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Reliability assessment of an aircraft propulsion system using IFS and OWA tree

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In conventional system reliability analysis, the failure probabilities of components of a system are treated as exact values when the failure probability of the entire system is estimated. However, it may be difficult or even impossible to precisely determine the failure probabilities of components as early as the product design phase. Therefore, an efficient and simplified algorithm to assess system reliability is needed. This article proposes a deductive top-down estimation methodology, which combines intuitionistic fuzzy set (IFS) and ordered weighted averaging (OWA) operators to evaluate system reliability. A case of an aircraft propulsion system from an aerospace company is presented to further illustrate the proposed approach. After comparing results from the proposed method and two other approaches, this research found that the proposed approach provides a more accurate and reasonable reliability assessment.

Keywords: reliability assessment; ordered weighted averaging; OWA tree; intuitionistic fuzzy set

1. Introduction

Product reliability is one of the most important factors in determining the quality and competitiveness of a product. In conventional reliability theories, it is assumed that components and systems have only two abrupt states: good or bad. This implies that success and failure are precisely defined and that there is no intermediate state between them. However, in real-world applications, the collected data or system parameters are often fuzzy or imprecise because of incomplete or unobtainable information. Under these circumstances, conventional probability-based reliability analysis is inadequate to account for such built-in uncertainties in data. In order to handle the hurdles that are posed by insufficient information, the fuzzy approach is often used to evaluate failure rate status.

Fuzzy reliability theory uses a fault-tree technique to obtain a system's reliability. Related studies include Tanaka *et al.* (1983), who use the trapezoidal fuzzy number to replace probability and apply fault-tree analysis to obtain the system's fault interval. Singer (1990) uses fuzzy numbers to represent the relative frequencies of basic events. He demonstrates use of the *n*-array possibilistic AND, OR and NEG operators to construct possible fault-trees. Recently, Chang *et al.* (2006) proposed a vague fault-tree analysis procedure to determine a weapon system's reliability. Their

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approach integrated experts' knowledge and experience in terms of providing the possibility of failure of bottom events, and used a triangular vague set to perform the calculation. In fault-tree analysis, AND-gates and OR-gates are used to represent independent and mutually exclusive relationships between events, respectively. To generalize this idea, Yager (1988) introduced a new family of aggregation techniques called the ordered weighted average (OWA) operators, which is a general mean type aggregator that provides a flexible aggregation operation that ranges between the minimum and maximum operators. The OWA tree is useful in illustrating the mode of occurrence of an accident logically. When the failure probabilities of system components are given, the probability of the whole system can be calculated.

O'Hagan (1988) developed a procedure to generate the OWA weights for a given degree of orness α , to maximize entropy. Many related studies have been published in recent years. For example, Fuller and Majlender (2001) used Lagrange multipliers to derive a polynomial equation to determine the optimal weighting vector by solving a constrained optimization problem. Sadiq and Tesfamariam (2007) utilized probability density functions to generate OWA weight distributions.

Schneier (1999) proposed attack trees to analyse the security of systems and sub-systems. The attack tree is a formal methodology that provides a way to consider security, capture and reuse expert expertise regarding security, and respond to changes in security. In the product design phase, when the malfunction data of the system elementary event are incomplete, the conventional approach to calculate reliability is no longer applicable. Huang *et al.* (2004) proposed the posbist fault-tree analysis method to find a system's reliability by redefining AND and OR operators based on the minimal cut of a posbist fault-tree. However, their method selects the maximal failure probability of the bottom event, which can result in biased conclusions. To solve this problem, this article proposes a new reliability method that collects expert knowledge and experience on the problem domain to build the possibility of failure of leaf nodes through combining intuitionistic fuzzy set (IFS) and OWA operators to evaluate system reliability. A malfunction of an aircraft propulsion system is presented as a case study to further illustrate the proposed method. This article also compares the proposed approaches with several other listed methods.

The rest of this article is organized as follows. Section 2 introduces the basic definition and some operations of the OWA operators, the methodology that is used to determine OWA weights and the OWA tree. In Section 3, the definition of the IFS and its operations are introduced. Section 4 presents the proposed approach, which combines the IFS and the OWA tree for reliability assessment. An example that is drawn from an aircraft propulsion system is used with the proposed approach for reliability assessment in Section 5. Section 6 concludes the article.

2. OWA operators and its operations

2.1. OWA operators

The concept of OWA operators was first introduced by Yager (1988) with regards to the problem of aggregating multi-criteria to form an overall decision function. OWA operators can provide an aggregation that lies between these two extremes (between the AND and OR situations), so it is a more compatible human thought than other operators (see Figure 1).

Yager (1988) proposed an OWA operator that was able to obtain optimal weights of attributes based on the rank of these weighting vectors after an aggregation process (see Definition 1).

