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A variables sampling plan based on C_{pmk} for product acceptance determination

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Abstract

Process capability indices are useful management tools, particularly in the manufacturing industry, which provide common quantitative measures on manufacturing capability and production quality. Most supplier certification manuals include a discussion of process capability analysis and describe the recommended procedure for computing a process capability index. Acceptance sampling plans have been one of the most practical tools used in classical quality control applications. It provides both vendors and buyers to reserve their own rights by compromising on a rule to judge a batch of products. Both sides may set their own safeguard line to protect their benefits. Two kinds of risks are balanced using a well-designed sampling plan. In this paper, we introduce a new variables sampling plan based on process capability index C_{pmk} to deal with product sentencing (acceptance determination). The proposed new sampling plan is developed based on the exact sampling distribution hence the decisions made are more accurate and reliable. For practical purpose, tables for the required sample sizes and the corresponding critical acceptance values for various producer's risk, the consumer's risk and the capability requirements acceptance quality level (AQL), and the lot tolerance percent defective (LTPD) are provided. A case study is also presented to illustrate how the proposed procedure can be constructed and applied to the real applications.
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Keywords: Acceptance sampling plans; Critical acceptance values; Process capability analysis; Process loss; Product acceptance determination

1. Introduction

Inspection of raw materials, semi-finished products, or finished products is one aspect of quality assurance. When inspection is for the purpose of acceptance or rejection of a product, based on adherence to a standard, the type of inspection procedure employed is usually called acceptance sampling (see, e.g., Schilling, 1982; Odeh and Owen, 1983; Montgomery, 2001). Acceptance sampling plans provide the vendor and the buyer

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a general criterion for lot sentencing while meeting their preset requirements on product order quality. However, it cannot avoid the risk of accepting bad product lots or rejecting good product lots unless 100% inspection is implemented. A well-designed sampling plan can significantly reduce the difference between the required and the actual supplied product quality. An acceptance sampling plan is a statement regarding the required sample size for product inspection and the associated acceptance or rejection criteria for sentencing individual lots. The criteria used for measuring the performance of an acceptance sampling plan, is usually based on the operating characteristic (OC) curve which quantifies the risks for producers and consumers. The OC curve plots the probability of accepting the lot versus the lot fraction nonconforming, which displays the discriminatory power of the sampling plan.

For product quality protection and company's profit, both the vendor and the buyer would focus on certain points on the OC curve to reflect their benchmarking risk. The vendor (producer) usually would focus on a specific level of product quality, traditionally called AQL (acceptable quality level), which would yield a high probability for accepting a lot. The AQL also represents the poorest level of quality for the vendor's process that the consumer would consider acceptable as a process average. The buyer (consumer) would also focus on another point at the other end of the OC curve, traditionally called LTPD (lot tolerance percent defective). Alternate names for the LTPD are the RQL (rejectable quality level) and LQL (limiting quality level). The LTPD is the poorest level of quality that the consumer is willing to accept for an individual lot. α is the probability of the Type I error, for a given sampling plan, of rejecting the product that has a defect level equal to the AQL. The producer suffers when this occurs because product with acceptable quality is rejected. β is the probability of the Type II error, for a given sampling plan, of accepting the product with defect level equal to the LTPD. The consumer suffers when this occurs, because product with unacceptable quality is accepted. Accordingly, a well-designed sampling plan must provide a probability of at least $1-\alpha$ of accepting a lot if the level of the product quality is at the contracted AQL. The sampling plan must also provide a probability of acceptance no more than β if the level of the product quality is at the LTPD level, the designated undesired level preset by the buyer. That is, the acceptance sampling plan must have its OC curve passing through those two designated points (AQL, $1-\alpha$) and (LTPD, β).

There are a number of different ways to classify acceptance sampling plans. One major classification is by attributes and variables. When a quality characteristic is measurable on a continuous scale and is known to have a distribution of a specified type, it may be possible to use as a substitute for an attributes sampling plan based on sample measurements such as the mean and the standard deviation of the sample. The variables sampling plan has the primary advantage that the same OC curve can be obtained with a smaller sample than is required by an attributes plan. The precise measurements required by a variables plan would probably cost more than the simple classification of items required by an attributes plan, but the reduction in sample size may more than offset this increased cost. Such saving may be especially marked if inspection is destructive and the item is expensive (see Schilling, 1982; Duncan, 1986; Montgomery, 2001). The basic concepts and models of statistically based on variables sampling plans were introduced by Jennett and Welch (1939). Lieberman and Resnikoff (1955) developed extensive tables and OC curves for various AQLs for MIL-STD-414 sampling plan. Das and Mitra (1964) have investigated the effect of non-normality on the performance of the sampling plans. Bender (1975) considered sampling plans for assuring the percent defective in the case of the product quality characteristics obeying a normal distribution with unknown standard deviation, and presented a procedure using iterative computer program calculating the non-central t-distribution. Govindaraju and Soundararajan (1986) developed variables sampling plans that match the OC curves of MIL-STD-105D. Other researches related to the classical acceptance sampling plans include Wallis (1947, 1950), Jacobson (1949), Owen (1967), Guenther (1969), Kao (1971), Stephens (1978), Hamaker (1979), Hailey (1980), Odeh and Owen (1980, 1983), Hald (1981), Moskowitz and Tang (1992), Tagaras (1994), Suresh and Ramanathan (1997) and Arizono et al. (1997). In addition to the graphical procedure for designing sampling plans with specified OC curves, tabular procedures are also available for the same purpose. Duncan (1986) gave a good description of these techniques.

