行政院國家科學委員會專題研究計畫結案報告

二維週期結構之導波特性研究（子計畫一）

Theoretical Study on Guidance characteristics of Two Dimensionally Periodic Structures

計畫主持人：彭松村 教授
共同主持人：黃瑞彬 研究副教授
交通大學電子與資訊研究中心
計畫編號：NSC 89-2213-E009-074

Guiding Characteristics of a coplanar waveguide with periodic variation in strip width

R.B. Hwang and S.T. Peng

Microelectronics and Information systems Research Center
National Chaio-Tung University
Hsinchu, Taiwan, R.O.C.

Abstract

In this paper, we present a systematic investigation of the guiding characteristics of a coplanar waveguide with periodic variation of the line profile. Based on the method of effective dielectric constant (EDC), simple formulas are obtained and the numerical results are expressed in the form of the Brillouin diagram, with both phase and attenuation constants included. Furthermore, the scattering of guided waves by a periodic structure of finite length is analyzed both theoretically and experimentally with results in excellent agreement, and the scattering results check very well with the dispersion characteristics. Thus, the method presented in this paper provides not only a clear physical picture for understanding the wave phenomena involved but also a simple yet accurate design criterion for practical considerations.

1. Introduction

The subject of periodic structures has attracted continuing interest in the literature [1, 2]. Due to the mathematical complexity of treating the metallic-type periodic structures, most of the research works in the past have been limited to experimental studies and simulations using various numerical methods. Although the scattering approach to a periodic structure of finite length is very useful for evaluating the performance of the periodic structure, it gives the cumulative effect of the overall structure, but not the insight of individual physical processes involved. On the other hand, the dispersion characteristics exhibit clearly the interactions of space harmonics and predict easily the locations of stopbands. In this paper, we present a study on the class of coplanar waveguides (CPW) with its width varying periodically along the waveguide axis. Specifically, we employ the method of effective dielectric constant, based on the dominant mode of CPW, to analyze theoretically the dispersion characteristics for an infinite periodic CPW. Furthermore, the dispersion characteristics are verified by both theoretical and experimental analysis of the scattering parameters of a periodic structure with a finite length, and the results by different approaches are in excellent agreement. This permits us to establish simple criteria for practical applications of the class of microwave planar circuits with periodic variation.

2. Statement of the problem

Fig. 1 depicts the configuration of a CPW with the width of the center strip varying periodically along the structure. The period of the structure is denoted by d, and the center strip may have an arbitrary variation of its width and its separations from the ground planes. The relative dielectric constant and thickness of the substrate are ε_s and h, respectively. Conceptually, the continuous variations of the edges of the strip and the ground planes may be discretized or approximated by a piecewise uniform profile, as shown in Fig. 1(b). In each uniform section of the discretized structure, the method of effective dielectric constant is employed to develop a network of cascaded transmission lines, each with a known propagation constant and a known characteristic impedance [3]. In general, the CPW in Fig. 1(a) can be reduced to a non-uniform transmission line with periodically varying propagation constant and impedance, $\kappa(z)$ and $Z(z)$. There are many ways to solve the problem of non-uniform transmission lines, depending on the profile of the non-uniformity. In particular, with the discretized structure, the input-output relation of a period can be easily constructed, such that a complete set of characteristic solutions for the periodic transmission-line can then be treated naturally by the eigenvalue problem, as will be explained in what follows.

3. Method of analysis

Returning to Fig. 1, we employ the building-block approach to simplify the analysis of this boundary value problem. Firstly, the input-output relation of the bifurcated PPWG, which is shown in Fig. 2, will be constructed by using the rigorous mode-matching method. Secondly, the input-output relation of the simple PPWG of finite length is well known and is skipped here. Finally, the field analysis of the cascaded sub-cells will be conducted, as given below.

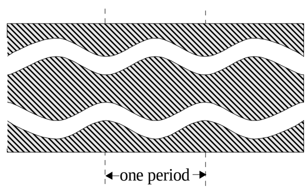

Fig. 1(a) Structural configuration for periodic CPW

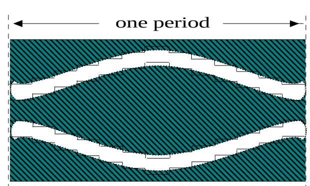

Fig. 1(b) Staircase approach for one period

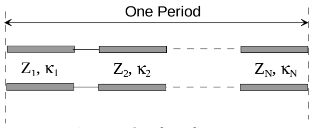

Fig. 1(C) Cascade of uniform transmission network

A. Dispersion relation of a periodic transmission-line network

Consider a periodic transmission line consisting of a finite number of uniform transmission-line sections in each unit cell,, as shown in Fig. 1(c). The total transfer matrix of the unit cell can then be written in terms of those of the constituent sections, as:

$$\tilde{T} = \prod_{i=1}^{N} T_i \quad (1)$$

where T_i is the transfer matrix for the i^{th} section in the unit cell to determine the output condition from that at the input, and is defined by:

$$T_i(d_i) = \begin{pmatrix} \cos\kappa_i d_i & jZ_i \sin\kappa_i d_i \\ jY_i \sin\kappa_i d_i & \cos\kappa_i d_i \end{pmatrix} \quad (2)$$