DEFINITION 1 *An OWA operator of dimension n is a mapping $F : R^n \rightarrow R$, which has an associated n weighting vector $W = [w_1, w_2, \dots, w_n]^T$ that has the properties*

$$\sum_i w_i = 1, \quad \forall w_i \in [0, 1], \quad i = 1, \dots, n,$$

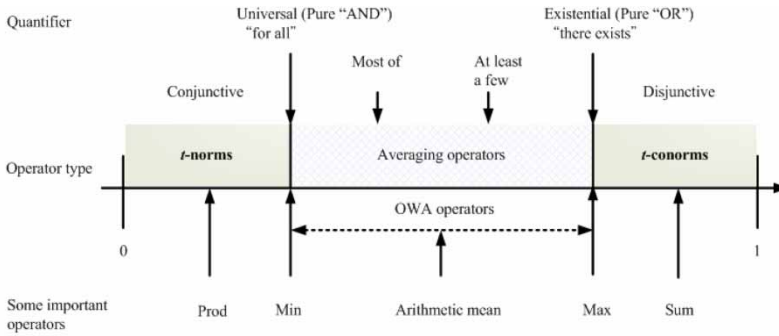


Figure 1. Common aggregation operators (Sadiq and Tesfamariam 2007).

such that

$$f(a_1, a_2, \dots, a_n) = \sum_{i=1}^n w_i b_i, \tag{1}$$

where b_i is the i th largest element in the vector (a_1, a_2, \dots, a_n) and $b_1 \geq b_2 \geq \dots \geq b_n$.

A number of special cases of this operator are illustrated in the following instances. If the components in W are such that $w_1 = 1$ and $w_j = 0$ for all $j \neq 1$, we get $OWA(a_1, a_2, \dots, a_n) = \text{Max}_j[a_j]$. This weighting vector is denoted as W^* . If the weights are $w_n = 1$ and $w_j = 0$ for $j \neq n$, one gets $OWA(a_1, a_2, \dots, a_n) = \text{Min}_j[a_j]$. This weighting vector is denoted W_* . If the weights are $w_j = 1/n$ for all j , denoted as W_n , then $OWA(a_1, a_2, \dots, a_n) = (1/2) \sum_{j=1}^n a_j$.

Yager (1988) introduced two important characterizing measures with respect to the weighting vector W of an OWA operator. One of these two measures is *orness* of the aggregation, which is defined as follows.

DEFINITION 2 Assume F is an OWA aggregation operator with weighting function $W = [w_1, w_2, \dots, w_n]$. The degree of orness associated with this operator is defined as

$$\text{orness}(W) = \frac{1}{n-1} \sum_{i=1}^n (n-i)w_i, \tag{2}$$

where $\text{orness}(W) = \alpha$ is a situation parameter.

The second is a measure of *dispersion* of the aggregation, defined in Definition 3.

DEFINITION 3 Let W be a weighting vector with elements $w_1 \dots w_n$. The measure of dispersion of W is defined as

$$\text{dispersion}(W) = - \sum_{i=1}^n w_i \ln w_i. \tag{3}$$

The dispersion measure of W takes into account all information in the aggregation. It is really a measure of entropy, which implies the following properties:

- (1) if $w_i = 1$ for some i , then the $\text{dispersion}(W) = 0$, a minimum value.
- (2) if $w_i = 1/n$ for all i , then the $\text{dispersion} = \ln n$, a maximum value.

O'Hagan (1988) combined the principle of maximum entropy and OWA operators to propose a particular OWA weight that has maximum entropy with a given level of orness. This approach is based on the solution of the following mathematical programming problem:

$$\begin{aligned}
 &\text{Maximize : } - \sum_{i=1}^n w_i \ln w_i \\
 &\text{Subject to : } \frac{1}{n-1} \sum_{i=1}^n (n-i)w_i = \alpha, \quad 0 \leq \alpha \leq 1, \\
 &\sum_{i=1}^n w_i = 1, \quad 0 \leq w_i \leq 1, \quad i = 1, \dots, n
 \end{aligned} \tag{4}$$

2.2. Determination of OWA weights

Fuller and Majlender (2001) used the method of Lagrange multipliers to transfer Yager's OWA equation to derive a polynomial equation that can determine the optimal weighting vector under maximal entropy. By their method, the associated weighting vector is obtained by Equations (5)–(7).

$$\ln w_j = \frac{j-1}{n-1} \ln w_n + \frac{n-j}{n-1} \ln w_1 \implies w_j = \sqrt[n-1]{w_1^{n-j} w_n^{j-1}} \tag{5}$$

and

$$w_n = \frac{((n-1)\alpha - n)w_1 + 1}{(n-1)\alpha + 1 - nw_1} \tag{6}$$

then

$$w_n[(n-1)\alpha + 1 - nw_1]^n = ((n-1)\alpha)^{n-1} \cdot [((n-1)\alpha - n)w_1 + 1] \tag{7}$$

where w is a weight vector, n is the number of attributes and α is the situation parameter.

The optimal value of w_1 should satisfy Equation (7). Once w_1 is obtained, then w_n can be determined from Equation (6), and the other weights are obtained from Equation (5). In a special case, when $w_1 = w_2 = \dots = w_n = 1/n$, then $dispersion(W) = \ln n$, which is the optimal solution to Equation (4) when $\alpha = 0.5$.

2.3. OWA tree

In many real-world applications, AND/OR nodes are not sufficient to represent the sophisticated relationship between a parent node and its children. Yager (2006) proposed an OWA node as an extension of the AND/OR node and its use to generalize AND/OR trees to OWA trees. As opposed to an AND/OR node, an OWA node is characterized by a vector W , called an OWA weighting vector. The dimension of vector W is equal to the number of children, n . Furthermore, the components of W , w_j , called the OWA weights, must satisfy the following two conditions: (1) $0 \leq w_j \leq 1$ and (2) $\sum_j w_j = 1$.

An OWA tree is generally initiated by a single node called the root node, and each path is terminated by a leaf node (no children). The OWA tree includes the AND node, OR node and OWA node. In an AND node (see Figure 2), accomplishment of the parent goal requires the success of all of the children, i.e. $P_{\text{and}} = \prod_{j=1}^n p_j$, which is the product of the probability of accomplishment of all children. In an OR node (see Figure 2), the accomplishment for the parent goal requires the success of any one of the children, i.e. $P_{\text{or}} = 1 - \prod_{j=1}^n (1 - p_j)$, which is the product of the probability of accomplishment of any one of the children.