Due to the sampling cannot guarantee that every defective item in a lot will be inspected, the sampling involves risks of not adequately reflecting the quality conditions of the lot. Such risk is even more significant as the rapid advancement of the manufacturing technology and stringent customers demand is enforced. Particularly, when the fraction of nonconforming products is required very low, such as measured in parts per

million (PPM), the required number of inspection items must be very large in order to adequately reflect the actual lot quality. As today's modern quality improvement philosophy, reduction of variation from the target value is the guiding principle as well as reducing the fraction of nonconformities. The capability index C_{pmk} is constructed by combining the yield-based index C_{pk} and the loss-based index C_{pm}, taking into account the process yield (meeting the manufacturing specifications) as well as the process loss (variation from the target). Therefore, in this paper we propose the use of the C_{pmk} capability index as a quality benchmark for an acceptance sampling scheme. The proposed sampling plan is developed based on analytical exact formulae hence the decisions made are reliable.

2. Process capability indices approach

2.1. Process capability indices and product quality

Process capability analysis has become an important and integrated part in the applications of statistical process control to continuous improvement of quality and productivity. The relationship between the actual process performance and the specification limits or tolerance may be quantified using appropriate process capability indices (PCIs). Based on analyzing the PCIs, a production department can trace and improve a poor process so that the quality level can be enhanced and the requirements of the customers can be satisfied. Several authors have promoted the use of various PCIs and examined with different degrees of completeness (see Pearn et al., 1992, 1998; Kotz and Lovelace, 1998; Kotz and Johnson, 2002; Spiring et al., 2003 for more details). Those indices have been defined explicitly as below, where μ is the process mean, σ is the process standard deviation, USL and LSL are the upper and lower specification limits, T is the target value preset by customers or product designers, $d = (\mathrm{USL} - \mathrm{LSL})/2$ is half of the length of the specification interval:

$$C_{\mathrm{p}} = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sigma}, \quad C_{\mathrm{pk}} = \min\left\{\frac{\mathrm{USL} - \mu}{3\sigma}, \frac{\mu - \mathrm{LSL}}{3\sigma}\right\},$$

$$C_{\mathrm{pm}} = \frac{\mathrm{USL} - \mathrm{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}}, \quad C_{\mathrm{pmk}} = \min\left\{\frac{\mathrm{USL} - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - \mathrm{LSL}}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}.$$

The index C_{p} measures only the distribution spread (process precision), which only reflects the consistency of the product quality characteristic. The index C_{pk} takes into account the magnitude of process variance as well as the departures of process mean from the mid-point of specification limits. The C_{p} and C_{pk} indices are appropriate measures of progress for quality improvement paradigms in which reduction of variability is the guiding principle and process yield is the primary measure of success. But, they are not related to the cost of failing to meet customer desire. Taguchi, on the other hand, emphasizes the loss in a product's worth when one of its characteristics departs from the customers' ideal value T. To help account for this Hsiang and Taguchi (1985) introduced the index C_{pm}, which was also proposed independently by Chan et al. (1988). The index is related to the idea of squared error loss, $\mathrm{loss}(X) = (X - T)^2$, and which incorporates two variation components: (i) variation to the process mean and (ii) deviation of the process mean from the target.

The index C_{pmk} is constructed by combining the yield-based index C_{pk} and the loss-based index C_{pm}, taking into account the process yield (meeting the manufacturing specifications) as well as the process loss (variation from the target). When the process mean μ departs from the target value T, the reduced value of C_{pmk} is more significant than those of C_{p}, C_{pk}, and C_{pm}. Hence, the index C_{pmk} responds to the departure of the process mean μ from the target value T faster than the other three basic indices C_{p}, C_{pk}, and C_{pm}, while it remains sensitive to the changes of process variation (see Pearn and Kotz, 1994–1995). We note that a process meeting the capability requirement "$C_{\mathrm{pk}} \geqslant C$" may not be meeting the capability requirement "$C_{\mathrm{pm}} \geqslant C$". On the other hand, a process meets the capability requirement "$C_{\mathrm{pm}} \geqslant C$" may not be meeting the capability requirement "$C_{\mathrm{pk}} \geqslant C$" either. But, if the process meets the capability requirement "$C_{\mathrm{pmk}} \geqslant C$", then the process must meet both capability requirements "$C_{\mathrm{pk}} \geqslant C$" and "$C_{\mathrm{pm}} \geqslant C$" since $C_{\mathrm{pmk}} \leqslant C_{\mathrm{pk}}$ and $C_{\mathrm{pmk}} \leqslant C_{\mathrm{pm}}$. Thus, the index C_{pmk} indeed provides more quality assurance with respective to process yield and process loss to the customers than the two indices C_{pk} and C_{pm}. Throughout the presentation, all developments are made

assuming the process is in a state of statistical control and the characteristic under investigation arise from normal distribution. Also, the target value is assumed to be the mid-point of the specification limits $T = M = (\mathrm{USL} + \mathrm{LSL})/2$ (which is quite common in practical situations) unless otherwise stated.

In addition to the above advantage, C_{pmk} reveals the most information about the location of the process average. Given a C_{pk} index of 1.0, all a practitioner can say about μ is that it is somewhere between the LSL and the USL, i.e., $T - d < \mu < T + d$, where d equals $(\mathrm{USL} - \mathrm{LSL})/2$. With the C_{pm} index, it can be shown (Bothe, 1997) that the distance between μ and T must be less than $d/(3C_{\mathrm{pm}})$. Therefore, given a C_{pm} index of 1.0, we know that $T - d/3 < \mu < T + d/3$. This is a much smaller interval than the one for C_{pk} equal to 1.0. For the C_{pmk} index, it can be shown that the distance between μ and T is less than $d/(3C_{\mathrm{pmk}} + 1)$. That is, for a C_{pmk} index of 1.0, one knows that $T - d/4 < \mu < T + d/4$, which is even a smaller interval than the one for C_{pm}. This is a desired property according to today's modern quality improvement theory, reduction of process loss is just as important as increasing process yield. Consequently, while C_{pk} remains the more popular and widely used index, the index C_{pmk} is considered be an advanced and useful index for processes with two-sided manufacturing specifications. Considering C_{pmk} as a mixture of C_{pk} and C_{pm}, C_{pmk} behaves "more like C_{pm}" if σ^2 is small, whereas C_{pmk} behaves "more like C_{pk}" if σ^2 is large (Jessenberger and Weihs, 2000). Particularly, for semiconductor or microelectronics manufacturing, the index C_{pmk} is appropriate for capability measure due to high standard and stringent requirement on product quality and reliability.