Here, κ_i is the propagation constant, Z_i is the characteristic impedance, Y_i is the characteristic admittance, and d_i is the length of the transmission-line section. The total transfer matrix holds for every unit cell; hence, the electromagnetic fields and their propagating characteristics can be conveniently expressed in terms of its eigenvalues and eigenvectors. Let λ be an eigenvalue of $\tilde{T}$ and f be its corresponding eigenvector, such that they satisfy the following relation:

$$\tilde{T}\underline{f} = \lambda \underline{f} \quad (3)$$

For such a second-order transfer matrix $\tilde{T}$, the eigenvalues are determined by the characteristic equation:

$$\lambda^2 - Tr(\tilde{T})\lambda + 1 = 0 \quad (4)$$

where $Tr(\tilde{T})$ represents the trace of $\tilde{T}$ and is equal to the sum of the two diagonal elements. In arriving at the last equation, we have made use of the fact that the determinant of the transfer matrix $\tilde{T}$ is always equal to unity. Therefore, the two eigenvalues of $\tilde{T}$ must be reciprocal to each other, and we may express the two eigenvalues in the exponential form,

$$\lambda_1 = exp(-j\kappa d) \quad (5)$$

$$\lambda_2 = exp(+j\kappa d) \quad (6)$$

where κ is called the characteristic exponent. Evidently, λ_1 and λ_2 represent the phase changes of the forward and backward waves traveling over a period d, respectively. The characteristic equation (5) can now be written in the alternative form:

$$2\cos\kappa d = Tr(\tilde{T}) \quad (7)$$

Thus, the characteristic exponent can now be determined easily with the straightforward calculation of the transfer matrix of the unit cell. Since the matrix $\tilde{T}$ depends on the frequency ω, we may plot the dispersion curves by determining κ as a function of the frequency ω from the last equation. The range of ω is a pass-band, if κ is real; it is a stop-band, if κ is complex. Finally, once an eigenvalue is determined, the two eigenvectors f_1 and f_2 are then determined from (3) for the forward and backward traveling waves.

B. Input-output relation for the periodic transmission line networks

In practice, a periodic structure contains only a finite number of unit cells. With the transfer matrix defined for a unit cell, as shown above, the input-output relation of the whole structure of p cells can be given by:

$$\begin{pmatrix} V(x=l) \\ I(x=l) \end{pmatrix} = \tilde{T}^p \begin{pmatrix} V(x=0) \\ I(x=0) \end{pmatrix} \quad (8)$$

The terminal condition at the output end may be specified as: $V(l) = Y_{out} I(l)$, where Y_{out} is the out admittance. The voltage and current at the output end can be related to the voltage at the input end, as:

$$\begin{pmatrix} V(l) \\ I(l) \end{pmatrix} = \frac{V(0)e^{-jp\kappa d}}{1 + Se^{-j2p\kappa d}} \begin{pmatrix} 1 \\ Y_{out} \end{pmatrix} \quad (9)$$

where S is supposed to be a known constant for a given system. Evidently, in a stopband, the output voltage and current will tend to diminish as the number of unit cells, p, becomes very large; this results in the vanishing transmission, as should be expected.

3. Numerical Results and Discussions

Fig. 2 shows the dispersion characteristics of a periodic CPW with two alternating widths and lengths in a period. The dimensions of the structure are w_1 = 4.1 mm, s_1=0.25 mm, d_1 = 21 mm; and w_2 = 2 mm, s_2=1.3 mm, d_2 = 9 mm. The relative dielectric constant and the thickness of the substrate used are 3.62 and 0.81mm, respectively. Also shown are the theoretical and measured results for the transmission coefficient (S_{21}) in Fig. 2(a) and (b), respectively, as will be further discussed later on. The propagation constant is normalized to the period *d/2®*, and is displayed in the form of the Brillouin diagram. The dispersion curves of the uniform CPW is obtained by the average width of the two alternating sections and is taken as the unperturbed ones, as displayed in the dashed lines in the same figure. The indices attached to the unperturbed dispersion curves indicate the orders of the space harmonics associated with the dominant mode of CPW. According to the theory of mode coupling, an actual dispersion curve of a periodically perturbed structure should differ only slightly from that of the unperturbed one, except in the stopband region where the propagation characteristics change qualitatively from a propagating to a decaying wave. Along the unperturbed dispersion curve for the fundamental mode, $n = 0$, there are strong contra-flow interactions or couplings resulting in stopbands around the intersections points with those of the higher space harmonics. Evidently from Fig. 2, the results meet the expectation of the theory of mode coupling.