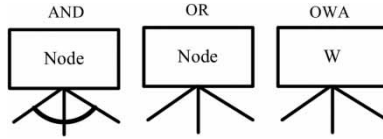


Figure 2. AND, OR and OWA nodes.

The OWA node allows this research to model situations in which there is some probabilistic uncertainty in the number of children that need be satisfied. An OWA node is defined as $P_{OWA} = \sum_{j=1}^n w_j R_j$ (see Figure 2). Here, w_j indicates the probability that it must accomplish j sub-tasks to satisfy the parent, and R_j indicates the probability that the required j sub-tasks are satisfied. To succinctly express the form of R_j , the following notation is introduced. With p_i being the probability of the i th child succeeding, it assumes $pid(k)$ to be the index of the child with the k th highest probability of success. It follows that $p_{pid(k)}$ is the k th largest probability of success of a child. Using this, one gets $R_j = \prod_{k=1}^j p_{pid(k)}$, which is the product of the j largest probabilities.

3. IFS and its operations

3.1. IFS

Zadeh (1965) proposed fuzzy sets to describe fuzzy phenomena under a specific attribute. A fuzzy set A of the universe of discourse U , $U = \{u_1, u_2, \dots, u_n\}$, is a set of ordered pairs, $\{(u_1, \mu_A(u_1)), (u_2, \mu_A(u_2)), \dots, (u_n, \mu_A(u_n))\}$, where μ_A is the membership function of the fuzzy set A , $\mu_A : U \rightarrow [0, 1]$, and $\mu_A(u_i)$ indicates the grade of membership of u_i in $A : \forall u_i \in U$. The membership value $\mu_A(u_i)$ is a single value between 0 and 1 that combines the evidence for $u_i \in U$ and the evidence against $u_i \in U$, without indicating how much there is of each. Thus, Atanassov (1983) presented the concepts of the IFS. The notion of the IFS was introduced for the first time by Atanassov in 1983 as a generalization of an ordinary Zadeh fuzzy set. An IFS A for a given underlying set E is represented by a pair $\langle \mu_A, \nu_A \rangle$ of functions $E \rightarrow [0, 1]$. For $x \in E$, $\mu_A(x)$ gives the degree of membership to A , and $\nu_A(x)$ gives the degree of non-membership; moreover, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ must hold.

The uncertainty of x can be described as the differential value of $(1 - \nu_A(x)) - \mu_A(x)$. If the differential value is small, the value of x is more certain. If the differential value is great, the computation is more uncertain about x . When $1 - \nu_A(x) = \mu_A(x)$, the IFS A regresses to a fuzzy

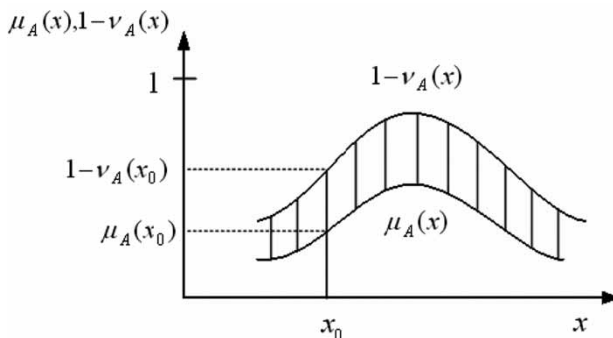


Figure 3. IFS explanation of a real number R .

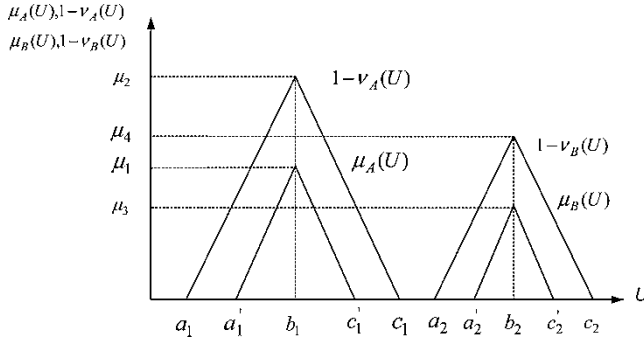


Figure 4. Triangle IFS A and B.

set. Obviously, when $1 - v_A(x) = \mu_A(x) = 1$ or $1 - v_A(x) = \mu_A(x) = 0$, the IFS A regresses to a crisp set. From the above results, crisp sets and fuzzy sets can be viewed as special cases of IFS. Figure 3 shows an IFS explanation of a real number R .

