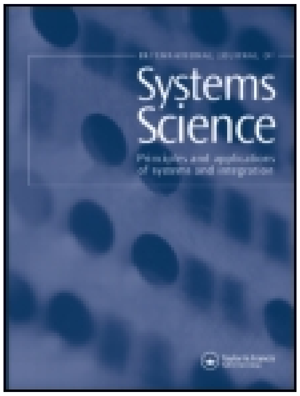

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Improved periodic review inventory model involving lead-time with crashing components and service level

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A periodic review inventory model with backorders and lost sales in which both lead-time and the periodic review length are decision variables and the production interval demand follows a normal distribution is explored in this article. Ouyang and Chuang discussed this problem in a recent paper published in the *International Journal of Systems Science*. However, their algorithms might not, although intended to, find the optimal solution due to questionable results in their solution procedure. The purpose of this study is 3-fold. First, the criteria for the existence and uniqueness of the critical solution for minimising the total expected annual cost are determined. Second, a correct and efficient algorithm to improve their method is constructed to find the optimal lead-time and periodic review length simultaneously. Finally, some numerical examples are provided to compare our solution procedure with that of Ouyang and Chuang's method to demonstrate their questionable results.

Keywords: Inventory; Lead-time; Backorder; Lost sales; Periodic review

1. Introduction

The irresistible trend of dropping gross profit and shortening product life cycle has been certain in recent years while entrepreneurs have been striving to pinpoint the needs of the market under the fierce competitive environment. Most business customers bear in mind four pivotal elements: quality, price, delivery time and service on the whole. Among which delivery time has become significantly crucial nowadays since it reflects efficiency of speed intensive era and demonstrates the capability of coping with high volatile changes of demand and shorter product life cycle. That is how this particular element grabs our concentration and in the meantime is widely recognised in most real-life applications as a decision variable. For fast delivery, suppliers work overtime, add manpower, renew equipment, reset layout or choose better logistic means in order to cut down the lead-time in order to gain appreciation from customers and competitive advantages.

Lead-time has recently been used as a decision variable in some models. Liao and Shyu (1991) first presented lead-time as negotiable and decomposed it into several components, each having a different piecewise linear crash cost function for lead time reduction. Ben-Daya and Raouf (1994) extended Liao and Shyu's (1991) work, considering both lead-time and order quantity as decision variables where shortages were neglected. Moon and Gallego (1994) assumed unfavourable lead-time demand distribution and solved both the continuous review and periodic review models with a mixture of backorders and lost sales using the minmax distribution free approach. Ouyang *et al*. (1996) generalised Ben-Daya and Raouf's (1994) assumption that shortages were allowed and constructed variable lead-time from a mixed inventory model with backorders and lost sales. Moon and Choi (1998) and Lan *et al*. (1999) pointed out the problem in Ouyang *et al*. (1996). They found optimal order quantities and

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optimal lead-time for a mixed inventory model, and developed a simplified solution procedure under different conditions. Chu *et al.* (1999) showed some drawbacks in the Ben-Daya and Raouf's (1994) solution procedure and used the Newton–Raphson method with an appropriate starting point to improve this problem. Wu and Ouyang (2000) assumed that an arrival order lot might contain some defective items. They derived a modified mixed inventory model in which the order quantity, order point and lead-time were decision variables. Wu and Tsai (2001) considered that the lead-time demands from different customers are not identical. They developed a mixed inventory model with backorders and lost sales for variable lead-time demand with a mixed normal distribution. Pan and Hsiao (2001) presented inventory models with backorder discount as inducement and variable lead-time to ensure that customers would be willing to wait for backorders. Pan *et al.* (2004) proposed integrated inventory systems with the objective to simultaneously optimising the order quantity, lead time, back ordering and reorder point. Following that Pan and Hsiao (2005) proposed two inventory models, one with normally distributed demand and another with generally distributed demand, in which lead-time crashing cost was represented as a function of reduced lead-time and the quantities in the order. Hoque and Goyal (2006) developed a heuristic solution procedure to minimise the total cost of setup or ordering, inventory holding and lead-time crashing for an integrated inventory system under controllable lead-time between a vendor and a buyer. Instead of a stock-out term in the objective function, Lee *et al.* (2006) added a service level constraint to the model and developed two computational algorithms to find optimal order quantity and optimal lead-time. Chang *et al.* (2006) developed an iterative procedure to find the optimal solution with the consideration that lead-time can be shortened at an extra crashing cost which depends on the lead-time length to be reduced and the ordering lot size. Lee *et al.* (2007) developed a computational algorithmic procedure to simultaneously optimise the order quantity, ordering cost, back-order discount and lead-time with the consideration that lead-time can be shortened at an extra crashing cost and allow the back-order rate as a control variable. Tempelmeier (2007) included the exact on hand inventory into the model formulation that minimises the setup and holding costs with respect to a constraint on the probability that the inventory at the end of any period does not become negative. As a result, the models are also applicable in situations with very low service levels. Wu *et al.* (2007) developed two algorithmic procedures to find optimal inventory policy with the consideration that the lead-time demand follows either the mixture of normal distribution or mixture of free distribution and the total crashing cost is related to the lead-time by a negative exponential function.

