigma$; $E_\Omega^\sigma \equiv G^S \mathbf{W}_j^S \cdots G^2 \mathbf{W}_j^2 G^1 \mathbf{W}_j^1$ and r_ς, m_ς, and $p_\varsigma(\varsigma = 1, 2, \ldots, R)$ represent the variables φ of the ςth neuron of the first, second, and Sth layer, respectively. Finally, according to (2.4), the dynamics of the jth $(j = 1, 2, \ldots, J)$ NN model (2.7) can be rewritten as the following LDI state-space representation:

$$\dot{X}_j(t) = \sum_{i=1}^{\phi_j} h_{ij}(t) E_{ij} \Lambda_j(t) \tag{2.8}$$

where $h_{ij}(t) \geq 0$, $\sum_{i=1}^{\phi_j} h_{ij}(t) = 1$, ϕ_j is a positive integer, and E_{ij} is a constant matrix with appropriate dimension associated with E_Ω^σ.

The LDI state-space representation (2.8) can be further rearranged as follows:

$$\dot{X}_j(t) = \sum_{i=1}^{\phi_j} h_{ij}(t) \Bigg\{ A_{ij}X_j(t) + B_{ij}U_j(t) + \sum_{k=1}^{L_j}\sum_{n=1}^{J} \overline{A}_{iknj} X_n(t-\tau_{knj}) + \sum_{\substack{n=1\\ n\neq j}}^{J} \hat{A}_{inj}X_n(t) \Bigg\} \tag{2.9}$$

where $\overline{A}_{iknj}$, A_{ij}, B_{ij}, and $\hat{A}_{inj}$ are the partitions of E_{ij} corresponding to the partition $\Lambda_j^T(t)$.

Remark 2: For simplicity of NN model construction, the NNs for all subsystems in this correspondence are assumed to have the same forms with respect to the numbers of layers and neurons. Therefore, we have

$$\phi_1 = \phi_2 = \cdots = \phi_J. \tag{2.10}$$

B. T–S Fuzzy Control

On the basis of the decentralized control scheme, a set of T–S fuzzy controllers is synthesized to stabilize the nonlinear multiple time-delay large-scale system N. The jth fuzzy controller takes the following form:

Rule β : IF $x_{1j}(t)$ is $M_{\beta 1j}$ and $\ldots$ and $x_{\delta_j j}(t)$ is $M_{\beta\delta_j j}$ THEN $U_j(t) = -C_{\beta j}X_j(t)$

$\beta = 1, 2, \ldots, \mu_j$, and μ_j is the number of IF–THEN rules of the fuzzy controller and $M_{\beta\theta j}(\theta = 1, 2, \ldots, \delta_j)$ are the fuzzy sets. Hence, the final output of this fuzzy controller is inferred as follows:

$$U_j(t) = -\frac{\sum_{\beta=1}^{\mu_j} w_{\beta j}(t) C_{\beta j} X_j(t)}{\sum_{\beta=1}^{\mu_j} w_{\beta j}(t)} = -\sum_{\beta=1}^{\mu_j} h_{\beta j}(t) C_{\beta j} X_j(t) \tag{2.11}$$

with

$$w_{\beta j}(t) \equiv \prod_{\theta=1}^{\delta_j} M_{\beta\theta j}\left(x_{\theta j}(t)\right) \quad h_{\beta j}(t) \equiv \frac{w_{\beta j}(t)}{\sum_{\beta=1}^{\mu_j} w_{\beta j}(t)}$$

in which $M_{\beta\theta j}(x_{\theta j}(t))$ is the grade of membership of $x_{\theta j}(t)$ in $M_{\beta\theta j}$. In this correspondence, it is also assumed that $w_{\beta j}(t) \geq 0$, $\beta = 1, 2, \ldots, \mu_j$; $j = 1, 2, \ldots, J$ and $\sum_{\beta=1}^{\mu_j} w_{\beta j}(t) > 0$ for all t. Therefore, $h_{\beta j}(t) \geq 0$ and $\sum_{\beta=1}^{\mu_j} h_{\beta j}(t) = 1$ for all t.