3.2. Arithmetic operations of triangle IFS

From the definition of the triangle IFS (Lee 1998), this article proposes four arithmetic operations for the triangle IFS. Let A and B be two IFSs, as shown in Figure 4 (Lee 1998). If $\mu_A \neq \mu_B$ and $v_A \neq v_B$, then the arithmetic operations are defined as:

$$A = \langle [(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle \tag{8}$$

$$B = \langle [(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle \tag{9}$$

$$\begin{aligned} A(+)B &= \langle [(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle (+) \langle [(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle \\ &= \langle [(a'_1 + a'_2, b_1 + b_2, c'_1 + c'_2); \min(\mu_1, \mu_3)], [(a_1 + a_2, b_1 + b_2, c_1 + c_2); \min(\mu_2, \mu_4)] \rangle \end{aligned} \tag{10}$$

$$\begin{aligned} A(-)B &= \langle [(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle (-) \langle [(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle \\ &= \langle [(a'_1 - a'_2, b_1 - b_2, c'_1 - c'_2); \min(\mu_1, \mu_3)], [(a_1 - a_2, b_1 - b_2, c_1 - c_2); \min(\mu_2, \mu_4)] \rangle \end{aligned} \tag{11}$$

$$\begin{aligned} A(\times)B &= \langle [(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle (\times) \langle [(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle \\ &= \langle [(a'_1 a'_2, b_1 b_2, c'_1 c'_2); \min(\mu_1, \mu_3)], [(a_1 a_2, b_1 b_2, c_1 c_2); \min(\mu_2, \mu_4)] \rangle \end{aligned} \tag{12}$$

$$\begin{aligned} A(/)B &= \langle [(a'_1, b_1, c'_1); \mu_1], [(a_1, b_1, c_1); \mu_2] \rangle (/) \langle [(a'_2, b_2, c'_2); \mu_3], [(a_2, b_2, c_2); \mu_4] \rangle \\ &= \langle [(a'_1/c'_2, b_1/b_2, c'_1/a'_2); \min(\mu_1, \mu_3)], [(a_1/c_2, b_1/b_2, c_1/c_2); \min(\mu_2, \mu_4)] \rangle \end{aligned} \tag{13}$$

When $a_1 = a'_1, c_1 = c'_1$ and $a_2 = a'_2, c_2 = c'_2$, the IFS of its four arithmetic operations will be easier.

3.3. Defuzzification of the triangle IFS

Defuzzification is the procedure that generates a crisp value out of one or more given fuzzy sets. There are several defuzzification algorithms that have been developed. According to Wang and Louh (2000), it has been demonstrated that for any triangular shape, the centroid position

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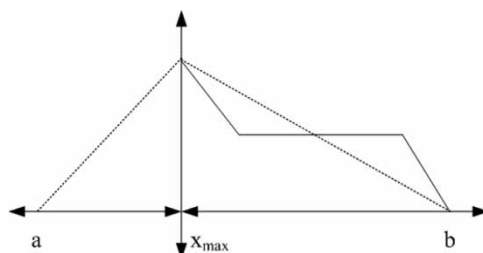


Figure 5. The output fitted to triangular shapes.

could be computed as depicted in Equation (14). This method reduces the computational time by decomposing the output in triangles and trapezoids, and then computes and combines its centroids. The triangular centroid approximation is shown in Figure 5.

$$\bar{x}_t = \frac{1}{3}(a + x_{\max} + b) \quad (14)$$

The proposed approach modifies the definition proposed by Wang and Luoh (2000) and redefines the centroid position of the triangular shape IFS as shown in Equation (15). The triangle IFS is shown in Figure 4.

$$\bar{x}_t = \frac{1}{6}(a + a' + 2b + c' + c). \quad (15)$$

4. Proposed combination of IFS and OWA tree approach

Huang *et al.* (2004) proposed a posbist fault-tree analysis based on posbist reliability theory. They define the AND operator and the OR operator based on the minimal cut of a posbist fault-tree to determine a system's reliability. This method has the advantages of evaluating the probability of failure in a system when historical data are scarce or the failure probability is extremely small. However, the method by Huang *et al.* selects the maximal failure probability of the bottom event and may obtain a biased conclusion. In the system design phase, the collected data or system parameters are often vague due to incomplete or unobtainable information, and the probabilistic approach that has been adopted in conventional reliability analysis is inadequate to account for such built-in uncertainties in data. To solve this problem, a more general approach that combines the IFS and the OWA tree model is proposed in this section.

4.1. The reason for using IFS

For a new product, due to uncertainties and imprecision of data, it may be difficult or even impossible to precisely determine the failure probabilities of components as early as the product design phase. Therefore, using an IFS and OWA tree can help to solve system reliability problems in the product design phase when the available information is incomplete. The major advantage of the IFS over the fuzzy set is that the IFS separate the positive (the degree of membership) and negative (the degree of non-membership) evidence for membership of an element in the set. Fuzzy sets are IFSs, but the converse is not necessarily true. For this reason, using the IFS, not the fuzzy set, in OWA tree diagrams is more suitable.

The concept of the IFS can be viewed as an alternative approach to define a fuzzy set in the case when available information is not sufficient or precise enough to define a conventional

fuzzy set. In the product design phase, in the situation of vague or incomplete information, an expert's experience that is given maximum membership degree might not be equal to one. It is corresponding to the IFS definition, but is not corresponding to the fuzzy set definition.

In the system design phase, the collected data or system parameters are often vague due to incomplete or unobtainable information. Incomplete failure data will increase the difficulty of reliability design and calculation. This issue cannot be fully resolved by traditional probability reliability and fuzzy reliability. This article proposes to use experts' opinions to speculate on the system reliability of vague phenomena. According to expert knowledge and experience, the intuitionistic fuzzy membership degree of each leaf node is obtained by reasonably giving different fault membership functions of possibilities of failure distributions for different leaf nodes. This is useful when the available information is imprecise, incomplete or uncertain in the design phase.

4.2. The reliability operation using the IFS and OWA tree

Based on the above discussion, this article now derives the reliability of an OWA tree in the AND node, OR node and OWA node condition.

4.2.1. AND node

A parallel system is composed of n elements that perform identical functions, the success of any of which will lead to system success. In other words, all of the components must fail in order to cause system failure.