2.2. Estimation of C_{pmk} and its sampling distribution

For a normally distributed process that is demonstrably stable (under statistical control), Pearn et al. (1992) suggested using the natural estimator, which is defined as

$$\hat{C}_{\mathrm{pmk}} = \min\left\{\frac{\mathrm{USL} - \bar{X}}{3\sqrt{S_n^2 + (\bar{X} - T)^2}}, \frac{\bar{X} - \mathrm{LSL}}{3\sqrt{S_n^2 + (\bar{X} - T)^2}}\right\} = \frac{d - |\bar{X} - M|}{3\sqrt{S_n^2 + (\bar{X} - T)^2}},$$

where $\bar{X} = \sum_{i=1}^{n} X_i/n$ and $S_n^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2/n$ are the maximum likelihood estimators (MLEs) of μ and σ^2, respectively. We note again that $S_n^2 + (\bar{X} - T)^2 = \sum_{i=1}^{n}(X_i - T)^2/n$ which is in the denominator of $\hat{C}_{\mathrm{pmk}}$ is the uniformly minimum variance unbiased estimator (UMVUE) of $\sigma^2 + (\mu - T)^2$ in the denominator of C_{pmk}. In fact, the estimator of C_{pmk} can be expressed as $\hat{C}_{\mathrm{pmk}} = (D - \sqrt{H})/(3\sqrt{K + H})$, where $D = n^{1/2}d/\sigma$, $K = nS_n^2/\sigma^2$, $H = n(\bar{X} - T)^2/\sigma^2$, and $\eta = n^{1/2}|\mu - T|/\sigma$. Under the assumption of normality, K is distributed as χ^2_{n-1}, a chi-square distribution with $n - 1$ degrees of freedom, H is distributed as $\chi'^2_{1,\lambda}$ a non-central chi-square distribution with one degree of freedom and the non-centrality parameter $\lambda = n(\mu - T)^2/\sigma^2$, and $\sqrt{H}$ is distributed as the normal distribution $N(\eta, 1)$ with mean η and variance 1. That is, the estimator $\hat{C}_{\mathrm{pmk}}$ is a mixture of the chi-square distribution and the non-central chi-square distribution, as expressed in the following (see Pearn et al., 1992):

$$\hat{C}_{\mathrm{pmk}} \sim \frac{\frac{d\sqrt{n}}{\sigma} - \chi'_{1,\lambda}}{3\sqrt{\chi^2_{n-1} + \chi'^2_{1,\lambda}}}.$$

Chen and Hsu (1995) investigated the asymptotic sampling distribution of the estimated C_{pmk} and showed that the estimator $\hat{C}_{\mathrm{pmk}}$ is consistent, asymptotically unbiased estimator of C_{pmk}, and if the fourth moment of the distribution of X is finite, then $\hat{C}_{\mathrm{pmk}}$ is asymptotically normal. Vännman and Kotz (1995) obtained the distribution of the estimated $C_{\mathrm{p}}(u, v)$ for cases with $T = M$. By taking $u = 1$ and $v = 1$, the distribution of $C_{\mathrm{p}}(1, 1) = C_{\mathrm{pmk}}$ can be obtained. Wright (1998) derived an explicit but rather complicated expression for the probability density function of the estimated C_{pmk}. Using variables transformation and the integration technique similar to that presented in Vännman (1997), the cumulative distribution function (CDF) and the probability density function (PDF) of the estimated index $\hat{C}_{\mathrm{pmk}}$ may be expressed alternatively in terms of a mixture of the chi-square distribution and the normal distribution. The explicit form of the CDF

considerably simplify the complexity for analyzing the statistical properties of the estimated index, which can be expressed below (see Pearn and Lin, 2002):

$$F_{\hat{C}_{\mathrm{pmk}}}(y) = 1 - \int_0^{b\sqrt{n}/(1+3y)} G\left(\frac{(b\sqrt{n}-t)^2}{9y^2} - t^2\right)[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})]\mathrm{d}t \quad (1)$$

for $y > 0$, where $b = d/\sigma$, $\xi = (\mu - T)/\sigma$, $G(\cdot)$ is the CDF of the chi-square distribution, χ^2_{n-1}, with $n-1$ degrees of freedom, $\phi(\cdot)$ is the PDF of the standard normal distribution $N(0,1)$, and it is noted that we would obtain an identical equation if we substitute ξ by $-\xi$ into (1) for fixed values of y and n. Noting that for $\mu > \mathrm{USL}$ or $\mu < \mathrm{LSL}$, the capability $C_{\mathrm{pmk}} < 0$, and for $\mu = \mathrm{USL}$ or $\mu = \mathrm{LSL}$, the capability $C_{\mathrm{pmk}} = 0$. The requirement with $\mathrm{LSL} < \mu < \mathrm{USL}$ has been a minimum capability requirement applies to most start-up engineering applications or new processes.

