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An Integrated Multi-objective Model to Determine the Optimal Rescue Path and Traffic Controlled Arcs for Disaster Relief Operations under Uncertainty Environments

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This study proposes an integrated multi-objective model to determine the optimal rescue path and traffic controlled arcs for disaster relief operations under uncertainty environments. The model consists of three sub-models: rescue shortest path model, post-disaster traffic assignment model, and traffic controlled arcs selection model to minimize four objectives: travel time of rescue path, total detour travel time, number of unconnected trips of non-victims, and number of police officers required. Since these sub-models are inter-related with each other, they are solved simultaneously. This study employs genetic algorithms incorporated with traffic assignment and K-shortest path methods to determine optimal rescue path and controlled arcs. To cope with uncertain information associated with the damaged network, fuzzy system reliability theory (weakest t-norm method) is used to measure the access reliability of rescue path. To investigate the validity and applicability of the proposed model, studies on an exemplified case and a field case of Chi-Chi earthquake in Taiwan are conducted. The performances of three rescue strategies: without traffic control, selective traffic control (i.e. the proposed model) and absolute traffic control are compared. The results show that the proposed model can maintain the efficiency of rescue activity with minimal impact to ordinary trips and number of police officers required.

Keywords: Chi-Chi earthquake, disaster relief operations, fuzzy access reliability, genetic algorithms, shortest path algorithm

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1. Introduction

The relief management of natural disasters, such as earthquakes, hurricanes, tsunami, floods and volcanic eruption, involves very critical and urgent decision issues regarding the well being of affected people. The decision-makers responsible for rescue activities should respond quickly and effectively to logistical problems in such complex emergency situations. To effectively mitigate the degree of damage that may be caused, the most important logistical problem is how to deliver rescue personnel, first-aid materials and equipments from supply depots to the affected areas efficiently. However, without the aid of an efficient decision support model, a reliable logistical solution is difficult to achieve, in particular at the very beginning of disasters wherein uncertainties and complex interactions between relief operations and ordinary economic activities do exist in the decision environments.

Undoubtedly, mathematical models are suitable for providing comprehensive solutions to the disaster relief operations. Numerous mathematical programming models related to disaster relief operations have been proposed. For instance, Haghani and Oh (1996) proposed a large-scale time-space network model for multi-commodity and multi-modal logistical problem of disaster relief management. Fiedrich *et al.* (2000) proposed a mathematical model for determining the optimal allocation of emergency resource after earthquake disasters to minimize the total number of fatalities. Bryson *et al*. (2002) developed a mathematical model to determine the optimal reconstruction projects after a disaster by considering the budget constraint and reliability of projects. Barbarosglu *et al.* (2002) proposed a multi-objective bi-level mathematical model for helicopter mission planning during a disaster relief operation. Considering the uncertainties in disaster relief operations, Esogbue (1996) proposed fuzzy mathematical dynamic programming models that further enhance reliability and utility of conventional crisp models with a case study of floods examined.

Most of the abovementioned studies mainly focus on the disaster relief logistical problem to determine the most efficient manner to transport the rescue personnel, first aid materials, equipments, and food from supply points to the affected areas, to transfer affected people to the closest heath care centers, or to vacuum unaffected people to safe areas. Only a few have considered the necessary traffic control measures to prevent ordinary activity trips from intervening the relief operations.

These traffic control measures may not be inevitably implemented to such a large extent that it impacts the day-to-day activities of non-victims and the overall economy. In the crucial traffic control measures, the controlled arcs of the network should be clearly indicated to prohibit usage by ordinary trips. Besides, the progress of relief operations would be impeded due to the damaged transportation network caused by the disaster and it should be taken into account in planning these operations. The degree of network performance degradation depends not only on the types and strength of disasters, but also on the structure of network itself. Although some studies have proposed and computed some indices to measure post-disaster transportation system performance on a specific disaster (e.g. Chang and Nojima, 2001), the information may not be easily acquired at the very beginning of disaster where the electrical power and communications network might be seriously affected. But the relief decisions should be immediately made without waiting for further confirmation. Therefore, it is imperative to study how to arrange these relief operations under such uncertain conditions. Moreover, in measuring the degree of impact to the non-victims caused by traffic control measures, a prediction of post-disaster ordinary trips distribution among the damaged network after implementation of traffic control measures is also necessary.

Considering the aforementioned key issues, this paper aims to propose a multi-objective mathematical programming model for determining an optimal rescue path with selective traffic controlled arcs under uncertainty environments. The proposed model is to maximize the efficiency and connectivity of rescue path and to minimize total detour travel time of ordinary trips, number of unconnected ordinary trips, and number of police officers required for the implementation of traffic control measures. For comparison, three control strategies are considered in the present paper: (1) selective traffic control, (2) without traffic control, and (3) absolute traffic control. The selective traffic control strategy (Strategy 1) is to implement traffic control only on the arcs suggested by the proposed model. The without traffic control strategy (Strategy 2) is not to implement any traffic control on the network. The absolute control strategy (Strategy 3) is to implement traffic control on all arcs in the rescue path. The rest of this paper is organized as follows. Section 2 presents details of the proposed model. Section 3 delineates the solution algorithm of the proposed model. Section 4 provides the validation results based on an exemplified case. Section 5 further applies the proposed model to the real case of Chi-Chi earthquake in Taiwan. Finally, the concluding remarks and suggestions for future studies follow.

2. Model Formulation

In determining a shortest rescue path satisfying certain level of reliability and corresponding traffic controlled arcs without causing too adversely effect on ordinary trips, three sub-problems are considered here. Firstly, the shortest path problem for solving a rescue path to transport relief personnel, first aid materials and equipments to the affected area in the most efficient and reliable manner is considered. However, the information regarding the degree of network performance degradation is vague at the beginning stage of the disasters; i.e., the shortest path might not be reliable. Thus, fuzzy set theory is employed to deal with the vague information in measuring the reliability of the path. In this study, *K*-shortest paths will be solved and only the path satisfying the preset fuzzy reliability level will be suggested. The second sub-problem is the selection problem for indicating the necessarily controlled arcs to prohibit the usage by ordinary trips. The last sub-problem is a traffic assignment problem to forecast the distribution of post-disaster ordinary trips. Non-victims might rearrange their routes according to the information regarding the damage and/or traffic control of the network. The systematic relationships among these sub-problems are depicted in Figure 1.