Example Suppose an AND node is composed of three assemblies, one with a reliability of 0.7 and the others with reliabilities of 0.8 and 0.9. Let Q_{AND} represent the system failure probability of the AND node, and Q_{AND} can be calculated as

$$Q_{AND} = \prod_{j=1}^3 q_j = (1 - 0.7)(1 - 0.8)(1 - 0.9) = 0.006.$$

The system reliability of the AND node, P_{AND} , is

$$P_{AND} = 1 - 0.006 = 0.994.$$

4.2.2. OR node

A series system is composed of n elements, the failure of any of which will cause a system failure.

Example Suppose an OR node is composed of three assemblies, one with a reliability of 0.7 and the others with reliabilities of 0.8 and 0.9. Let Q_{OR} represent the system failure probability of the OR node, then Q_{OR} can be calculated as

$$Q_{OR} = 1 - \prod_{j=1}^3 (1 - q_j) = 1 - (1 - 0.3)(1 - 0.2)(1 - 0.1) = 0.496.$$

The system reliability of the OR node, P_{OR} , is

$$P_{OR} = 1 - 0.496 = 0.504. \tag{16}$$

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4.2.3. OWA node

An OWA node is composed of n elements, in which there is some probabilistic uncertainty in the number of components that needs be satisfied.

Example Suppose an OWA node is composed of three assemblies, one with a reliability of 0.7 and the others with reliabilities of 0.8 and 0.9. If $w_1 = 0.826294$, $w_2 = 0.146973$ and $w_3 = 0.026306$, the system reliability of the OWA node, P_{OWA} , is calculated as

$$P_{OWA} = (0.826494 \times 0.9) + (0.146973 \times 0.9 \times 0.8) + (0.026306 \times 0.9 \times 0.8 \times 0.7) = 0.862743.$$

4.3. Procedure of the proposed approach

The procedure of the proposed approach is organized into seven steps.

Step 1 Construct the OWA tree diagram

Construct the OWA tree diagram by the AND node, OR node and OWA node, tracing back the whole process from the main goal to the physical tasks (an example is illustrated in Figure 6).

Step 2 Obtain the intuitionistic fuzzy membership degree of leaf nodes

Obtain the intuitionistic fuzzy membership degree of leaf nodes according to expert knowledge and experience.

Step 3 Defuzzification

Use Equation (15) to obtain crisp values.

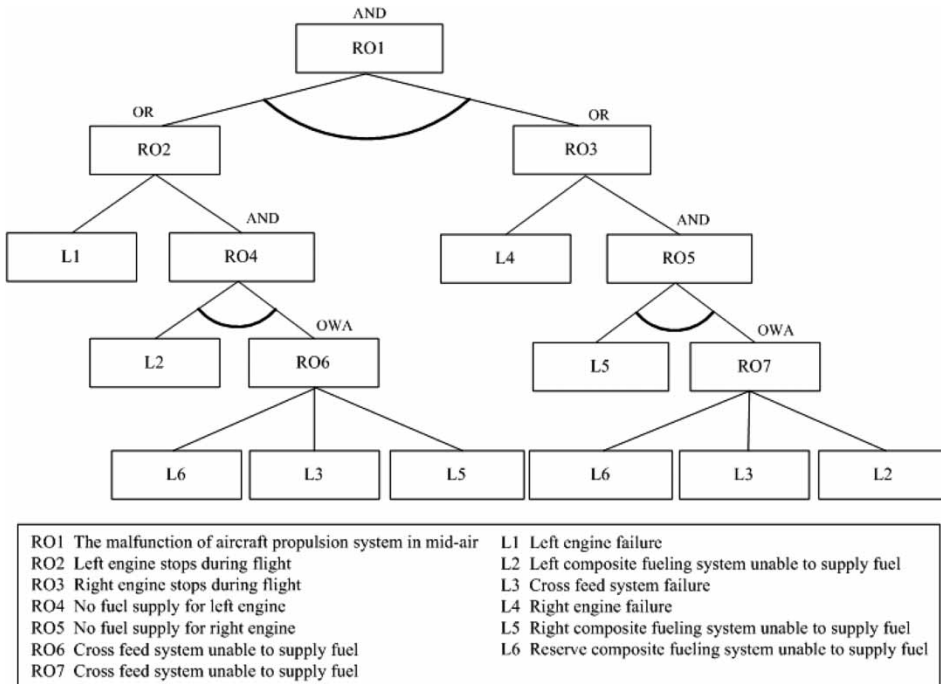


Figure 6. OWA tree of the aircraft propulsion system.

Step 4 Calculate the OWA weights

From Section 2.2, use Equations (5)–(7) to calculate the OWA weights.

Step 5 Calculate the possible malfunction probability of the main goal

From Section 2.3, use the OWA tree diagram and IFS arithmetic operations to calculate the possible malfunction probability of the main goal.

Step 6 Calculate the reliability of the main goal

The reliability of the main goal is equal to one minus the possible malfunction probability of the main goal.

Step 7 Analyse the results and provide suggestions

5. Case study

A real case of an aircraft propulsion system, drawn from an aerospace company, is used to demonstrate the proposed approach. First of all, an OWA tree is constructed that includes the main goal (‘the malfunction of the aircraft propulsion system in mid-air’), the sub-goals (left engine stops during flight, right engine stops during flight), the sub-tasks (left engine failure, no fuel supply for left engine, right engine failure, no fuel supply for right engine), and the physical tasks (left composite fuelling system unable to supply fuel, right composite fuelling system unable to supply fuel, reserve composite fuelling system unable to supply fuel, cross-feed system failure). As shown in Figure 6, the OWA tree integrates the main goal, the sub-goals, the sub-tasks, and the physical tasks with the AND, OR and OWA nodes. After the OWA tree was constructed, this article collected the failure possibility interval of leaf failure from expert knowledge and experience and organized the data in Table 1.