3. Designing C_{pmk} variables sampling plans

Consider an acceptance sampling plan by variables to control the lot fraction of nonconformities and assure the loss caused by the deviation from its target value simultaneously. Suppose the quality characteristic of interest is measurable on a continuous scale and is normally distributed with mean μ and standard deviation σ, the USL and the LSL can be defined as the acceptable values of this parameter. According to today's modern quality improvement theory, reduction of the process loss is as important as the process yield, C_{pmk} can be used as a quality benchmark for acceptance of a product lot. Thus, it is easy to design a variables sampling plan with a specified OC curve. Let $(C_{\mathrm{AQL}},\ 1-\alpha)$ and $(C_{\mathrm{LTPD}}, \beta)$ be the two points on the OC curve of interest, where C_{AQL} and C_{LTPD} represent the capability requirements corresponding to the AQL and the LTPD based on C_{pmk} index, respectively. The concept of the new variables sampling plan may be constructed as:

If $C_{\mathrm{pmk}} \geqslant C_{\mathrm{AQL}}$, then the lot should be accepted with producer's risk α, and
if $C_{\mathrm{pmk}} \leqslant C_{\mathrm{LTPD}}$, then the lot should be rejected with consumer's risk β.

That is, if the C_{pmk} value of producer's product is greater than C_{AQL}, then the probability of consumer accepting the lots will be larger than $100(1-\alpha)\%$. On the other hand, if the C_{pmk} value of producer's product is less than C_{LTPD}, then the probability of consumer would accept no more than $100\beta\%$. As described previously, owing to the sampling distribution of $\hat{C}_{\mathrm{pmk}}$ is expressed in terms of a mixture of the chi-square and the normal distributions. For processes with $T = M$, the index may be rewritten as $C_{\mathrm{pmk}} = (d/\sigma - |\xi|)/[3(1+\xi^2)^{1/2}]$, where $\xi = (\mu - T)/\sigma$. Further, given $C_{\mathrm{pmk}} = C$, $b = d/\sigma$ can be rewritten as $b = 3C(1+\xi^2)^{1/2} + |\xi|$. The probability of accepting the lot can be expressed as

$$\begin{aligned}\pi_A(C_{\mathrm{pmk}}) &= P(\hat{C}_{\mathrm{pmk}} \geqslant C_0 | C_{\mathrm{pmk}} = C)\\ &= \int_0^{b\sqrt{n}/(1+3C_0)} G\left(\frac{(b\sqrt{n}-t)^2}{9C_0^2} - t^2\right)[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})]\mathrm{d}t. \quad (2)\end{aligned}$$

Therefore, the required inspection sample size n and critical acceptance value C_0 of $\hat{C}_{\mathrm{pmk}}$ for the sampling plans are the solution to the following two nonlinear simultaneous Eqs. (3) and (4):

$$1-\alpha \leqslant \int_0^{b_1\sqrt{n}/(1+3C_0)} G\left(\frac{(b_1\sqrt{n}-t)^2}{9C_0^2} - t^2\right)[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})]\mathrm{d}t, \quad (3)$$

$$\beta \geqslant \int_0^{b_2\sqrt{n}/(1+3C_0)} G\left(\frac{(b_2\sqrt{n}-t)^2}{9C_0^2} - t^2\right)[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})]\mathrm{d}t, \quad (4)$$

where $b_1 = 3C_{\mathrm{AQL}}(1+\xi^2)^{1/2} + |\xi|$ and $b_2 = 3C_{\mathrm{LTPD}}(1+\xi^2)^{1/2} + |\xi|$, $C_{\mathrm{AQL}} > C_{\mathrm{LTPD}}$. Note that the required sample size n is the smallest possible value of n satisfying (3) and (4), and determining the $\lceil n \rceil$ as sample size, where $\lceil n \rceil$ means the least integer greater than or equal to n.

3.1. The behavior of the critical acceptance value C_0 and the sample size n

Generally, the lot or process mean μ and standard deviation σ are unknown, then the distribution characteristic parameter, $\xi = (\mu - T)/\sigma$ is also unknown, which has to be estimated in real applications. However, such approach introduces additional sampling errors from estimating ξ in finding the critical acceptance values and the inspected sample sizes. To eliminate the need for estimating the distribution characteristic parameter ξ, we examine the behavior of the critical acceptance values C_0 and the sample size n against the parameter ξ.

We perform extensive calculations to obtain the critical acceptance values C_0 and the sample size n for $\xi = 0(0.05)3.00$, with various levels of C_{AQL} and C_{LTPD}. Fig. 1a plots the critical acceptance value C_0, versus ξ value for $C_{\mathrm{AQL}} = 1.33, 1.50, 1.67, 2.00$, $C_{\mathrm{LTPD}} = 1.00$ with $\alpha = 0.05$, $\beta = 0.05$. Fig. 1b plots the required sample size n versus ξ value for $C_{\mathrm{AQL}} = 1.33, 1.50, 1.67, 2.00$, $C_{\mathrm{LTPD}} = 1.00$ with $\alpha = 0.05$, $\beta = 0.05$. Noting that parameter values we investigated, $\xi = 0(0.05)3.00$, cover a wide range of applications and it has the same result when ξ is replaced by $-\xi$. We find that the sample size n obtains its maximum either at $\xi = 0.50$ (for most cases), or at 0.45 (in a few cases). Further, we find that the critical acceptance value C_0 does not change a lot as the ξ increases and stays the same acceptance value with accuracy up to 10^{-2} in all cases (and for $n \geqslant 100$, $\xi = 0.5$ with accuracy up to 10^{-3}). Hence, for practical purpose we may solve equations with $\xi = 0.5$ to obtain the criterion of $\hat{C}_{\mathrm{pmk}}$ and the required sample size n, without having to estimate the parameter ξ. This approach ensures that the decisions made based on those criteria are more reliable than all existing methods. We note the above result is almost impossible to prove theoretically.