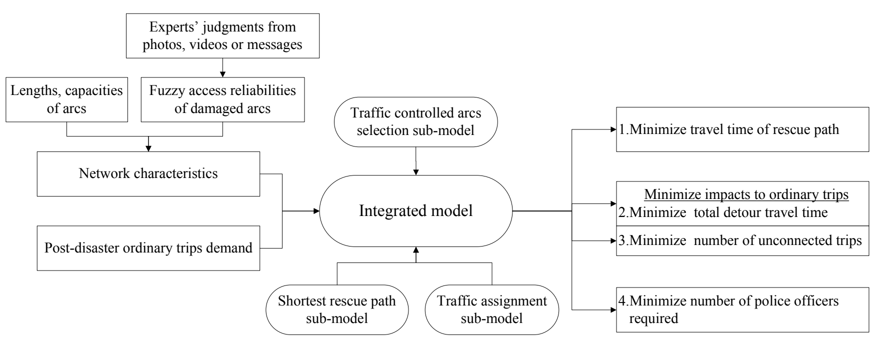

Figure 1. Framework of the proposed model

Since these three sub-problems are inter-related with each other and cannot be solved independently, we propose an integrated multi-objective mathematical programming model to determine an optimal rescue path with necessary controlled arcs by considering four objectives: the efficiency and reliability of rescue path, the adverse effect on non-victims including the number of unconnected trips and total detour time caused by implementing traffic control measures, and the number of police officers required to enforce traffic control measures. The model is presented as follows.

[MOP]

$$Min\ \ Z_1 = \sum_i \sum_j c_{ij}(y_{ij}, z_{ij}) x_{ij} \tag{1}$$

$$Min\ \ Z_2 = \sum_i \sum_j \int_0^{y_{ij}} d_{ij}(w, z_{ij}) dw \text{ - } L \tag{2}$$

$$Min\ \ Z_3 = \sum_u \sum_v T_{uv},\ if\ P_{uv} = \Phi \tag{3}$$

$$Min\ \ Z_4 = \sum_i \sum_j e_{ij} z_{ij} \tag{4}$$

$s.t.$

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(i,j)\in A} x_{ji} = \begin{cases} 1 & \text{if } i = s \\ 0 & \text{for all } i \in N - \{s,t\} \\ -1 & \text{if } i = t \end{cases} \tag{5}$$

$$\widetilde{\Omega} \geq \widetilde{\Omega}_r \tag{6}$$

$$\sum_{k \in P_{uv}} f_{uv}^{k} = T_{uv}, \forall u, v \tag{7}$$

$$y_{ij} = \sum_u \sum_v \sum_k \delta_{uv}^{(i,j)k} f_{uv}^{k}, \forall (i,j) \in A \tag{8}$$

$$x_{ij} = \{0,1\} \quad for\ \ (i,j) \in A \tag{9}$$

$$f_{uv}^{k} \geq 0, \forall u, v, k \tag{10}$$

$$z_{ij} = \{0,1\} \quad for\ \ (i,j) \in A \tag{11}$$

where, G(N, A) stands for the network under consideration. N is the set of all nodes of the network. A is the set of all arcs of the network. Two categories of decision variables are defined: x_{ij}=1 represents the arc (i, j) belonging to rescue path, $x_{ij}=0$ otherwise. z_{ij}=1 stands for implementation of traffic control measures on arc (i, j), $z_{ij}=0$ otherwise. According to the values of z_{ij}, the arcs of the network can be further divided into two sets: the set of controlled arcs, A', and the set of uncontrolled arcs, A''. Of course, $A = A' \cup A''$. y_{ij} represents the flow on arc (i, j). f_{uv}^{k} represents the ratio of traffic demand from node u to node v choosing the k^{th}-alternative routes between nodes u and v.

Equation (1) is the first objective function that minimizes the travel time of rescue path. $c_{ij}(y_{ij},z_{ij})$ stands for the travel time of arc (i, j) under the flow of y_{ij} and traffic control condition of z_{ij}. $c_{ij}(y_{ij}, z_{ij})$ is computed according to the commonly used BPR function: $c_{ij}(y_{ij}, z_{ij}) = t_{ij}^{f}(1+\alpha(\frac{(1-z_{ij})y_{ij}}{C_{ij}})^{\beta})$, where t_{ij}^{f} is the travel time of arc (i, j) under free-flow condition. C_{ij} stands for the remained capacity after the disaster. α and β are parameters. Accordingly, if the traffic control is implemented at arc (i, j), *i.e.* z_{ij}=1, the flow of arc (i, j) equals 0 and travel time equals t_{ij}^{f}. In contrast, if arc (i, j) is not controlled, *i.e.* $z_{ij}=0$, the travel time of arc (i, j) can be computed accordingly under flow rate of y_{ij}.

Equation (2) is the second objective function that minimizes the total detour travel time of non-victims caused by the traffic control measures. $d_{ij}(w, z_{ij})$ represents the travel time function of arc (i, j) under flow of w and traffic control condition of z_{ij}. $d_{ij}(w,z_{ij})$ is also computed by BPR function: $d_{ij}(w,z_{ij})$= $t_{ij}^{f}(1+\alpha(\frac{w}{(1-z_{ij}+\varepsilon)C_{ij}})^{\beta})$ with an arbitrary value ε being added to avoid calculation errors. If arc (i, j) is controlled, *i.e.* z_{ij}=1, the capacity of arc (i, j) is set to be nearly 0 for ordinary traffic. On the contrary, if arc (i, j) is uncontrolled, *i.e.* z_{ij}=0, the capacity of arc (i, j) is set to be C_{ij} for ordinary traffic. L is total travel time of non-victims after the disaster if traffic control measures are not implemented (*i.e.* z_{ij} is set as 0 for all i, j), which can be computed by the following traffic assignment model:

$$\text{[TAP]} \qquad Min\ \ L = \sum_{i}\sum_{j}\int_{0}^{y_{ij}} d_{ij}(w, z_{ij})dw \tag{12}$$

$$s.t.$$

$$\sum_{k \in P_{uv}} f_{uv}^{k} = T_{uv}, \forall u, v \tag{13}$$

$$y_{ij} = \sum_{u} \sum_{v} \sum_{k} \delta_{uv}^{(i,j)k} f_{uv}^{k}, \forall (i,j) \in A \tag{14}$$

$$f_{uv}^{k} \geq 0, \forall u, v, k \tag{15}$$

Equation (3) is the third objective function that minimizes the number of unconnected ordinary trips caused by the implementation of traffic control measures. T_{uv} is the travel demand of non-victims of O-D pair (*u, v*). P_{uv} is the set of all possible paths from node *u* to node *v*. $P_{uv} = \Phi$ implies that there is no path connecting node *u* to node *v*.

Equation (4) is the fourth objective function that minimizes the total number of police officers required to enforce the traffic control measures. e_{ij} represents the number of police officers needed on arc (*i, j*).