5.1. Crisp failure possibility method (Kales 1988)

The conventional crisp failure possibility method (Kales 1988) is used to deal with heterogeneous problems, and the probability can only show the success or failure events in random. This method is constrained by its usage on the condition of a large amount of data samples, and all of the event outcomes are certain. However, a lot of uncertainty factors may cause fuzziness in the procedure of evaluating an aircraft propulsion system, such as statistical uncertainty, model uncertainty, and data uncertainty. These uncertainty factors will limit the understanding of system component failure. Another drawback of the conventional reliability method is the lack of the ability to make statistical estimates. Therefore, use of the conventional crisp failure possibility method is hard to calculate the malfunction possibility of a system and its components in a precise way because of incomplete data. This research calculated the malfunction possibility of an aircraft propulsion

Table 1. The possible range of leaf node failure.

Failure possibility	a_i	a'_i	b_i	c'_i	c_i	$\mu_A(U)$	$1 - v_A(U)$
L1 (left engine failure)	0.0004	0.0009	0.0012	0.0015	0.0020	0.9	0.8
L2 (left composite fuelling system unable to supply fuel)	0.00009	0.0001	0.0004	0.0008	0.0009	1.0	0.9
L3 (cross feed system failure)	0.0002	0.0004	0.0007	0.0010	0.0013	0.9	0.9
L4 (right engine failure)	0.0004	0.0009	0.0012	0.0015	0.0020	0.9	0.8
L5 (right composite fuelling system unable to supply fuel)	0.00009	0.0001	0.0004	0.0008	0.0009	1.0	0.9
L6 (reserve composite fuelling system unable to supply fuel)	0.0003	0.0006	0.0009	0.0013	0.0015	0.9	0.8

system based on data of Table 1 (column b_i) by the conventional probability reliability method as follows.

If the composite fuelling system that is unable to supply fuel is serial, then

$$\begin{aligned} Q_{RO1} &= \{1 - (1 - Q_{L1})[1 - Q_{L2} \cdot (1 - (1 - Q_{L6}) \cdot (1 - Q_{L3}) \cdot (1 - Q_{L5}))]\} \\ &\quad \times \{1 - (1 - Q_{L4})[1 - Q_{L5} \cdot (1 - (1 - Q_{L6}) \cdot (1 - Q_{L3}) \cdot (1 - Q_{L2}))]\} \\ &= 0.0000014419 \\ P_{RO1} &= 1 - Q_{RO1} = 0.9999985581 \end{aligned}$$

If the composite fuelling system that is unable to supply fuel is parallel, then

$$\begin{aligned} Q_{RO1} &= \{1 - (1 - Q_{L1})(1 - Q_{L2} \times Q_{L6} \times Q_{L3} \times Q_{L5})\} \\ &\quad \times \{1 - (1 - Q_{L4})(1 - Q_{L5} \times Q_{L6} \times Q_{L3} \times Q_{L2})\} \\ &= 0.0000014400 \\ P_{RO1} &= 1 - Q_{RO1} = 0.9999985600 \end{aligned}$$

After the above calculation, the reliability of the 'aircraft propulsion system' is between 0.9999985581 and 0.9999985600.

5.2. Huang et al. method (2004)

When the failure probability of a system is extremely small or when essential statistical data are scarce, the posbist fault-tree analysis proposed by Huang *et al.* (2004) could be applied to predict and diagnose a system's failures and evaluate its reliability and safety. Calculations of the failure possibility of an 'aircraft propulsion system malfunction' based on the crisp failure possibilities are listed in Table 1 (column b_1), per the following:

If the composite fuelling system that is unable to supply fuel is serial, then

$$\begin{aligned} P_{oss}(RO7) &= \min(P_{oss}(L6), P_{oss}(L3), P_{oss}(L2)) = \min(0.9991, 0.9993, 0.9996) = 0.9991 \\ P_{oss}(RO6) &= \min(P_{oss}(L6), P_{oss}(L3), P_{oss}(L5)) = \min(0.9991, 0.9993, 0.9996) = 0.9991 \\ P_{oss}(RO5) &= \max(P_{oss}(L5), P_{oss}(RO7)) = \max(0.9996, 0.9991) = 0.9996 \\ P_{oss}(RO4) &= \max(P_{oss}(L2), P_{oss}(RO6)) = \max(0.9996, 0.9991) = 0.9996 \\ P_{oss}(RO3) &= \min(P_{oss}(L4), P_{oss}(RO5)) = \min(0.9988, 0.9996) = 0.9988 \\ P_{oss}(RO2) &= \min(P_{oss}(L1), P_{oss}(RO4)) = \min(0.9988, 0.9996) = 0.9988 \\ P_{oss}(RO1) &= \max(P_{oss}(RO2), P_{oss}(RO3)) = \max(0.9988, 0.9988) = 0.9988 \end{aligned}$$