3.2. Solving nonlinear simultaneous equations

In order to solve the above two nonlinear simultaneous Eqs. (3) and (4), we let

$$S_1(n, C_0) = \int_0^{b_1\sqrt{n}/(1+3C_0)} G\left(\frac{(b_1\sqrt{n}-t)^2}{9C_0^2} - t^2\right)[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})]\mathrm{d}t - (1-\alpha), \quad (5)$$

$$S_2(n, C_0) = \int_0^{b_2\sqrt{n}/(1+3C_0)} G\left(\frac{(b\sqrt{n}-t)^2}{9C_0^2} - t^2\right)[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})]\mathrm{d}t - \beta. \quad (6)$$

For $C_{\mathrm{AQL}} = 1.33$ and $C_{\mathrm{LTPD}} = 1.00$, the surface and contour plots of (5) and (6) with α-risk $= 0.10$ and β-risk $= 0.05$ are displayed in Figs. 2a, b and 3a, b, respectively.

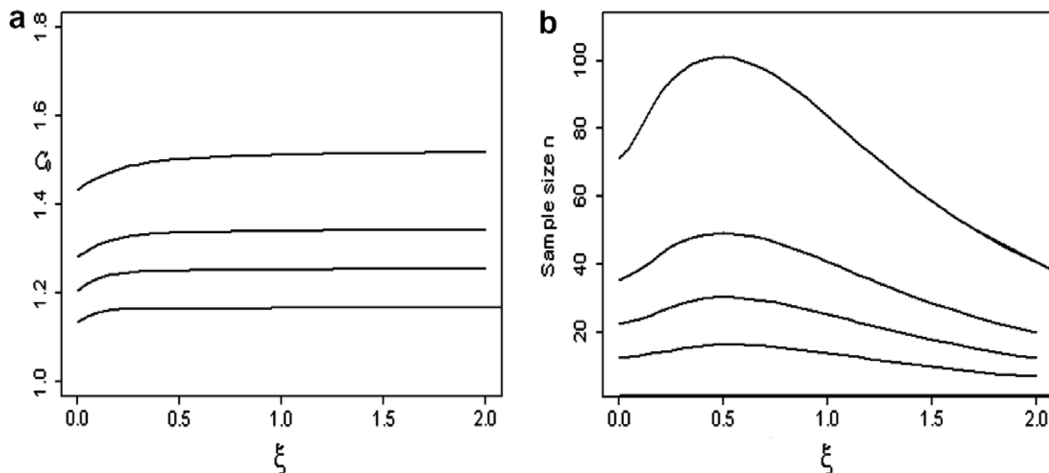

Fig. 1. (a) Plot of C_0 versus ξ value for $C_{\mathrm{AQL}} = 1.33, 1.50, 1.67, 2.00$, $C_{\mathrm{LTPD}} = 1.00$ with $\alpha = 0.05$, $\beta = 0.05$ (from bottom to top in plot). (b) Plot of sample sizes n versus ξ value for $C_{\mathrm{AQL}} = 1.33, 1.50, 1.67, 2.00$, $C_{\mathrm{LTPD}} = 1.00$ with $\alpha = 0.05$, $\beta = 0.05$ (from bottom to top in plot).

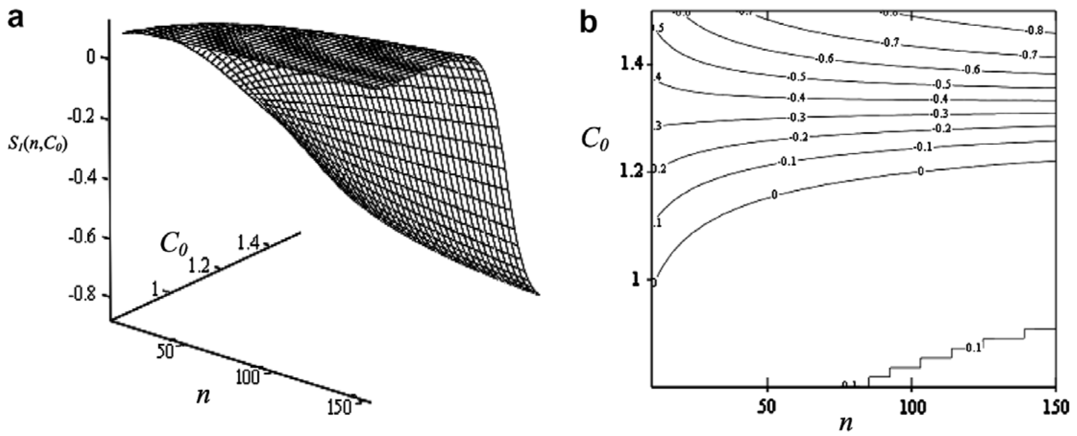

Fig. 2. (a) Surface plot of $S_1(n, C_0)$. (b) Contour plot of $S_1(n, C_0)$.