Equation (5) is a flow conversation constraint. The net flow of origin node from which relief personnel departs is set as 1 and the net flow of destination node of affected area is set as -1. The net flows of other nodes are set as 0. $(i, j) \in A$ represents arc (*i, j*) belonging to set *A*. $j : (i, j) \in A$ stands for the node *j* of arc (*i, j*) belonging to set *A*. Equation (6) ensures that the fuzzy access reliability $\widetilde{\Omega}$ of the rescue path is larger than the required level $\widetilde{\Omega}_r$ preset by decision makers. Equation (7) ensures that total trips on all alternative paths of O-D pair (*u, v*) are equal to corresponding traffic demands. Equation (8) represents the definitional incidence relationships expressing the arc flow in terms of path flows. $\delta_{uv}^{(i,j)k}$ is a dummy variable representing whether arc (*i, j*) belongs to k^{th}-alternative routes from node *u* to node *v*. If so, $\delta_{uv}^{(i,j)k} = 1$; otherwise, $\delta_{uv}^{(i,j)k} = 0$.

Since it is almost impossible to gather sufficient data to construct a conventional probability reliability of a specific path right after a disaster, this paper employs the fuzzy system reliability theory to measure the access reliability of a rescue path. The degree of damage to the transportation facilities on every arc near the affected area can be evaluated by experts from the photos or video films taken by helicopters or even satellites (Zimmermann, 1995; Tobias *et al.*, 2000). These measurements of access reliability of all arcs, of course, are based on human judgments, and should be better modeled by fuzzy set theory. Then, the accessibility of a path can be represented by summarizing the degree of accessibility of all the constituent arcs. In formulating that, we adopt the fuzzy reliability operator of a serial system proposed by Hong and Do (1997), based on the weakest t-norm (*Tw*), which can be expressed as follows:

$$\tilde{\Omega} = \tilde{R}_1 \otimes \tilde{R}_2 \otimes \ldots \otimes \tilde{R}_n = (a_1, \alpha_1, \beta_1)_{LR} \otimes (a_2, \alpha_2, \beta_2)_{LR} \otimes \ldots \otimes (a_n, \alpha_n, \beta_n)_{LR}$$

$$= \left(\prod_{i=1}^{n} a_i, \max\left\{ \alpha_i \prod_{j=1, j\neq i}^{n} a_j \middle| i = 1,2,\ldots,n \right\}, \max\left\{ \beta_i \prod_{j=1, j\neq i}^{n} a_j \middle| i = 1,2,\ldots,n \right\} \right)_{LR} \quad (16)$$

This implies that the access reliability of a path will be very low, once it contains an arc with very low reliability. The decision maker can preset a specific level of required access reliability ($\tilde{\Omega}_r$) for selecting the shortest rescue path that satisfies the required reliability level. However, since both sides of Equation (16) are fuzzy numbers, the total score method proposed by Chen and Hwang (1989) is used to judge whether the constraint is satisfied or not. The total scores of two arbitrary fuzzy numbers ($\tilde{M}_1$ and $\tilde{M}_2$) are calculated as follows:

$$\mu_T(i) = \frac{\left| \mu_R(i) + 1 - \mu_L(i) \right|}{2} \quad (17)$$

where, $\mu_T(i)$ represents the total score of the i^{th} fuzzy number, i =1, 2. If $\mu_T(1) \geq \mu_T(2)$, then $\tilde{M}_1 \geq \tilde{M}_2$. $\mu_R(i)$ and $\mu_L(i)$ stand for the right score and left score of the i^{th} fuzzy number, respectively, i =1, 2, which can be calculated as follows:

$$\mu_R(i) = \sup_x \min[\mu_{\max}(x), \mu_{M_i}(x)] \quad (18)$$

$$\mu_L(i) = \sup_x \min[\mu_{\min}(x), \mu_{M_i}(x)] \quad (19)$$

$\mu_{max}(x)$ stands for a straight line with slope 1 and its function can be expressed as $\mu_{max}(x) = x$. $\mu_{min}(x)$ stands for a straight line with slope -1, which can be expressed as $\mu_{min}(x) = 1\text{-}x$. $\mu_{Mi}(x)$ is the membership function of the i^{th} fuzzy number, i =1, 2. The relationship between left score and right score can be depicted in Figure 2.

In addition, the fuzzy reliability of an arc also affects its capacity. This paper employs the defuzzified value of the fuzzy access reliability to adjust its capacity that would be incorporated into the traffic assignment and rescue route selection procedures.

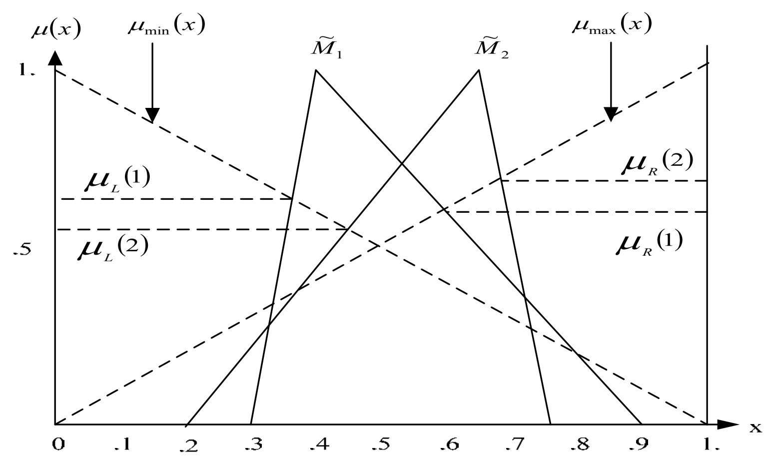

Figure 2. The relationship between left score and right score

3. Solution algorithm

The above [MOP] model is a multi-objective 0-1 mixed programming problem with nonlinear and fuzzy constraints. It is difficult to solve by conventional mathematical programming techniques. Based on the characteristics of each sub-problem, this paper proposes three algorithms to solve them, respectively, and subsequently they are integrated into a whole algorithm. Since there are four objective functions in the proposed model, in solving the problem, a weighted-sum of these four objective functions is computed. To avoid the difference in units and amplitude of the four objectives adversely affecting the decision, however, these four objectives are normalized before weighted by the following equation:

$$Z_{ij}'=\frac{Z_{ij}-Z_i^{\min}}{Z_i^{\max}-Z_i^{\min}} \tag{20}$$

where, Z_{ij}' is the normalized value of the i^{th} objective of the j^{th} chromosome. Z_{ij} is the value of the i^{th} objective of the j^{th} chromosome. $Z_i^{\min}$ is the minimal value of the i^{th} objective of all chromosomes. $Z_i^{\max}$ is the maximal value of the i^{th} objective of all chromosomes. The weighted-sum of these four normalized objectives can be computed by the following equation:

$$O_j = w_1 Z_{1j}'+w_2 Z_{2j}'+w_3 Z_{3j}'+w_4 Z_{4j}' \tag{21}$$

where, w_i is the weight of the i^{th} objective and $\sum_{i=1}^{4} w_i = 1$.