If the composite fuelling system that is unable to supply fuel is parallel, then

$$\begin{aligned} P_{oss}(RO7) &= \max(P_{oss}(L6), P_{oss}(L3), P_{oss}(L2)) = \max(0.9991, 0.9993, 0.9996) = 0.9996 \\ P_{oss}(RO6) &= \min(P_{oss}(L6), P_{oss}(L3), P_{oss}(L5)) = \max(0.9991, 0.9993, 0.9996) = 0.9996 \\ P_{oss}(RO5) &= \max(P_{oss}(L5), P_{oss}(RO7)) = \max(0.9996, 0.9996) = 0.9996 \\ P_{oss}(RO4) &= \max(P_{oss}(L2), P_{oss}(RO6)) = \max(0.9996, 0.9996) = 0.9996 \\ P_{oss}(RO3) &= \min(P_{oss}(L4), P_{oss}(RO5)) = \min(0.9988, 0.9996) = 0.9988 \\ P_{oss}(RO2) &= \min(P_{oss}(L1), P_{oss}(RO4)) = \min(0.9988, 0.9996) = 0.9988 \\ P_{oss}(RO1) &= \max(P_{oss}(RO2), P_{oss}(RO3)) = \max(0.9988, 0.9988) = 0.9988 \end{aligned}$$

Table 2. The optimal weighting vector under maximal entropy ($n = 3$).

α	w_1	w_2	w_3
0.5	0.333333	0.333333	0.333333
0.6	0.438355	0.323242	0.238392
0.7	0.553955	0.291992	0.153999
0.8	0.681854	0.235840	0.081892
0.9	0.826294	0.146973	0.026306
1	1	0	0

After the above calculation, it is shown that the reliability of the ‘aircraft propulsion system’ is 0.9988.

5.3. Proposed method

Fuzzy logic provides a tool for directly working with the linguistic terms that are used in making the reliability assessment. The analysis uses expert knowledge and experience to define the intuitionistic fuzzy membership degree of the leaf nodes. These inputs are then ‘fuzzified’ to determine the degree of membership in each input class. In the malfunction of the aircraft propulsion system, the triangle IFS of each leaf node failure is defined by the expert’s experience, as shown in Table 1. Sensitivity analysis enables the identification of the impact on system reliability by different choices of the value of α . According to Equations (5)–(7), the optimal weighting vector under the maximal entropy for $n = 3$ is calculated and organized in Table 2.

Based on the arithmetic operations of the AND, OR and OWA nodes, the reliability of the ‘aircraft propulsion system’ can be described as:

When $n = 3$ and $\alpha = 1$, from Table 2, it is found that $w_1 = 1$, $w_2 = 0$ and $w_3 = 0$. Therefore,

$$\begin{aligned}
 P_{OWA}(RO7) &= 1 \times (0.99955) + 0 \times (0.99883) + 0 \times (0.99791) = 0.99955 \\
 P_{OWA}(RO6) &= 1 \times (0.99955) + 0 \times (0.99883) + 0 \times (0.99791) = 0.99955 \\
 P_{OWA}(RO1) &= 1 - \{1 - P_{L1}[1 - (1 - P_{L2})(1 - P_{RO6})]\} \\
 &\quad \times \{1 - P_{L4}[1 - (1 - P_{L5})(1 - P_{RO7})]\} = 0.9999985595
 \end{aligned}$$

When $n = 3$ and $\alpha = 0.9$, then from Table 2, it is found that $w_1 = 0.826294$, $w_2 = 0.146973$ and $w_3 = 0.026306$. It follows that

$$\begin{aligned}
 P_{OWA}(RO7) &= 0.826294 \times (0.99955) + 0.146973 \times (0.99883) + 0.026306 \\
 &\quad \times (0.99791) = 0.998974 \\
 P_{OWA}(RO6) &= 0.826294 \times (0.99955) + 0.146973 \times (0.99883) + 0.026306 \\
 &\quad \times (0.99791) = 0.998974 \\
 P_{AND}(RO1) &= 1 - \{1 - P_{L1}[1 - (1 - P_{L2})(1 - P_{RO6})]\} \\
 &\quad \times \{1 - P_{L4}[1 - (1 - P_{L5})(1 - P_{RO7})]\} = 0.9999985589
 \end{aligned}$$

When $n = 3$ and $\alpha = 0.8$, then from Table 2, it is found that $w_1 = 0.681854$, $w_2 = 0.235840$ and $w_3 = 0.081892$. Therefore,

$$P_{OWA}(RO7) = 0.681854 \times (0.99955) + 0.235840 \times (0.99883) + 0.081892 \\ \times (0.99791) = 0.998832$$

$$P_{OWA}(RO6) = 0.681854 \times (0.99955) + 0.235840 \times (0.99883) + 0.081892 \\ \times (0.99791) = 0.998832$$

$$P_{AND}(RO1) = 1 - \{1 - P_{L1}[1 - (1 - P_{L2})(1 - P_{RO6})]\} \\ \times \{1 - P_{L4}[1 - (1 - P_{L5})(1 - P_{RO7})]\} = 0.9999985587$$

When $n = 3$ and $\alpha = 0.7$, then from Table 2, it is found that $w_1 = 0.553955$, $w_2 = 0.291992$ and $w_3 = 0.153999$. Hence,

$$P_{OWA}(RO7) = 0.553955 \times (0.99955) + 0.291992 \times (0.99883) + 0.153999 \\ \times (0.99791) = 0.999033$$

$$P_{OWA}(RO6) = 0.553955 \times (0.99955) + 0.291992 \times (0.99883) + 0.153999 \\ \times (0.99791) = 0.999033$$

$$P_{AND}(RO1) = 1 - \{1 - P_{L1}[1 - (1 - P_{L2})(1 - P_{RO6})]\} \\ \times \{1 - P_{L4}[1 - (1 - P_{L5})(1 - P_{RO7})]\} = 0.9999985590$$