III. Robustness Design of Fuzzy Control and Stability Analysis

In this section, the stability of the nonlinear multiple time-delay large-scale system $\boldsymbol{N}$ is examined under the influence of modeling error.

A. Modeling Error

Substituting (2.11) into (2.1) and (2.9) yields the jth closed-loop subsystem $\overline{N}_j$ as follows:

$$\begin{aligned}
\dot{X}_j(t) = &\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t) \\
&\times \Bigg[(A_{ij} - B_{ij}C_{\beta j})X_j(t) + \sum_{k=1}^{L_j}\sum_{n=1}^{J} \bar{A}_{iknj}X_n \\
&\times\ (t-\tau_{knj}) + \sum_{\substack{n=1\\ n\neq j}}^{J} \hat{A}_{inj}X_n(t) \Bigg] + \Gamma_j\left(X_j(t)\right) \\
&+ \sum_{k=1}^{L_j}\sum_{n=1}^{J} \rho_{knj}\left(X_n(t-\tau_{knj})\right) + \sum_{\substack{n=1\\ n\neq j}}^{J} b_{nj}\left(X_n(t)\right) \\
&- \sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t) \\
&\times \Bigg[(A_{ij} - B_{ij}C_{\beta j})X_j(t) + \sum_{k=1}^{L_j}\sum_{n=1}^{J} \bar{A}_{iknj}X_n \\
&\times (t-\tau_{knj}) + \sum_{\substack{n=1\\ n\neq j}}^{J} \hat{A}_{inj}X_n(t) \Bigg] \\
= &\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t) \\
&\times \Bigg[(A_{ij} - B_{ij}C_{\beta j})X_j(t) + \sum_{k=1}^{L_j}\sum_{n=1}^{J} \bar{A}_{iknj}X_n \\
&\times\ (t-\tau_{knj}) + \sum_{\substack{n=1\\ n\neq j}}^{J} \hat{A}_{inj}X_n(t) \Bigg] + \Delta\Phi_j(t)
\end{aligned} \tag{3.1}$$

where $f_j(X_j(t), U_j(t)) \equiv \Gamma_j(X_j(t))$ with

$$U_j(t) = -\sum_{\beta=1}^{\mu_j} h_{\beta j}(t)C_{\beta j}X_j(t)$$

$$\Delta\Phi_j(t) \equiv e_j(t) + \sum_{k=1}^{L_j}\sum_{n=1}^{J} \overline{e}_{knj}(t-\tau_{knj}) + \sum_{\substack{n=1\\ n\neq j}}^{J} \hat{e}_{nj}(t)$$

with

$$e_j(t) \equiv \Bigg[\Gamma_j\left(X_j(t)\right) - \sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t) \times (A_{ij} - B_{ij}C_{\beta j})X_j(t) \Bigg] \tag{3.2}$$

$$\begin{aligned}\bar{e}_{knj}(t-\tau_{knj}) \equiv & \left[\rho_{knj}\left(X_n(t-\tau_{knj})\right)\right. \\ & \left. - \sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t)\overline{A}_{iknj}X_n(t-\tau_{knj})\right] \\ = & \left[\rho_{knj}\left(X_n(t-\tau_{knj})\right)\right. \\ & \left. - \sum_{i=1}^{\phi_j} h_{ij}(t)\overline{A}_{iknj}X_n(t-\tau_{knj})\right] \end{aligned} \tag{3.3}$$

$$\begin{aligned}\hat{e}_{nj}(t) \equiv & \left[b_{nj}\left(X_n(t)\right) - \sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t)\hat{A}_{inj}X_n(t)\right] \\ = & \left[b_{nj}\left(X_n(t)\right) - \sum_{i=1}^{\phi_j} h_{ij}(t)\hat{A}_{inj}X_n(t)\right] \end{aligned} \tag{3.4}$$

and $\Delta\Phi_j(t)$ denotes the modeling error between the jth closed-loop nonlinear subsystem (3.1) and the closed-loop NN model [(2.8) + (2.9)].