Since Strategy 2 (without traffic control) and Strategy 3 (absolute traffic control) are two extreme cases of Strategy 1, the minimal and maximal values of the first objective ($Z_1^{\min}$ and $Z_1^{\max}$) are set as the rescue travel time under Strategy 3 and under Strategy 2, respectively. In contrast, the minimal and maximal values of the second, third and fourth objectives ($Z_i^{\min}$ and $Z_i^{\max}$, i = 2, 3, 4) are set according to the corresponding objectives of Strategy 2 and Strategy 3, respectively.

3.1. Traffic control sub-problem

Since the sub-problem is a 0-1 integer programming problem, this paper employs simple genetic algorithms (SGA) to solve it. Each gene is used to represent the implementation or non-implementation of traffic control on the corresponding arc. If the value of the corresponding gene is 1, the arc is traffic controlled, *i.e.* prohibition of the usage by ordinary trips. Otherwise, the arc is not controlled. Three genetic operators of selection, crossover, and mutation are respectively set as: the Monte-Carlo selection method, two-point crossover method and gene mutation method (changing the value of a randomly chosen gene, *i.e.* $1\rightarrow 0$ or $0\rightarrow 1$). The algorithm is stated as follows:

Step 0: **Initialization.** Generate an initial population with p chromosomes. Each chromosome has $|A|$ genes and each gene randomly takes one integer of 0 or 1.

Step 1: **Fitness calculation.** Calculate the fitness values of all chromosomes. The fitness is set to be the reciprocal of weighted sum of normalized values of four objectives.

Step 2: **Selection.**

Step 3: **Crossover.**

Step 4: **Mutation.**

Step 5: **Stop condition test.** If the stop condition is satisfied, the incumbent solution is the optimal solution. If not, go back to Step 3.

3.2. Traffic assignment model of post-disaster ordinary trips

It is assumed that the ordinary trips will choose their routes according to the information of network damage and traffic control arcs. These traffic flows will also cause a burden of network and further affect the decision of selection of rescue path and controlled arcs. Therefore, it should be concurrently taken into account. This paper employs the convex combinations method (also known as Frank-Wolfe method) to conduct the traffic assignment for each O-D pair.

In conducting the traffic assignment, the capacity of controlled arc is set to 0. As for the capacity of the damaged arc, it is multiplied by the defuzzified value of fuzzy reliability. The lower access reliability the arc is, the higher degree its capacity will be affected. The travel time of each arc is calculated basing on BPR equation. The algorithm of traffic assignment is delineated as follows (Sheffi, 1985):

Step 0: **Initialization.** Apply all-or-nothing assignment based on $d_{ij}(0, z_{ij})$ for all *i, j*. This yields $\{ y_{ij}^{1} \}$. Set counter n:=1.

Step 1: **Update**. Set $d_{ij}^{n} = d_{ij}(y_{ij}^{n}, z_{ij})$, for all *i, j*.

Step 2: **Direction finding**. Perform all-or-nothing assignment based on $\{ y_{ij}^{n} \}$. This yields a set of (auxiliary) flows $\{ g_{ij}^{n} \}$.

Step 3: **Line search**. Find η_n that solves $\min_{0 \le \eta_n \le 1} \Phi = \sum_i \sum_j \int_0^{y_{ij}^n + \eta_n (g_{ij}^n - y_{ij}^n)} d(w, z_{ij}) dw$.

Step 4: **Move**. Set $y_{ij}^{n+1} = y_{ij}^{n} + \eta_n^{*} (g_{ij}^{n} - y_{ij}^{n})$ for all *i, j*.

Step 5: **Convergence test**. If $\eta_n^{*} \le \varepsilon$, then stop (the current solution, $\{ y_{ij}^{n+1} \}$, is the set of equilibrium arc flows); otherwise, set *n*:=*n*+1 and go to step 1. ε is an arbitrary small number.

In the above step 2, since $d_{ij}(w,z_{ij}) = t_{ij}^f(1+\alpha(\frac{w}{(1-z_{ij}+\varepsilon)C_{ij}})^\beta)$, Φ becomes:

$$\sum_i\sum_j t_{ij}^f w + \frac{\alpha t_{ij}^f}{((1-z_{ij}+\varepsilon)C_{ij})^\beta(1+\beta)} w^{1+\beta}\Bigg|_0^{y_{ij}^n+\eta_n(g_{ij}^n-y_{ij}^n)} =$$
$$\sum_i\sum_j t_{ij}^f(y_{ij}^n+\eta_n(g_{ij}^n-y_{ij}^n)) + \frac{\alpha t_{ij}^f}{((1-z_{ij}+\varepsilon)C_{ij})^\beta(1+\beta)}(y_{ij}^n+\eta_n(g_{ij}^n-y_{ij}^n))^{1+\beta}$$

The first-order condition of above equation can be derived as:

$$\sum_i\sum_j t_{ij}^f(g_{ij}^n-y_{ij}^n)(1+\alpha(\frac{y_{ij}^n+\eta_n(g_{ij}^n-y_{ij}^n)}{(1-z_{ij}+\varepsilon)C_{ij}})^\beta)=0$$

However, it is hard to derive a closed form in terms of η_n from above equation, thus the optimal value η_n^* is approximately obtained by Newton-Raphson numerical method.

3.3 Shortest rescue path model

This model aims to find the shortest path satisfying the preset fuzzy reliability level. The fuzzy access reliability of an arc is assumed a symmetrical triangle fuzzy number, expressed by:

$$\mu_i(x) = \begin{cases} 1-\dfrac{|\mathrm{x}-\mathrm{c}|}{\delta} & \mathrm{c}-\delta \le \mathrm{x} \le \mathrm{c}+\delta \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

where, c is the center, δ is the spread, and i is the degree of linguistic variable. Considering five degrees, for instance, the linguistic variables can be very poor, poor, normal, good and excellent corresponding to the five degrees of damage that are disastrous damage, severe damage, moderate damage, slight damage, and no damage, respectively. In practice, the values of c and δ, even the shapes of the membership functions, of various degrees of linguistic variables can be calibrated in advance by interviewing the experts. Usually, a lower value of c indicates the roadway more seriously damaged and a larger value of δ reflects the higher uncertainty in evaluating the level of damage of roadway.

Numerous algorithms have been proposed to find the shortest path based on additional constraints, also known as the "constrained shortest path problem" (CSPP) (Handler and Zang, 1980; Aneja *et al.*, 1983; van der Zijpp and Catalano, 2005). However, it is hard to incorporate the fuzzy constraint into the path finding procedure. In this paper, an exact algorithm of finding the K-shortest paths between a single origin and a single destination proposed by Azevedo *et al.* (1993) is employed. This algorithm is revised from the algorithm of Martins (1984). The core logic of Martins' algorithm is based on a path deletion algorithm used to remove a shortest path (only the very first arc in the shortest path) from the network once it is determined. At the same time, new nodes (the intermediate nodes in the shortest path) with outgoing arcs are added to the network. This results in an enlarged network where all the paths but the deleted one can be determined. As such, a sequence $\{G_1, G_2,\ldots, G_K\}$ of networks is defined. G_1 is the original network, $G(N, A)$, and its K^{th}-shortest path is determined from the shortest path of G_K. Basically, Azevedo's algorithm follows the core logic of Martins' algorithm, but it deletes the last arc of shortest path and adds the new nodes with incoming arcs to the network. This can avoid the K-1 executions of the shortest path algorithm and thus largely enhance the solving efficiency. To the best of author's knowledge, Azevedo's algorithm is the most efficient shortest path, it is employed to compute certain number (K) of shortest paths and the shortest one that satisfies the constraint defined by fuzzy number ranking method (*i.e.* total score method adopted in this paper) is selected.