When $n = 3$ and $\alpha = 0.6$, from Table 2, it is found that $w_1 = 0.438355$, $w_2 = 0.323242$ and $w_3 = 0.238392$. Therefore,

$$P_{OWA}(RO7) = 0.438355 \times (0.99955) + 0.323242 \times (0.99883) + 0.238392 \\ \times (0.99791) = 0.998915$$

$$P_{OWA}(RO6) = 0.438355 \times (0.99955) + 0.323242 \times (0.99883) + 0.238392 \\ \times (0.99791) = 0.998915$$

$$P_{AND}(RO1) = 1 - \{1 - P_{L1}[1 - (1 - P_{L2})(1 - P_{RO6})]\} \\ \times \{1 - P_{L4}[1 - (1 - P_{L5})(1 - P_{RO7})]\} = 0.9999985588$$

When $n = 3$ and $\alpha = 0.5$, then from Table 2, it is found that $w_1 = 0.333333$, $w_2 = 0.333333$ and $w_3 = 0.333333$. Therefore,

$$P_{OWA}(RO7) = 0.333333 \times (0.99955) + 0.333333 \times (0.99883) + 0.333333 \\ \times (0.99791) = 0.998762$$

$$P_{OWA}(RO6) = 0.333333 \times (0.99955) + 0.333333 \times (0.99883) + 0.333333 \\ \times (0.99791) = 0.998762$$

$$P_{AND}(RO1) = 1 - \{1 - P_{L1}[1 - (1 - P_{L2})(1 - P_{RO6})]\} \\ \times \{1 - P_{L4}[1 - (1 - P_{L5})(1 - P_{RO7})]\} = 0.9999985587$$

After the above calculations, the reliability of the 'aircraft propulsion system' is between 0.9999985587 and 0.9999985595.

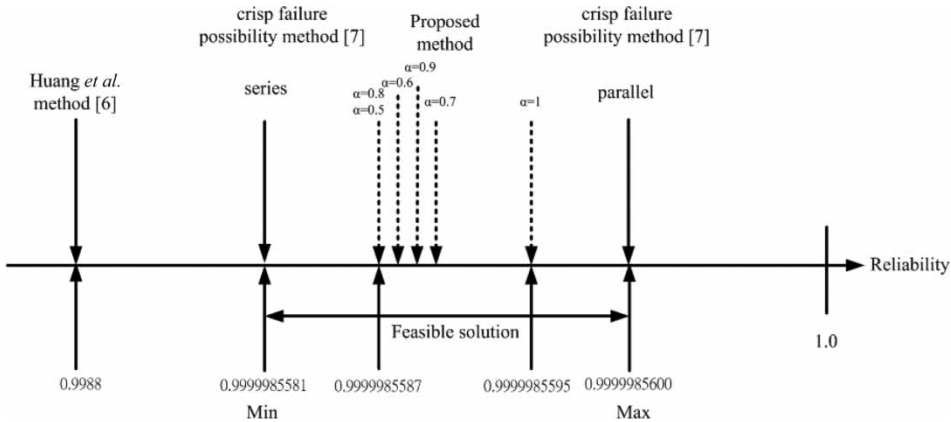


Figure 7. The reliability of the main goal for the aircraft propulsion system.

5.4. Comparisons and discussion

In order to evaluate the proposed method, a case study verification compares the proposed approach with other methods (crisp failure possibility method and the method of Huang *et al.* (2004)). The input data of these methods are shown in Table 1. In comparing the results of the three methods, the differences between the proposed method and the listing methods can be clearly seen in Figure 7. From Figure 7, there are two findings: (1) the simulation results of the proposed method correspond to the common aggregation operator’s definition (as illustrated in Figure 1); (2) in posbist fault-tree analysis, the reliability of the ‘aircraft propulsion system’ is 0.9988, which falls outside of the range of the [min, max] interval. This is because the Huang *et al.* method selects the maximal failure probability of leaf nodes, which results in biased conclusions.

From the comparison, it is clear that the intuitionistic fuzzy OWA tree analysis technique outlined in this study provides the following advantages. Firstly, the failure information is described by intuitionistic fuzzy variables; this results in a more realistic and flexible reflection of the real situation. Secondly, the proposed method gives a more flexible structure in combining the AND, OR and OWA nodes to an OWA tree of the aircraft propulsion system. Finally, the proposed approach can indeed help to solve system reliability problems in a product’s design phase when the available information is incomplete.

6. Conclusion

This article collects experts’ knowledge and experience on the aircraft propulsion system problem domain and builds the intuitionistic fuzzy number for representing possibilities of failure leaf nodes of an aircraft propulsion system. A new approach has been proposed, which combines the IFS and OWA tree approach, to evaluate the reliability of an aircraft propulsion system. This is useful when evaluating system reliability using available information and expert knowledge, which are often uncertain or vague in the design phase. The proposed approach provides a more flexible structure for combining the AND, OR and OWA nodes to an OWA tree of the aircraft propulsion system. This approach can help users solve reliability assessment problems under situations of vague or incomplete information.

In order to further illustrate the proposed method and compare with other techniques of system reliability analysis, an OWA tree for the aircraft propulsion system is adopted as a simulation example. This research compares the simulation results with the crisp failure possibility

(Kales 1988) and the method proposed by Huang *et al.* (2004). The results show that the proposed approach could provide a more accurate and reasonable reliable assessment, which can assist designers in making correct decisions for a safer and more reliable product design. Furthermore, the presented approach can be helpful for solving system reliability problems in product design phase.

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