Suppose that there exists bounding matrix $\Delta H_{i\beta j}$ such that

$$\|\Delta\Phi_j(t)\| \leq \left\|\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t)\Delta H_{i\beta j}X_j(t)\right\| \tag{3.5}$$

for trajectory $X_j(t)$, and the bounding matrix $\Delta H_{i\beta j}$ can be described as follows:

$$\Delta H_{i\beta j} = \varepsilon_{i\beta j}H_j \tag{3.6}$$

where H_j is the specified structured bounding matrix and $\|\varepsilon_{i\beta j}\| \leq 1$ for $i = 1, 2, \ldots, \phi_j$; $\beta = 1, 2, \ldots, \mu_j$; and $j = 1, 2, \ldots, J$. From (3.5) and (3.6), we have

$$\begin{aligned}\Delta\Phi_j^T(t)\Delta\Phi_j(t) \leq & \left[\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t)\Delta H_{i\beta j}X_j(t)\right]^T \\ & \times \left[\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t)\Delta H_{i\beta j}X_j(t)\right] \\ \leq & \sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t)\left\|H_jX_j(t)\right\|\left\|\varepsilon_{i\beta j}\right\| \\ & \times \sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t)\|\varepsilon_{i\beta j}\|\left\|H_jX_j(t)\right\| \\ \leq & \left[H_jX_j(t)\right]^T\left[H_jX_j(t)\right]. \end{aligned} \tag{3.7}$$

That is, the modeling error $\Delta\Phi_j(t)$ is bounded by the specified structured bounding matrix H_j.

Remark 3 [18]: The procedures for determining $\varepsilon_{i\beta j}$ and H_j are described by the following simple example. Assume that the possible bounds for all elements in $\Delta H_{i\beta j}$ are

$$\Delta H_{i\beta j} = \begin{bmatrix} \Delta h_{i\beta j}^{11} & \Delta h_{i\beta j}^{12} \\ \Delta h_{i\beta j}^{21} & \Delta h_{i\beta j}^{22} \end{bmatrix}$$

where $-\gamma_j^{rs} \leq \Delta h_{i\beta j}^{rs} \leq \gamma_j^{rs}$ for some $\gamma_{i\beta j}^{rs}$, with $r, s = 1, 2$; $i = 1, 2, \ldots, \phi_j$; $\beta = 1, 2, \ldots, \mu_j$; and $j = 1, 2, \ldots, J$.

One possible description for the bounding matrix $\Delta H_{i\beta j}$ is

$$\Delta H_{i\beta j} = \begin{bmatrix} \varepsilon_{i\beta j}^{11} & 0 \\ 0 & \varepsilon_{i\beta j}^{22} \end{bmatrix}\begin{bmatrix} \gamma_j^{11} & \gamma_j^{12} \\ \gamma_j^{21} & \gamma_j^{22} \end{bmatrix} = \varepsilon_{i\beta j}H_j$$

where $-1 \leq \varepsilon_{i\beta j}^{rr} \leq 1$ for $r = 1, 2$. It is noticed that $\varepsilon_{i\beta j}$ can be chosen by other forms as long as $\|\varepsilon_{i\beta j}\| \leq 1$. Then, we check the validity of (3.5); if it is not satisfied, we can expand the bounds for all elements in $\Delta H_{i\beta j}$ and repeat the design procedures until (3.5) holds.

B. Stability in the Presence of Modeling Error

In the following, a stability criterion is proposed to guarantee the asymptotic stability of the closed-loop nonlinear multiple time-delay large-scale system $\overline{N}$, which consists of J closed-loop subsystems described in (3.1). Prior to examination of the asymptotic stability of $\overline{N}$, a useful concept is given here.

Lemma 1 [31]: For real matrices A and B with appropriate dimensions, we have

$$A^TB + B^TA \leq \lambda A^TA + \lambda^{-1}B^TB$$

where λ is a positive constant.