Azevedo's algorithm is narrated as follows:

Step 0: **Initialization.** Determine a shortest path $P_1=\{s=s_0, (s_0, s_1), s_1,\ldots, s_{j-1},(s_{j-1}, s_j), s_j=t\}$ in $G_1(N, A)$. Set k =1. s and t are origin and destination terminal nodes, respectively. $\{s_1, s_2,\ldots, s_{j-1}\}$ are intermediate nodes.

Step 1: **Path deleting**. Delete P_k from $G_k(N_k, A_k)$ and create an enlarged $G_{k+1}(N, A)$ by adding the set of $\{s_1', s_2',\ldots, s_{j-1}'\}$ intermediate (new) nodes to the network. The sets of the incoming arcs of each new node are defined as:

$I(s_1') \leftarrow \{(v,s_1')|(v,s_1) \in I(s_1)\ and\ v \neq s_0\}$ and

$I(s_i') \leftarrow \{(v,s_i')|(v,s_i) \in I(s_i)\ and\ v \neq s_{i-1}\} \cup \{s_{i-1}',s_i'\}$, for i=2,…,j-1.

The set of incoming arcs of node t is updated as:

$I(t) \leftarrow I(t) - \{(s_{j-1},t)\} \cup \{(s_{j-1}',t)\}$

Step 2: **Label updating**. After deleting P_k and creating an enlarged network G_k, the labels of primed nodes and new nodes are updated according to each one of the equations:

$\pi(s'_i) = \min\{\pi(v) + c(v, s'_i) | \forall (v, s'_i) \in I(s'_i)\}$,

$\forall i \in \{1, ..., j-1\}$ and $\pi(t) = \min\{\pi(v) + c(v,t) | \forall (v,t) \in I(t)\}$

where, $\pi(v)$ is the shortest distance (or travel time, or travel cost) from s to v. $\xi(v)$ is the predecessor node of v in the corresponding shortest path. $c(v,t)$ is the distance of arc (v, t). Compute $\{\pi(s'_i), \xi(s'_j)\}$ and $\{\pi(t), \xi(t)\}$.

Step 3: **Shortest path finding.** If ($\pi(t) < \infty$ and $k < K$), then the shortest path p in G_k corresponds to p_k in G_1. Set $k = k+1$ and go back to Step 1. Otherwise, terminate. The K-shortest paths have been found (if $k = K$), or there do not exist K-shortest paths in the network (if $k < K$).

3.4. The integrated algorithm

Since these three sub-problems are closely inter-related, they cannot be solved independently and have to be integrated. The integrated algorithm is as follows:

Step 1: **Generate an initial population with *p* chromosomes.** Each chromosome has $|A|$ genes, A is a set of all arcs in the network under consideration, and each gene randomly takes one integer of 0 or 1.

Step 2: **Calculate the fitness value for each chromosome.**

Step 2-1: Determine the number of police officers required according to the number of controlled arcs.

Step 2-2: Determine the remained capacity of each arc. Let the capacity of controlled arc equal to 0 and capacity of damaged arc equal to its original capacity multiplied by defuzzified reliability value. Conduct the traffic assignment of ordinary trips and determine the total detour travel time and number of unconnected trips.

Step 2-3: Based on the traffic assignment results, determine the shortest path from K-shortest paths with satisfaction of preset required reliability level.

Step 2-4: The fitness is set to be the reciprocal of weighted sum of normalized values of four objectives.

Step 3: **Selection.**

Step 4: **Crossover.**

Step 5: **Mutation.**

Step 6: **Stop condition test.** If the stop condition is satisfied, the incumbent solution is the optimal solution. If not, go back to Step 2.

4. Computational experiments

4.1. Exemplified case and parameter settings

To evaluate the performance of proposed model, a study on an exemplified case is performed. A network composed of 50 nodes and 85 arcs, as depicted in Figure 3, is tested. Let node 1 be the relief center from which the rescue personnel depart and let node 50 represent the affected area. The capacity of the arc with bold line is assumed 3500 pcu/hr and the arc with narrow line is 2500 pcu/hr. The fuzzy reliabilities of all arcs after the disaster are systematically set according to the principle "the closer the arc to the affected area, the lower center and the larger spread of its fuzzy reliability." The free flow speed of all arcs is assumed 60 km/hr. The length of each arc is randomly set within a range of 1 to 10 km. The parameters of BPR function are set as α= 0.15 and β = 4.0. The weights of four objectives are set as w_1=0.6, w_2=0.2, w_3=0.1, and w_4=0.1. The number of police officers required to enforce traffic control on every arc is set as 2 persons. The required fuzzy reliability for rescue path is set as "good." The stop condition of the integrated algorithm is set as 200 generations.

To select a reasonable setting of the GA parameters, the model performances with different combinations of crossover rate and mutation rate settings are compared in Table 1. Note that the model outperforms at crossover rate=0.8 and mutation rate=0.01 and 0.02; thus, crossover rate of 0.8 and mutation rate of 0.01 are used for the following analysis.

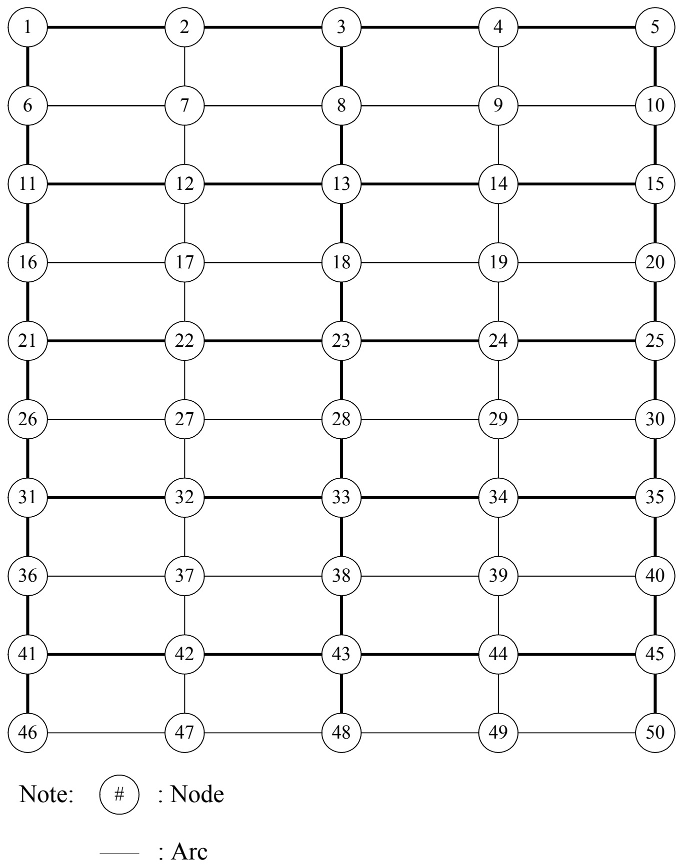

Figure 3. The network of exemplified case (not to scale)