This study examines the same inventory model as Ouyang and Chuang (2000). They considered both lead-time and periodic review length as decision variables for a mixture periodic review inventory model. Ouyang and Chuang (2000) thought that it is often difficult to determine the stock-out cost in inventory systems. They added a service level constraint in the model instead of a stock-out cost term in the objective function. Their method predicted that (a) the expected annual cost is a convex function and (b) sometimes, this inventory model does not have feasible solutions. This study will show that the expected annual cost is not a convex function and this inventory model always has an optimal solution. This study also constructs correct and efficient algorithms to find the optimum order quantity and lead-time simultaneously when the protection interval demand distribution is normal. Two numerical examples are provided to explain the questionable results in Ouyang and Chuang's algorithm.

2. Notation and assumption

We use the same notations and assumptions as Ouyang and Chuang (2000).

Notation:

A = Fixed ordering cost per order.
D = Average demand per year.
h = Inventory holding cost per item per year.
L = Length of lead-time, a decision variable.
T = Length of periodic review, a decision variable.
X = Demand of production interval, $T+L$, which has probability density function f_X, finite mean $D(T+L)$ and standard derivation $\sigma\sqrt{T+L}$.
x^+ = Maximum value of x and 0, i.e. $x^+ = \max\{x, 0\}$.
α = Proportion of demands that are not met from stock so $1-\alpha$ is the service level.
β = Fraction of the demand backordered during the stock-out period.
$C(L)$ = Lead-time crashing cost.
$EAC(T, L)$ = Total expected annual cost.

Assumptions:

(1) *The inventory level is reviewed every T units of time. A sufficient ordering quantity is ordered up*

to the target level R, and the ordering quantity is arrived at after L units of time.

(2) *The length of the lead-time L is not greater than the review period length T so that there is never more than a single order taking place in any cycle.*

(3) *The target level $R =$ expected demand during protection interval + safety stock (SS), and $SS = k\sigma\sqrt{T+L}$, that is $R = D(T+L) + k\sigma\sqrt{T+L}$ where k is the safety factor and satisfies $P(X > R) = q$, in which q given represents the allowable stock-out probability during the protection interval.*

(4) *$E[X-R]^+$ is the expected demand short at the end of cycle. Hence, $\beta E[X-R]^+$ are the backordered quantities and $(1-\beta)E[X-R]^+$ are the lost sales. Therefore, the expected net inventory level at the beginning of the period is $R - DL + (1-\beta)E[X-R]^+$ and the expected net inventory level at the end of the period is $R - D(T+L) + (1-\beta)E[X-R]^+$. Hence, the expected holding cost per year is $h[R - D((T/2)+L) + (1-\beta)E[X-R]^+]$.*

(5) *If X has a normal distribution function $F(x)$, then $E[X-R]^+ = \sigma\sqrt{T+L}G(k)$ where $G(k) = \int_k^\infty (z-k)f_Z(z)\mathrm{d}z$ and $f_Z(z)$ is the probability density function of the standard normal random variable Z.*

(6) *The lead-time L includes n mutually independent components. The ith component has a minimum duration a_i, and normal duration b_i, and a crashing cost per unit time c_i. Further, we assume that $c_1 \le c_2 \le \cdots \le c_n$. The lead-time components are crashed one at a time starting with the component of least c_i and so on.*

(7) *If we let $L_0 = \sum_{j=1}^n b_j$ and L_i be the length of lead-time with components $1, 2, \ldots, i$ crashed to their minimum duration, then $L_i = \sum_{j=i+1}^n b_j + \sum_{j=1}^i a_j$. The lead-time crashing cost $C(L)$ per cycle for a given $L \in [L_i, L_{i-1}]$, is given by $C(L) = c_i(L_{i-1} - L) + \sum_{j=1}^{i-1} c_j(b_j - a_j)$.*

(8) *When X has a normal distribution function, the service level constraint becomes*

$$\frac{E[X-R]^+}{D(T+L)} = \frac{\sigma G(k)}{D\sqrt{T+L}} \le \alpha.$$

3. Review of Ouyang and Chuang's result

We review the inventory model in which the protection interval demand follows normal distribution. The total expected annual costs are the sum of the ordering cost, holding cost and lead-time crashing cost, subject to a constraint on the service level. Hence, the problem can be formulated as

$$\text{Min}\, EAC(T,L) = \frac{A}{T} + h\left[\frac{DT}{2} + \sigma\sqrt{T+L}\big(k + (1-\beta)G(k)\big) + \right] + \frac{C(L)}{T} \tag{1}$$

$$\text{subject to } \frac{\sigma G(k)}{D\sqrt{T+L}} \le \alpha, \tag{2}$$

where $0 < T < \infty$ and $L \in [L_i, L_{i-1}]$ for $i = 1, 2, \ldots, n$.