Theorem 1: The closed-loop nonlinear multiple time-delay large-scale system $\overline{N}$ is asymptotically stable if there exist symmetric positive definite matrices P_j, ψ_{knj} and positive constants α_j, z_j, and $\eta_j (j = 1, 2, \ldots, J)$, and feedback gains $C_{\beta j}$s, as shown in (2.11), are chosen such that the following inequalities hold:

$$Q_{i\beta nj} \equiv \sum_{k=1}^{L_j} \tau_{knj}\xi_{i\beta kj} + \varpi_{inj} < 0$$
$$\text{for } i = 1, 2, \ldots, \phi_j,\ \beta = 1, 2, \ldots, \mu_j;\ n, j = 1, 2, \ldots, J \tag{3.8a}$$

$$\nabla_{iknj} \equiv \alpha_j^{-1}L_j\overline{A}_{iknj}^T\overline{A}_{iknj} - \psi_{knj} < 0$$
$$\text{for } i = 1, 2, \ldots, \phi_j,\ k = 1, 2, \ldots, L_j;\ n, j = 1, 2, \ldots, J \tag{3.8b}$$

where

$$\xi_{i\beta kj} = (A_{ij} - B_{ij}C_{\beta j})^TP_j + P_j(A_{ij} - B_{ij}C_{\beta j}) + z_j\tau_{knj}H_j^TH_j \tag{3.9a}$$

$$\begin{aligned}\varpi_{inj} = & \alpha_jL_j\sum_{k=1}^{L_j}\tau_{knj}^2P_j^2 + L_jz_j^{-1}P_j^2 + \sum_{k=1}^{L_j}\psi_{kjn} \\ & + \eta_j^{-1}\sum_{k=1}^{L_j}\tau_{knj}^2P_j\hat{A}_{inj}\hat{A}_{inj}^TP_j + \kappa_{inj}L_nI \end{aligned} \tag{3.9b}$$

with $\kappa_{inj} = (h_{in}(t)/h_{ij}(t))\eta_n$.

Proof: See the Appendix.

Remark 4: In physical conditions, it is not an easy task to divide the large-scale system into many obviously different subsystems based on the concept of theory. In this correspondence, the high similarities among all the subsystems are assumed. Therefore, (3.8a) with time-dependent functions $h_{in}(t)$ and $h_{ij}(t)$ can be solved by letting $(h_{in}(t)/h_{ij}(t)) \approx 1$.

Remark 5: Since the matrices ϖ_{inj} in (3.9b) are positive definite, the matrices $\xi_{i\beta kj}$s must be chosen to be negative definite to meet the stability condition (3.8a). Hence, based on (3.9a), we have that the larger delay τ_{knj} will make Theorem 1 more difficult to be satisfied.

Remark 6: Based on (3.5), the modeling error $\Delta\Phi_j(t)$ is assumed to be bounded by the specified structured bounding matrix H_j; then, the larger modeling error results in larger H_j. According to the same corollary shown in Remark 5, the larger modeling error will make Theorem 1 more difficult to be satisfied.

Remark 7: Equation (3.8b) is a linear matrix inequality (LMI), and it can be rewritten as follows:

$$\begin{bmatrix} \overline{\psi}_{kjn} & W_j\overline{A}^T_{iknj} \\ \overline{A}_{iknj}W_j & \frac{\alpha_j}{L_j}I \end{bmatrix} > 0$$
$$\text{for } i=1,2,\ldots,\phi_j;\ n,j=1,2,\ldots,J;\text{ and } k=1,2,\ldots,L_j \tag{3.10}$$

where $\overline{\psi}_{kjn} = W_j\psi_{kjn}W_j$ with $W_j = P_j^{-1}$. Moreover, (3.8a) can be reformulated into LMI via the following procedure.

By introducing new variables $Y_{\beta j} = C_{\beta j}W_j$ and $\overline{z}_j = z_j^{-1}$, (3.8a) is rewritten as follows:

$$\begin{aligned}
&\sum_{k=1}^{L_j}\tau_{knj}\Big\{W_jA^T_{ij} - (B_{ij}Y_{\beta j})^T + A_{ij}W_j - B_{ij}Y_{\beta j} \\
&\qquad + \overline{z}_j^{-1}\tau_{knj}W_jH_j^TH_jW_j\Big\} + \alpha_jL_j\sum_{k=1}^{L_j}\tau^2_{knj}I \\
&\quad + L_j\overline{z}_jI + \sum_{k=1}^{L_j}\overline{\psi}_{kjn} + \eta_j^{-1}\sum_{k=1}^{L_j}\tau^2_{knj}\hat{A}_{inj}\hat{A}^T_{inj} \\
&\quad + \kappa_{inj}L_nW_jW_j < 0
\end{aligned} \tag{3.11}$$