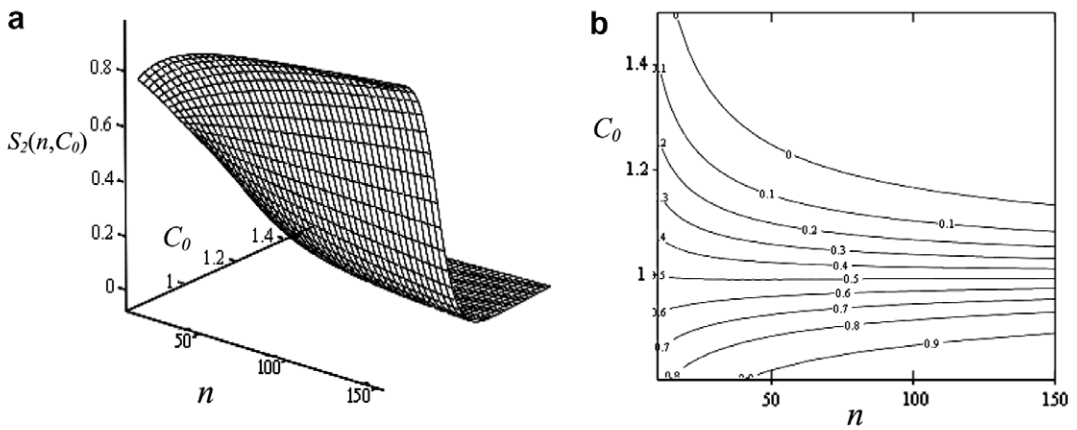

Fig. 3. (a) Surface plot of $S_2(n, C_0)$. (b) Contour plot of $S_2(n, C_0)$.

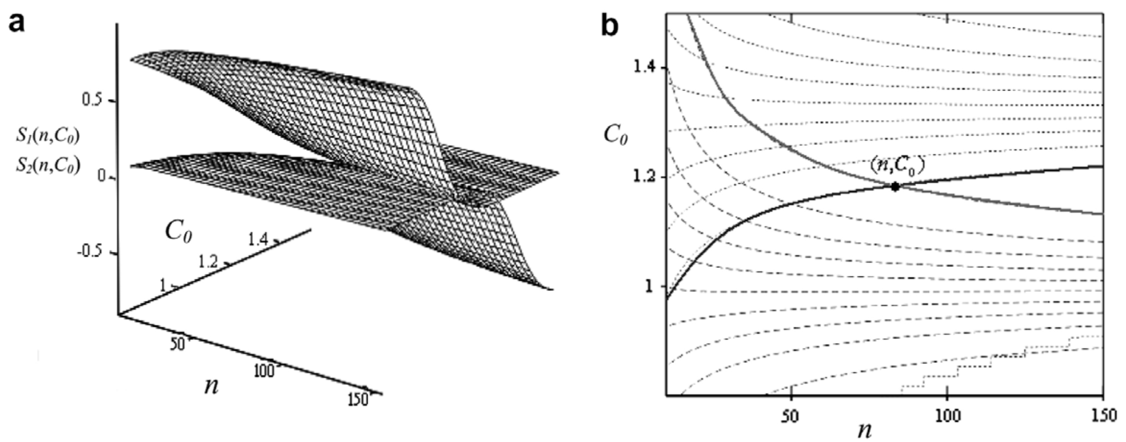

Fig. 4. (a) Surface plot of S_1 and S_2. (b) Contour plot of S_1 and S_2.

Fig. 4a and b display the surface and contour plots of (5) and (6) simultaneously with α-risk $= 0.10$ and β-risk $= 0.05$ under $C_{\text{AQL}} = 1.33$ and $C_{\text{LTPD}} = 1.00$, respectively. From the Fig. 4b, we can see that the interaction of $S_1(n, C_0)$ and $S_2(n, C_0)$ contour curves at level 0 is $(n, C_0) = (82, 1.1870)$, which is the solution to

nonlinear simultaneous Eqs. (3) and (4). That is, in this case, the minimum required sample size $n = 82$ and the corresponding critical acceptance value $C_0 = 1.1870$ of sampling plan based on the capability index C_{pmk}.

To investigate the behavior of the critical acceptance values and the required sample sizes with various parameters, we perform extensive calculations to obtain the solution of (3) and (4). We observe that the larger of the risks which producer or consumer could suffer, the smaller is the required sample size n. This phenomenon can be explained intuitively: if we expect that the chance of wrongly concluding a bad process as good or good lots as bad ones is smaller, the more sample information need to judge the lots. Further, for fixed α-, β-risks and C_{LTPD}, the required sample sizes become smaller when the C_{AQL} becomes larger. This can also be explained by the same reasoning as above, since the judgment will be more accurate with a larger difference between the C_{AQL} and C_{LTPD}.

4. Sampling procedure and decision making

Selection of a meaningful critical value for a capability test requires specification of an AQL and a LTPD for the C_{pmk} value. The AQL is simply a standard against which to judge the lots. It is hoped that the producer's process will operate at a fallout level that is considerably better than the AQL. Both producer and consumer will lay down their requirements in the contract: the former demands that not too many "good" lots shall be rejected by the sampling inspection, while the latter demands that not too many "bad" lots shall be accepted. In choosing a sampling plan attempts will be made to meet these somewhat opposing requirements. Thus, both producers and consumers may set their own safeguard line to protect their benefits. Two kinds of risks are balanced using a well-designed sampling plan. That is, if product process capability with $C_{\text{pmk}} \geqslant C_{\text{AQL}}$ (in high quality), the probability of acceptance must be larger than $1 - \alpha$. If producer's capability is only with $C_{\text{pmk}} \leqslant C_{\text{LTPD}}$ (in low quality), consumer would accept no more than β.

For practical applications purpose, we calculate and tabulate the critical acceptance values and the sample sizes required for the sampling plans, with commonly used α, β, C_{AQL} and C_{LTPD}. Table 1 displays (n, C_0) values for producer's α-risk = 0.01, 0.025(0.025)0.10, consumer's β-risk = 0.01, 0.025(0.025)0.10, with various benchmarking quality levels, $(C_{\text{AQL}}, C_{\text{LPTD}}) = (1.33, 1.00)$, $(1.50, 1.00)$, $(1.50, 1.33)$, $(1.67, 1.33)$, $(1.67, 1.50)$, $(2.00, 1.67)$. For example, if the benchmarking quality level $(C_{\text{AQL}}, C_{\text{LPTD}})$ set to (1.33, 1.00) with producer's α-risk= 0.01 and consumer's β-risk = 0.05, then the corresponding sample size and critical acceptance value can be calculated as $(n, C_0) = (144, 1.1360)$. That is, the lot will be accepted if the 144 inspected product items yield measurements with $\hat{C}_{\text{pmk}} \geqslant 1.1360$. Otherwise, we do not have sufficient information to conclude that the process meets the present capability requirement. In this case, we would believe that $C_{\text{pmk}} \leqslant C_{\text{LTPD}}$. The consumer will reject the lot.