Table 1. Performances of the model with different combinations of parameters settings on exemplified case

Parameters		Objective values	Ranking
Crossover rate	Mutation rate		
0.7	0.01	0.283	8
0.7	0.02	0.281	6
0.7	0.03	0.272	4
0.8	0.01	0.265	1
0.8	0.02	0.265	1
0.8	0.03	0.266	3
0.9	0.01	0.279	5
0.9	0.02	0.292	9
0.9	0.03	0.281	6

4.2. Results and Comparisons

The evolution progress of the proposed algorithm is depicted in Figure 4. Note that the proposed algorithm converges after 100 generations progressed and the weighted-sum objective value decreases from 1.591 to 0.265. The CPU searching time, on Intel Core 2 Duo E 6400 2.13G, is 1347.21 seconds (approximately 22.5 minutes), which is in the tolerance of disaster relief operations.

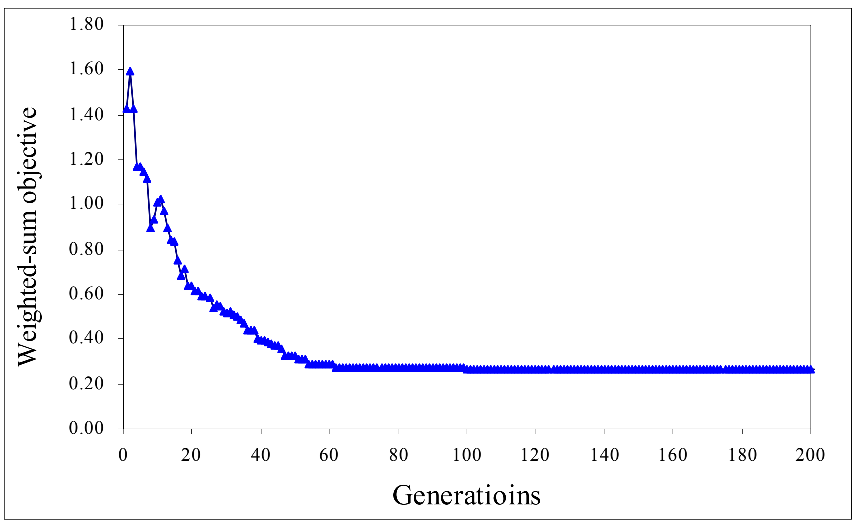

Figure 4. The evolution progress of proposed algorithm on exemplified case

The optimal rescue path and controlled arcs are depicted in Figure 5. The optimal rescue path consists of 13 arcs: node 1→ node 2→ node 3→ node 8→ node 9→ node 14→ node 15→ node 20→ node 25→ node 30→ node 35→ node 40→ node 45→ node 50 with travel time of 49.71 minutes. A total of 9 controlled arcs are selected for traffic control, which are (2, 3), (3, 8), (8, 9), (9, 14), (14, 15), (15, 20), (25, 30), (30, 35), and (35, 40) wherein 18 police officers are required. It is noteworthy that not all arcs on rescue path require traffic control (only 9 out of 13 arcs, approximately 69%) and that all controlled arcs are in the rescue path. The total of detour travel time caused by traffic control is 504.11 vehicle-hours. No unconnected trips caused by traffic control are found.

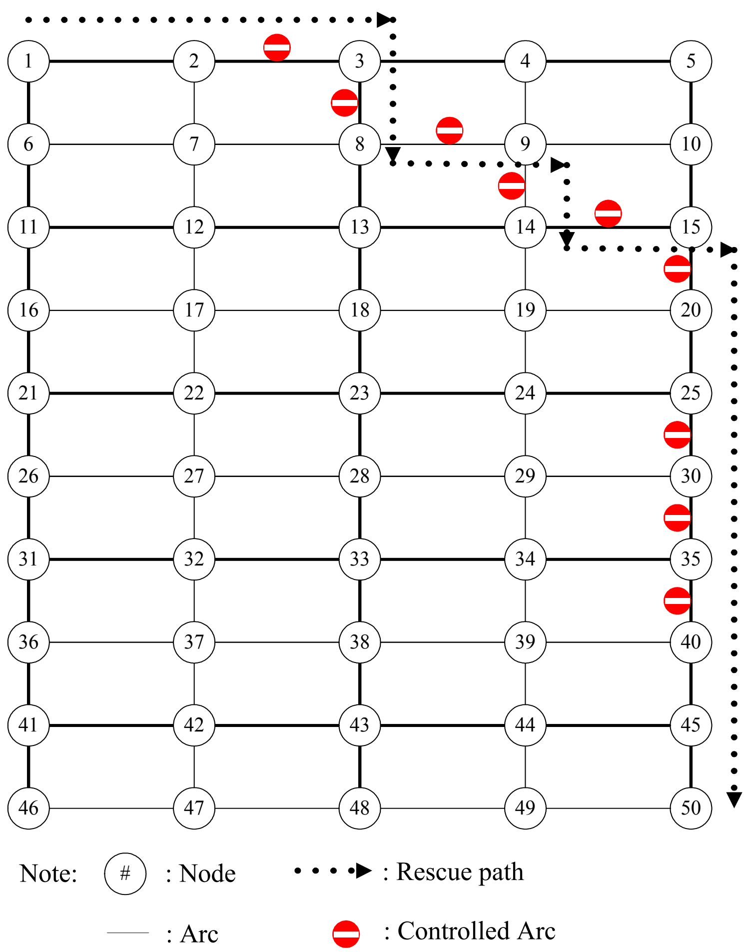

Figure 5. Optimal rescue path and controlled arcs of exemplified case

The performances of three strategies are summarized in Table 2. In terms of rescue travel time, Strategy 3 outperforms with 49.10 minutes, followed by Strategy 1 with only a slightly higher value of 49.71 minutes (only 1.24% increased). However, in case of rescue operations, the travel time for Strategy 2 will be dramatically increased from 49.10 minutes to 132.55 minutes, suggesting the importance of traffic control. Strategy 2 does not implement any traffic control, it will not cause any inconvenience to the ordinary trips. In contrast, Strategy 3 with 13 controlled arcs requiring 26 police officers will cause 527.81 veh-hr delays for the ordinary trips. Strategy 1 with only 9 controlled arcs necessitating 18 police officers will lead to lower delays (504.11 veh-hr) to the ordinary trips. In sum, Strategy 1 is the optimal strategy with highest fitness value of 3.78, followed by Strategy 3 with fitness value of 3.33. Strategy 2 is the worst one with fitness value of only 1.67.