for $i=1,2,\ldots,\phi_j$; $\beta=1,2,\ldots,\mu_j$; and $n,j=1,2,\ldots,J$. Furthermore, based on Schur's complement [2], [18], [23], [30], it is easy to find that the matrix inequality in (3.11) is equivalent to the following LMI:

$$\begin{bmatrix} \Gamma & H_jW_j & W_j \\ (H_jW_j)^T & -\left(\sum_{k=1}^{L_j}\tau^2_{knj}\overline{z}_j^{-1}\right)^{-1}I & 0 \\ W_j & 0 & -(\kappa_{inj}L_n)^{-1}I \end{bmatrix} < 0$$
$$\text{for } i=1,2,\ldots,\phi_j;\ \beta=1,2,\ldots,\mu_j;\ n,j=1,2,\ldots,J \tag{3.12}$$

where

$$\begin{aligned}
\Gamma = &\sum_{k=1}^{L_j}\tau_{knj}\left\{W_jA^T_{ij} - (B_{ij}Y_{\beta j})^T + A_{ij}W_j - B_{ij}Y_{\beta j}\right\} \\
&+ \alpha_jL_j\sum_{k=1}^{L_j}\tau^2_{knj}I + L_j\overline{z}_jI + \sum_{k=1}^{L_j}\overline{\psi}_{kjn} + \eta_j^{-1}\sum_{k=1}^{L_j}\tau^2_{knj}\hat{A}_{inj}\hat{A}^T_{inj}.
\end{aligned}$$

IV. Conclusion

A robustness design of fuzzy control via NN-based approach is proposed to overcome the influence of modeling error. First, the NN model is employed to approximate each subsystem. Then, the dynamics of each NN model is converted into LDI representation. Next, a delay-dependent stability criterion is derived from Lyapunov's direct method to ensure the asymptotic stability of nonlinear multiple time-delay large-scale systems. According to this criterion and the decentralized control scheme, a set of model-based fuzzy controllers is synthesized to stabilize the nonlinear multiple time-delay large-scale system.

Appendix
Proof of Theorem 1

Let the Lyapunov function for $\overline{\boldsymbol{N}}$ be defined as

$$\begin{aligned}
V(t) &= \sum_{j=1}^{J}V_j(t) \\
&= \sum_{j=1}^{J}\Bigg\{\sum_{k=1}^{L_j}\sum_{n=1}^{J}X_j^T(t)\tau_{knj}P_jX_j(t) \\
&\qquad + \sum_{k=1}^{L_j}\sum_{n=1}^{J}\int_0^{\tau_{knj}}X_n^T(t-\pi)\psi_{knj}X_n(t-\pi)d\pi\Bigg\}
\end{aligned} \tag{A1}$$

where the weighting matrices $P_j = P_j^T > 0$ and $\psi_{knj} = \psi^T_{knj} > 0$. We then evaluate the time derivative of $V(t)$ on the trajectories of (3.1) to get (A2) and (A3), shown on the next page. In (A2), based on the concept of interconnection, matrix $\hat{A}_{ijj}$ is set to be zero.

According to (3.7) and (A3), we have

$$\begin{aligned}
\dot{V}(t) \le &\sum_{j=1}^{J}\sum_{n=1}^{J}\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j}h_{ij}(t)h_{\beta j}(t)X_j^T(t) \\
&\times\Bigg\{(A_{ij}-B_{ij}C_{\beta j})^T\sum_{k=1}^{L_j}\tau_{knj}P_j \\
&\qquad + P_j\sum_{k=1}^{L_j}\tau_{knj}(A_{ij}-B_{ij}C_{\beta j}) + \alpha_jL_j\sum_{k=1}^{L_j}\tau^2_{knj}P_j^2 \\
&\qquad + L_jz_j^{-1}P_j^2 + z_j\sum_{k=1}^{L_j}\tau^2_{knj}H_j^TH_j + \sum_{k=1}^{L_j}\psi_{kjn} \\
&\qquad + \eta_j^{-1}\sum_{k=1}^{L_j}\tau^2_{knj}P_j\hat{A}_{inj}\hat{A}^T_{inj}P_j + \kappa_{inj}L_nI\Bigg\}X_j(t) \\
&+ \sum_{j=1}^{J}\sum_{i=1}^{\phi_j}\sum_{k=1}^{L_j}\sum_{n=1}^{J}h_{ij}(t)X_n^T(t-\tau_{knj}) \\
&\times\left\{\alpha_j^{-1}L_j\overline{A}^T_{iknj}\overline{A}_{iknj} - \psi_{knj}\right\}X_n(t-\tau_{knj})
\end{aligned} \tag{A4}$$