For the proposed sampling plan to be practical and convenient to use, a step-by-step procedure is provided as below.

Step 1: Decide the process capability requirements (i.e. set the values of C_{AQL} and C_{LTPD}), and choose the α-risk, the chance of wrongly concluding a capable process as incapable, and the β-risk, the chance of wrongly concluding a bad lot as good.
Step 2: Check Table 1 to find the critical value (or acceptance criterion) C_0 and the sample size n required for inspection based on given values of α, β, C_{AQL} and C_{LTPD}.
Step 3: Calculate the value of $\hat{C}_{\text{pmk}}$ from these n inspected samples.
Step 4: Make decisions to accept the entire lot if the estimated $\hat{C}_{\text{pmk}}$ value is greater than the critical value C_0 ($\hat{C}_{\text{pmk}} > C_0$), otherwise, reject the entire lot.

5. Application example

Liquid crystals have been used for display applications with various configurations. Most of the produced displays recently involve the use of either twisted nematic (TN), or super twisted nematic (STN) liquid crystals. The technology of the STN display was introduced recently to improve the performance of LCD as an alternative to the TFT. A larger twist angle can lead to a significantly larger electro-optical distortion. This leads to

Table 1
(n, C_0) values for α-risk = 0.01, 0.025(0.025)0.10, β-risk = 0.01, 0.025(0.025)0.10 with various $(C_{\text{AQL}}, C_{\text{LTPD}})$

α	β	$C_{\text{AQL}} = 1.33$		$C_{\text{AQL}} = 1.50$		$C_{\text{AQL}} = 1.50$		$C_{\text{AQL}} = 1.67$		$C_{\text{AQL}} = 1.67$		$C_{\text{AQL}} = 2.00$	
		$C_{\text{LTPD}} = 1.00$		$C_{\text{LTPD}} = 1.00$		$C_{\text{LTPD}} = 1.33$		$C_{\text{LTPD}} = 1.33$		$C_{\text{LTPD}} = 1.50$		$C_{\text{LTPD}} = 1.67$	
		n	C_0	n	C_0	n	C_0	n	C_0	n	C_0	n	C_0
0.010	0.010	202	1.1634	98	1.2466	1039	1.4147	286	1.4988	1253	1.5847	426	1.8341
	0.025	170	1.1497	82	1.2261	877	1.4075	240	1.4846	1058	1.5775	359	1.8203
	0.050	144	1.1360	70	1.2057	749	1.4003	204	1.4704	904	1.5703	305	1.8065
	0.075	129	1.1259	62	1.1907	671	1.3950	182	1.4599	810	1.5650	273	1.7962
	0.100	117	1.1173	56	1.1780	614	1.3904	166	1.4511	742	1.5605	249	1.7875
0.025	0.010	174	1.1779	85	1.2688	887	1.4220	245	1.5137	1068	1.5921	365	1.8485
	0.025	144	1.1642	70	1.2484	738	1.4149	203	1.4995	890	1.5849	303	1.8347
	0.050	120	1.1504	58	1.2279	621	1.4076	170	1.4852	749	1.5776	254	1.8207
	0.075	106	1.1400	51	1.2125	550	1.4021	150	1.4744	664	1.5721	224	1.8102
	0.100	96	1.1312	46	1.1995	499	1.3974	136	1.4653	602	1.5674	203	1.8013
0.050	0.010	151	1.1925	74	1.2913	765	1.4295	213	1.5287	922	1.5995	316	1.8630
	0.025	123	1.1792	60	1.2714	627	1.4224	174	1.5149	756	1.5924	258	1.8495
	0.050	102	1.1654	50	1.2511	520	1.4152	143	1.5006	627	1.5852	213	1.8356
	0.075	89	1.1551	43	1.2357	455	1.4096	125	1.4898	549	1.5762	187	1.8250
	0.100	79	1.1461	39	1.2226	409	1.4049	112	1.4805	493	1.5748	167	1.8159
0.075	0.010	137	1.2035	68	1.3081	691	1.4350	193	1.5399	832	1.6050	286	1.8739
	0.025	111	1.1905	54	1.2889	560	1.4281	156	1.5264	675	1.5981	231	1.8607
	0.050	90	1.1770	44	1.2689	459	1.4209	127	1.5124	553	1.5909	189	1.8470
	0.075	78	1.1667	38	1.2538	399	1.4154	110	1.5017	480	1.5854	164	1.8365
	0.100	69	1.1578	34	1.2407	355	1.4107	98	1.4924	428	1.5806	146	1.8274
0.100	0.010	127	1.2128	63	1.3225	636	1.4397	178	1.5494	766	1.6097	264	1.8831
	0.025	101	1.2002	50	1.3039	511	1.4330	143	1.5364	616	1.6030	211	1.8703
	0.050	82	1.1870	40	1.2845	415	1.4260	115	1.5227	500	1.5959	171	1.8569
	0.075	70	1.1769	35	1.2697	357	1.4205	99	1.5122	431	1.5905	147	1.8465
	0.100	62	1.1681	31	1.2568	316	1.4158	87	1.5030	381	1.5857	130	1.8375