Table 2. Performances of three strategies on exemplified example

Performances	Strategy 1 (selective control)	Strategy 2 (without control)	Strategy 3 (absolute control)
Rescue travel time (Length of rescue path)	49.71 min (49.1 km)	132.55 min (54.3 km)	49.10 min (49.1 km)
Detour travel time	504.11 veh-hr	0.00 veh-hr	527.81 veh-hr
Number of unconnected trips	0 trip	0 trip	0 trip
Number of police officers required (Number of controlled arcs)	18 persons (9 arcs)	0 person (0 arc)	26 persons (13 arcs)
Fitness value (Objective value)	3.78 (0.265)	1.67 (0.600)	3.33 (0.300)

5. Real case study

To demonstrate the applicability of the proposed model, a real case study based on the Chi-Chi earthquake in Taiwan is conducted. Three relief strategies are also compared.

5.1 Data explanation and parameter settings

The Chi-Chi earthquake, measuring 7.3 on the Richter scale, struck the middle region of Taiwan on September 21, 1999, killed more than 2500 people and damaged numerous bridges, highways and buildings. The proposed model is applied to the seriously damaged area of Nantou County (including Nantou City and Caotun Township). The network, with a total of 55 nodes and 91 arcs, is depicted in Figure 6. It is given that the rescue teams depart from node O (in Caotun Township) to node D (in Nantou City). The information of damaged network is assumed based on the impedance risk coefficient (η) proposed by Lu (2000). The centers of the fuzzy connective reliability of corresponding arcs are assumed 1-η. The spread of the fuzzy connective reliability are assumed α(1-η), where α is a parameter between [0,1]. Higher value of α represents closer to node D in reflecting higher uncertain information as the arc is closer to the affected area.

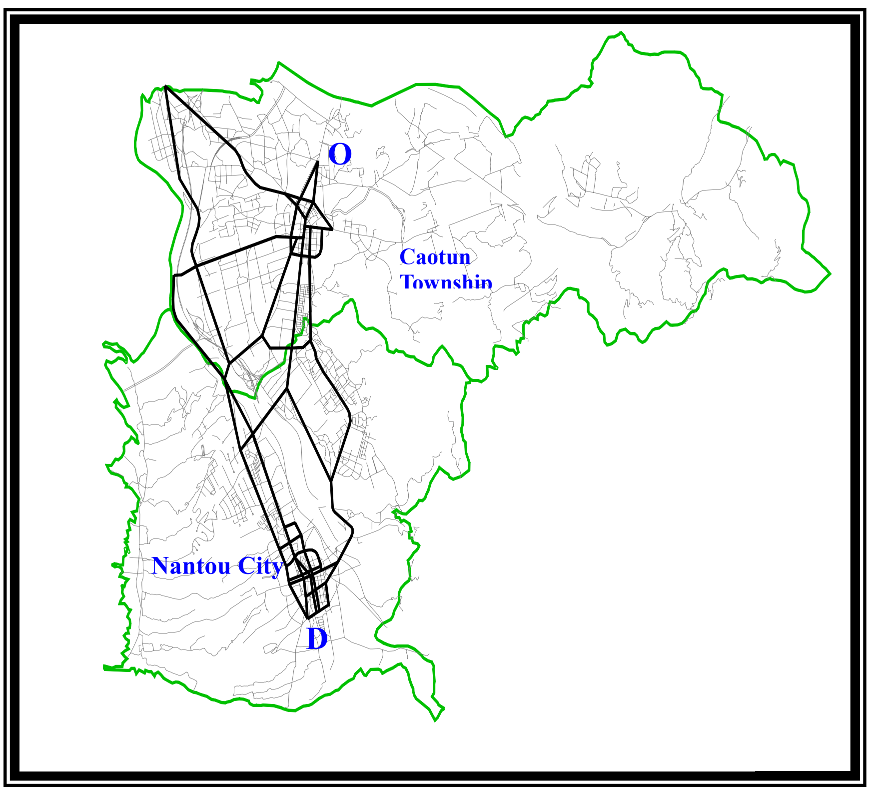

Figure 6. The network of central area of Nantou County

Different parameter settings on crossover and mutation rates are also tested in Table 3. Note that there are four settings which can achieve the best results: P_c=0.7 and P_m=0.02, P_c=0.8 and P_m=0.01, P_c=0.8 and P_m=0.02, P_c=0.9 and P_m=0.01, thus the parameters are set as P_c=0.8 and P_m=0.01 in the following analysis.

Table 3. Performances of Chi-Chi earthquake case under different parameters settings

Parameters		Objective value	Rank
Crossover rate	Mutation rate		
0.7	0.01	0.292	8
0.7	0.02	0.274	1
0.7	0.03	0.281	5
0.8	0.01	0.274	1
0.8	0.02	0.274	1
0.8	0.03	0.292	8
0.9	0.01	0.274	1
0.9	0.02	0.284	6
0.9	0.03	0.287	7

5.2 Results and comparisons

The CPU computation time for obtaining the optimal solution of the real case is 955.44 seconds (approximately 15.9 minutes). The results are shown in Figure 7 and Table 4. The travel time of rescue path is 7.12 minutes and 10 arcs are controlled by 20 police officers causing a total of 498.25 veh-hr detour travel time. The performances of three strategies are also compared in Table 4. Although the travel distances are almost equal in these three strategies, Strategy 1 and Strategy 3 have significantly shorter travel times (7.02 and 7.12 minutes) as compared to Strategy 2 (42.13 minutes). In terms of detour travel time and the number of police officers required, Strategy 3 has caused 511.32 veh-hr detour travel time and required 26 police officers to enforce traffic control, whereas Strategy 1 has caused detour travel time of 498.25 veh-hr and only 20 police officers are required. Overall, Strategy 1 outperforms with a fitness value of 3.66, followed by Strategy 3 with fitness value of 3.33. Strategy 2 has the least fitness value of 1.67.