$$\begin{aligned}
= &\sum_{j=1}^{J}\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j}\sum_{n=1}^{J}h_{ij}(t)h_{\beta j}(t)X_j^T(t)Q_{i\beta nj}X_j(t) \\
&+ \sum_{j=1}^{J}\sum_{i=1}^{\phi_j}\sum_{k=1}^{L_j}\sum_{n=1}^{J}h_{ij}(t)X_n^T(t-\tau_{knj})\nabla_{iknj}X_n(t-\tau_{knj}).
\end{aligned} \tag{A5}$$

Based on (3.8a) and (3.8b), we have $\dot{V}(t) < 0$, and the proof is thereby completed.

$$
\begin{aligned}
\dot{V}(t) =& \sum_{j=1}^{J} \dot{V}_j(t) = \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\left[\tau_{knj}\left(\dot{X}_j^T(t)P_jX_j(t) + X_j^T(t)P_j\dot{X}_j(t)\right)\right] + \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\left(X_n^T(t)\psi_{knj}X_n(t) - X_n^T(t-\tau_{knj})\psi_{knj}X_n(t-\tau_{knj})\right) \\
=& \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\tau_{knj}\left\{\left[\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t)\left((A_{ij} - B_{ij}C_{\beta j})X_j(t) + \sum_{d=1}^{L_j}\overline{A}_{idnj}X_n(t-\tau_{dnj}) + \sum_{n=1}^{J}\hat{A}_{inj}X_n(t)\right) + \Delta\Phi_j(t)\right]^T\right. \\
& \times P_jX_j(t) + X_j^T(t)P_j\left[\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t)\left((A_{ij} - B_{ij}C_{\beta j})X_j(t) + \sum_{d=1}^{L_j}\sum_{n=1}^{J}\overline{A}_{idnj}X_n(t-\tau_{dnj})\right.\right. \\
& \left.\left.\left. + \sum_{n=1}^{J}\hat{A}_{inj}X_n(t)\right) + \Delta\Phi_j(t)\right]\right\} \\
& + \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\left[X_n^T(t)\psi_{knj}X_n(t) - X_n^T(t-\tau_{knj})\psi_{knj}X_n(t-\tau_{knj})\right] \\
=& \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j} h_{ij}(t)h_{\beta j}(t)X_j^T(t)\left[(A_{ij} - B_{ij}C_{\beta j})^T\tau_{knj}P_j + P_j\tau_{knj}(A_{ij} - B_{ij}C_{\beta j})\right]X_j(t) \\
& + \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{i=1}^{\phi_j}\sum_{d=1}^{L_j}\sum_{n=1}^{J} h_{ij}(t)\left[X_n^T(t-\tau_{dnj})\overline{A}_{idnj}^T P_j\tau_{knj}X_j(t) + X_j^T(t)\tau_{knj}P_j\overline{A}_{idnj}X_n(t-\tau_{dnj})\right] \\
& + \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\sum_{i=1}^{\phi_j} h_{ij}(t)\left[X_n^T(t)\hat{A}_{inj}^T\tau_{knj}P_jX_j(t) + X_j^T(t)P_j\tau_{knj}\hat{A}_{inj}X_n(t)\right] \\
& + \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\left[\Delta\Phi_j^T(t)P_j\tau_{knj}X_j(t) + X_j^T(t)\tau_{knj}P_j\Delta\Phi_j(t)\right] \\
& + \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\left(X_n^T(t)\psi_{knj}X_n(t) - X_n^T(t-\tau_{knj})\psi_{knj}X_n(t-\tau_{knj})\right) \\
\le& \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j}\sum_{n=1}^{J} h_{ij}(t)h_{\beta j}(t)X_j^T(t)\left[(A_{ij} - B_{ij}C_{\beta j})^T\tau_{knj}P_j + P_j\tau_{knj}(A_{ij} - B_{ij}C_{\beta j})\right]X_j(t) \\
& + \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{i=1}^{\phi_j}\sum_{d=1}^{L_j}\sum_{n=1}^{J} h_{ij}(t)\left[\alpha_jX_j^T(t)\tau_{knj}^2P_j^2X_j(t) + \alpha_j^{-1}X_n^T(t-\tau_{dnj})\overline{A}_{idj}^T\overline{A}_{idj}X_n(t-\tau_{dnj})\right] \\
& + \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\sum_{i=1}^{\phi_j} h_{ij}(t)\left[\eta_jX_n^T(t)X_n(t) + \eta_j^{-1}\tau_{knj}^2X_j^T(t)P_j\hat{A}_{inj}\hat{A}_{inj}^TP_jX_j(t)\right] \\
& + \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\left[z_j\Delta\Phi_j^T(t)\tau_{knj}^2\Delta\Phi_j(t) + z_j^{-1}X_j^T(t)P_j^2X_j(t)\right] \quad \text{(by Lemma 1)} \\
& + \sum_{j=1}^{J}\sum_{k=1}^{L_j}\sum_{n=1}^{J}\left[X_n^T(t)\psi_{knj}X_n(t) - X_n^T(t-\tau_{knj})\psi_{knj}X_n(t-\tau_{knj})\right]
\end{aligned} \tag{A2}
$$