a substantial improvement in the contract and viewing angles over TN displays. An increasing number of personal computers are now network-ready and multimedia-capable and are equipped with CD-ROM drives. Due to advances in telecommunications' technology, simple monochromatic displays are no longer in popular demand. The next generation of telecommunication products will require displays with rich, graphic quality images and personal interfaces. Therefore, future displays must be clearer and sharper to meet these demands. Until this point, STN-LCD has been used mainly to display still images, and because of the slow response time needed to process still images, STN-LCD has not been able to reproduce animated images at an adequate contrast level. Thus, with the growing popularity of multimedia applications, there is a need for PCs equipped with color STN-LCD capable of processing animated pictures instead of still images. The space between the glass substrate is filled with liquid crystal material and the thickness of the liquid crystal is kept uniform with glass fibers or plastic balls as spacers. Thus, the STN-LCD is sensitive to the thickness of the glass substrates.

5.1. Capability requirements

In a purchasing contract, a minimum value of the PCI is usually specified. If the prescribed minimum value of the PCI fails to be met, the process is determined to be incapable. Otherwise, the process will be determined to be capable. Montgomery (2001) recommended some guidelines of minimum capability requirements for some special types of processes. In particular, it is recommended that 1.33 for existing processes, and 1.50 for new processes; 1.50 also for existing processes on safety, strength, or critical parameter, and 1.67 for new processes on safety, strength, or critical parameter. In recent years, many companies have adopted criteria for evaluating their processes that include process capability objectives that are more stringent than those recommended minimums above. For example, the "Six-Sigma" program pioneered by Motorola essentially

Table 2
The sample data with 79 observations (unit: mm)

0.717	0.698	0.726	0.684	0.727	0.688	0.708	0.703	0.694	0.713
0.730	0.699	0.710	0.688	0.665	0.704	0.725	0.729	0.716	0.685
0.712	0.716	0.712	0.733	0.709	0.703	0.730	0.716	0.688	0.688
0.712	0.702	0.726	0.669	0.718	0.714	0.726	0.683	0.713	0.737
0.740	0.706	0.726	0.688	0.715	0.704	0.724	0.713	0.694	0.742
0.690	0.704	0.697	0.705	0.707	0.687	0.718	0.718	0.724	0.706
0.687	0.673	0.730	0.732	0.720	0.688	0.710	0.707	0.706	0.709
0.729	0.729	0.685	0.686	0.722	0.720	0.715	0.727	0.696	

requires that when the process mean is in control, it will not be closer than six standard deviations from the nearest specification limit. Thus, in effect, it requires that the process capability will be at least 2.0 to accommodate the possible 1.5σ process shift (see Harry, 1988), and no more than 3.4 PPM of nonconformities.

To illustrate how the sampling plan can be established and applied to the actual data collected from the factories, we present a case study on STN-LCD manufacturing process, STN-LCD is popularly used in making the PDA (personal digital assistant), Notebook personal computer, Word Processor, and other Peripherals. The factory manufactures various types of the LCD. For a particular model of the STN-LCD investigated, the target value is set to $T = 0.70$ mm, and the tolerance of thickness is 0.07 mm, that is, the USL of a glass substrate's thickness is 0.77 mm, the LSL of a glass substrate's thickness is 0.63 mm. If the product characteristic does not fall within the tolerance (LSL, USL), the lifetime or reliability of the STN-LCD will be discounted. In the contract, the AQL and LTPD are set to 1.33 and 1.00 based on C_{pmk} index with the α-risk $= 0.05$ and β-risk $= 0.10$, respectively. Therefore, the problem for quality practitioners is to determine the critical acceptance value and the required sample size of the sampling plan that provide the desired levels of protection for both the producer and the consumer. Based on the proposed procedure, we can obtain the critical acceptance value and inspected sample size are $(n, C_0) = (79, 1.1461)$ from Table 1. Hence, the inspected samples are taken from the lot randomly and the observed measurements are showed in Table 2. Based on these inspections, we obtain that

$$\bar{X} = 0.7088, \quad S_n = 0.0171, \quad \hat{C}_{\text{pmk}} = \left\{ \frac{\text{USL} - \bar{X}}{3\sqrt{S_n^2 + (\bar{X} - T)^2}}, \frac{\bar{X} - \text{LSL}}{3\sqrt{S_n^2 + (\bar{X} - T)^2}} \right\} = 1.0621.$$

Therefore, in this case, the consumer would reject the entire lot, since the sample estimator from the inspections, 1.0621, is smaller than the acceptance value 1.1461 of the sampling plan. We note that if the existing sampling plans are applied here, it is almost certain that any sample of 79 STN-LCDs taken from each lot will contain no defective items. All the lots therefore will be accepted, which obviously provides no protection to the consumer at all.

6. Conclusions

Capability indices are becoming the standard tools for quality report, particularly, at the management level around the world. Proper understanding and accurate estimating them is essential for the company to maintain a capable supplier. According to today's modern quality improvement philosophy, reduction of process loss (variation from the target) is just as important as increasing process yield (meeting the specifications). The index C_{pmk} indeed provides more quality assurance with respective to process yield and process loss to the consumers. In this paper, we develop a new variables sampling plan based on the process capability index C_{pmk} to deal with lot sentencing problem even when the lot or process fraction of nonconformities is very low and reaches the PPM level. We develop a method for obtaining the required sample size for inspection and the corresponding critical acceptance values based on the exact sampling distribution, which provide the desired levels of protection for both producers and consumers. To make the proposed procedure practical for in-plant applications, a case study on STN-LCD manufacturing process is presented and tables of the

required sample sizes for inspection and the corresponding critical acceptance values for various producer's risks, the consumer's risks with the capability requirements AQL and the LTPD are provided.

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