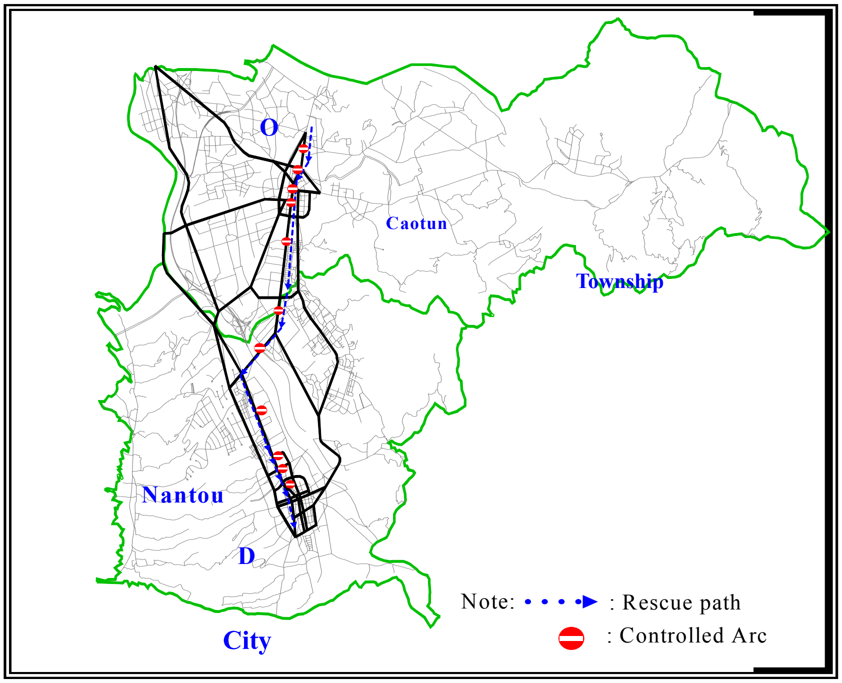

Figure 7. Optimal rescue path and controlled arcs of Chi-Chi earthquake study

Table 4. Performances of three strategies on Chi-Chi earthquake study

Performances	Strategy 1	Strategy 2	Strategy 3
Rescue travel time (Length of rescue path)	7.12 min (7.02 km)	42.13 min (8.4 km)	7.02 min (7.02 km)
Detour travel time	498.25 veh-hr	0.00 veh-hr	511.32 veh-hr
Number of unconnected trips	0 trip	0 trip	0 trip
Number of police officers required (Number of controlled arcs)	20 persons (10 arcs)	0 person (0 arc)	26 persons (13 arcs)
Fitness value (Objective value)	3.66 (0.274)	1.67 (0.600)	3.33 (0.300)

5.3 Weights analysis

To analyze the sensitivity of the weights, we hold the weights of the third and fourth objectives unchanged (w_3=w_4=0.1) and vary the weights of the first objective from 0.8 to 0.0. The key performance indexes are reported in Table 5. Note that as the weight of the first objective is greater than or equal to 0.5, the rescue path and the number of controlled arcs remain the same as Strategy 1 with original weights of (0.6, 0.2, 0.1, 0.1). Once the weight is smaller than or equal to 0.3, the rescue path and the number of controlled arcs are the same as Strategy 2 with original weights. It is worthy to notice that with the weights of (0.4, 0.4, 0.1, 0.1) the optimal rescue path remains the same as Strategy 1, but only 8 arcs are required for traffic control. The Pareto solutions provided in Table 5 are useful to decision makers of relief operations than a single solution solved in the previous subsection.

Table 5. Sensitivity analysis of various weights of the first two objectives

Perform-ances	w_1: w_2 (w_3=w_4=0.1)								
	0.8:0.0	0.7:0.1	0.6:0.2	0.5:0.3	0.4:0.4	0.3:0.5	0.2:0.6	0.1:0.7	0.0:0.8
Rescue travel time (min)	7.12	7.12	7.12	7.12	12.57	42.13	42.13	42.13	42.13
Detour travel time (veh-hr)	498.25	498.25	498.25	498.25	384.13	0	0	0	0
Number of unconnected trips (trips)	0	0	0	0	0	0	0	0	0
Number of controlled arcs (arcs)	10	10	10	10	8	0	0	0	0

6. Concluding remarks

This study developed an integrated multi-objective model to determine the optimal rescue path and controlled arcs under uncertainty environments. Our model is composed of three sub-models: rescue shortest path model, post-disaster traffic assignment model, and control arcs selection model to minimize four objectives: travel time of rescue path, detour travel time, number of unconnected trips of non-victims, and number of required police officers. A GA-based algorithm is proposed to solve the problem. An exemplified case and a real case on the Chi-Chi earthquake in Taiwan were conducted to demonstrate the applicability of the model. The model performances of selective traffic control strategy and two extreme strategies, without traffic control and absolute traffic control, are compared. The results consistently demonstrate that the proposed model can not only curtail the demand for police officers in enforcing the traffic control but also reduce the impacts to the ordinary trips, with only a small increment in travel time of rescue team as compared to the absolute traffic control strategy. This corroborates the applicability of the proposed model and its efficiency. Besides, a sensitive analysis on various weights of the weighted objective function is also conducted. The results show that as the weight of the first objective is greater than or equal to 0.5, the rescue travel time is 7.12 minutes and 10 arcs are controlled (same as Strategy 1). As the weight is smaller than or equal to 0.3, the rescue travel time is 42.13 minutes and no traffic control is needed (same as Strategy 2). The Pareto optimum is at the weight of 0.4, wherein the rescue travel time is 12.57 minutes and only 8 arcs require traffic control.

Several directions for future studies can be identified. Firstly, the fuzzy access reliabilities of arcs are systematically provided in this paper. The information collection and synthesis from different resources, such as victims, road authority, and rescue team, are urgently required and should be completed in a more efficient way right after the disaster happens. Secondly, the ordinary O-D trips are given externally and fixed in this paper, but in fact the number of O-D trips can be largely affected by the disaster. It is worthy to develop a model to forecast the post-disaster traffic demand and distribution. Thirdly, the affected capacities of arcs are determined by multiplying the defuzzified value of fuzzy reliabilities of corresponding arcs in the procedure of traffic assignment. A traffic assignment model with consideration of fuzzy reliability deserves further exploring. Fourthly, for simplification, this paper considers only the one-to-one rescue operation, i.e., from one supply point to a specific affected area. In real-world circumstances, the rescue resources may come from several supply points and the affected points can be geographically scattered over the disaster region. Thus, many-to-many shortest rescue paths should be further studied. Fifthly, the decision makers of the disaster relief operations are often concerned with the Pareto (or most effective) solutions instead of a single optimal solution. Multi-objective genetic algorithms (*e.g.* niched Pareto genetic algorithm, Horn *et al.*, 1994; non-dominated sorting genetic algorithms, Srinivas and Deb, 1994 and Deb *et al.*, 2002) can be easily applied to this problem to obtain the Pareto solutions for relief operations. Sixthly, the CPU times of the proposed algorithm are 1347.21 sec and 955.44 sec for the exemplified case and the real case, respectively. For the promptness of relief operations, improving the efficiency of the proposed algorithm should be further attempted. Last but not least, the information of the degrees of damages and the travel patterns of ordinary trips might change as time evolves, thus a dynamic mathematical programming model combined with dynamic traffic assignment and dynamic information requires further exploration.

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