$$
\begin{aligned}
=& \sum_{j=1}^{J}\sum_{i=1}^{\phi_j}\sum_{\beta=1}^{\mu_j}\sum_{n=1}^{J} h_{ij}(t)h_{\beta j}(t)X_j^T(t)\left[(A_{ij} - B_{ij}C_{\beta j})^T\sum_{k=1}^{L_j}\tau_{knj}P_j + P_j\sum_{k=1}^{L_j}\tau_{knj}(A_{ij} - B_{ij}C_{\beta j})\right]X_j(t) \\
& + \sum_{j=1}^{J}\sum_{i=1}^{\phi_j}\sum_{n=1}^{J} h_{ij}(t)\left[\alpha_jL_jX_j^T(t)\sum_{k=1}^{L_j}\tau_{knj}^2P_j^2X_j(t) + \alpha_j^{-1}L_j\sum_{k=1}^{L_j}X_n^T(t-\tau_{knj})\overline{A}_{iknj}^T\overline{A}_{iknj}X_n(t-\tau_{knj})\right] \\
& + \sum_{j=1}^{J}\sum_{n=1}^{J}\sum_{i=1}^{\phi_j} h_{ij}(t)\left[\frac{h_{in}(t)}{h_{ij}(t)}\eta_nL_nX_j^T(t)X_j(t) + \eta_j^{-1}\sum_{k=1}^{L_j}\tau_{knj}^2X_j^T(t)P_j\hat{A}_{inj}\hat{A}_{inj}^TP_jX_j(t)\right] \quad \text{(by (2.10))} \\
& + \sum_{j=1}^{J}\sum_{n=1}^{J}\left[z_j\Delta\Phi_j^T(t)\sum_{k=1}^{L_j}\tau_{knj}^2\Delta\Phi_j(t) + z_j^{-1}L_jX_j^T(t)P_j^2X_j(t)\right] + \sum_{n=1}^{J}\sum_{j=1}^{J}\left[X_j^T(t)\sum_{k=1}^{L_j}\psi_{kjn}X_j(t)\right] \\
& - \sum_{j=1}^{J}\sum_{n=1}^{J}\left[\sum_{k=1}^{L_j}X_n^T(t-\tau_{knj})\psi_{knj}X_n(t-\tau_{knj})\right]
\end{aligned} \tag{A3}
$$

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