Returning to the theoretical calculation and measurement of the transmission coefficient (S_{21}) of a periodic CPW with a finite number of unit cells, we have carried out both theoretical and experimental analysis of a periodic CPW for the three cases of 5, 6 and 7 periods. The results are superimposed on the dispersion curves in Fig. 2(a) and (b), respectively. It is noted that in order for comparison between theory and measurement, we have taken into account the dielectric loss and conductor loss in our numerical analysis. We observe that the stopbands can be accurately determined from either the dispersion curves for a periodic structure of infinite length or the transmission coefficient of a periodic structure of finite length. Furthermore, the increase in the number of unit cells will enlarge the width of stopbands; at the same time, it will enhance the attenuation of wave in the stopbands. Comparing the scattering parameters between theory and measurement in Fig. 2, we observe that they agree with each other very well. Furthermore, the dependence of the attenuation constant on the number of periods checks the simple formula in (9). Evidently, the dispersion characteristics provide not only a clear physical picture for understanding the wave propagation involved but also a simple, yet accurate, design criterion for practical design purpose.

4. Conclusions

We have presented the dispersion characteristics of CPW with periodic variation, based on the single mode approximation. We have verified with excellent agreement the stopbands by both theoretical computations and experimental measurements of the transmission coefficient through a periodic CPW of a finite length. Thus, the simple model presented not only provides the physical pictures of the wave propagation involved but also establish a design criterion for practical consideration.

5. References

1. S.T. Peng, T. Tamir and H.L. Bertoni, "Theory of dielectric grating waveguides," IEEE Trans. MTT, Vol. MTT-23, p123-133, 1975.
2. Yongxi Qian and Tatsuo Itoh, "Planar Periodic Structures for Microwave and Millimeter Wave Circuit Applications," IEEE MTT-S Digest, 1533-1536, 1999.

K.C. Gupta, Ramesh Garg and I.J. Bahl, *Coplanar waveguides and Slotlines*, Artech House, 1979

Acknowledges

We acknowledge with gratitude the support by the National Science Council under the contract number NSC 89-2213-E009-074.

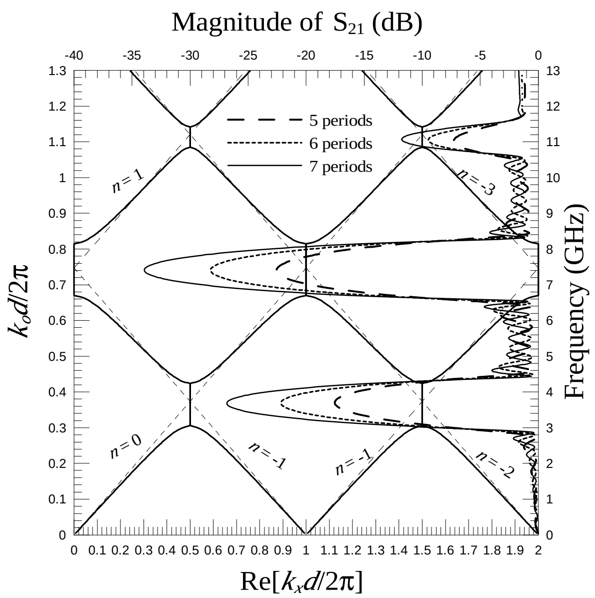

Fig. 2: Brillouin diagram of a infinite length periodic CPW and the scattering parameters (S21) of a finit length periodic CPW (numerical simulation)

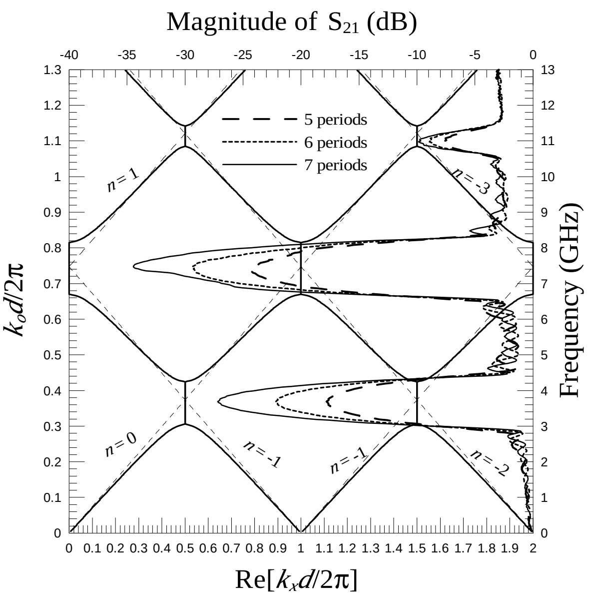

Fig. 3: Brillouin diagram of a infinite length periodic CPW and the scattering parameters (S21) of a finit length periodic CPW (measurement)