Ouyang and Chuang (2000) derived that for $L \in (L_i, L_{i-1})$, $i = 1, 2, \ldots, n$,

$$\frac{\partial^2}{\partial L^2} EAC(T,L) = \frac{-h\sigma}{4\sqrt{(T+L)^3}}(k + (1-\beta)G(k)) < 0. \tag{3}$$

They implied that $EAC(T, L)$ is a concave function of L. They obtained

$$\frac{\partial}{\partial T} EAC(T,L) = \frac{h\sigma(k + (1-\beta)G(k))}{2\sqrt{T+L}} + \frac{hD}{2} - \frac{A + C(L)}{T^2}, \tag{4}$$

and

$$\frac{\partial^2}{\partial T^2} EAC(T,L) = \left(\frac{3T+4L}{4T}\right)\frac{h\sigma(k + (1-\beta)G(k))}{\sqrt{(T+L)^3}} + \frac{hD}{T}. \tag{5}$$

From $(\partial^2/\partial T^2)EAC(T,L) > 0$, Ouyang and Chuang (2000) claimed that $EAC(T, L)$ is a convex function of T. From equation (3), they knew that the minimum would occur at $L = L_i$ for $i = 0, 1, 2, \ldots, n$. For a given L_i, setting equation (4) equal to zero, they solved

$$\frac{h\sigma(k + (1-\beta)G(k))}{2\sqrt{T_i + L_i}} + \frac{hD}{2} = \frac{A + C(L_i)}{T_i^2} \tag{6}$$

to find T_i. For each pair (T_i, L_i), they computed $EAC(T_i, L_i)$. Ouyang and Chuang (2000) first relaxed the service level constraint and found $\min_{i=0,\ldots,n} EAC(T_i, L_i)$.

If $EAC(T_s, L_s) = \min_{i=0,\ldots,n} EAC(T_i, L_i)$ and the service level constraint $(\sigma G(k)/D\sqrt{T_s + L_s}) \le \alpha$ is satisfied, they then accepted that (T_s, L_s) is the optimal solution. Conversely, if the service level constraint $(\sigma G(k)/D\sqrt{T_s + L_s}) \le \alpha$ is not satisfied, they then assumed that $EAC(T_t, L_t) =$ next

$\min_{i=0,\ldots,n} EAC(T_i, L_i)$ and checked whether the service level constraint $(\sigma G(k)/D\sqrt{T_t+L_t}) \le \alpha$ is satisfied? If it is satisfied, (T_t, L_t) is the optimal solution. Otherwise, they continued to search for a solution until the service level constraint was satisfied. They judged that this inventory model has no feasible solution if no solution for equation (6) satisfies the service level constraint.

4. Improved mathematical analysis

We begin by studying the normal distribution model. The errors in Ouyang and Chuang's (2000) algorithm are then discussed. From equation (3), we know that for fixed T, $EAC(T, L)$ is concave in $L \in [L_i, L_{i-1}]$. Hence, the problem can be simplified and formulated as

$$\text{Min } EAC(T, L_i) = \frac{A}{T} + h\left[\frac{DT}{2} + \sigma\sqrt{T+L_i}(k + (1-\beta)G(k)) + \right] + \frac{C(L_i)}{T} \quad (7)$$

$$\text{subject to } \frac{\sigma G(k)}{D\sqrt{T+L_i}} \le \alpha, \ \text{for } i = 0, 1, \ldots, n \quad (8)$$

We derive that

$$\frac{\partial}{\partial T} EAC(T, L_i) = \frac{h\sigma(k + (1-\beta)G(k))}{2\sqrt{T+L_i}} + \frac{hD}{2} - \frac{A + C(L_i)}{T^2}, \quad (9)$$

and

$$\frac{\partial^2}{\partial T^2} EAC(T, L_i) = 2\frac{A + C(L_i)}{T^3} - \frac{h\sigma(k + (1-\beta)G(k))}{4\sqrt{(T+L_i)^3}}. \quad (10)$$

Comparing equations (5) and (10), we know that the result of Ouyang and Chuang (2000) for $(\partial^2/\partial T^2)EAC(T, L)$ is questionable. To examine their result, we suppose that $T_i^{\#}$ satisfies $(\partial/\partial T)EAC(T_i^{\#}, L_i) = 0$. Then we compute $(\partial^2/\partial T^2)EAC(T_i^{\#}, L_i)$. It shows that

$$\frac{\partial^2}{\partial T^2} EAC\left(T_i^{\#}, L_i\right) = \left(\frac{3T_i^{\#} + 4L_i}{4T_i^{\#}}\right)\frac{h\sigma(k + (1-\beta)G(k))}{\sqrt{\left(T_i^{\#} + L_i\right)^3}} + \frac{hD}{T_i^{\#}}. \quad (11)$$

Hence, Ouyang and Chuang (2000) showed that at the critical solution of $(\partial/\partial T)EAC(T, L_i) = 0$, say $T_i^{\#}$, the second partial derivative of $EAC(T, L)$ with respect to T is positive. Their result only implies that $T_i^{\#}$ (if it exists) is a local minimum solution but they did not prove that $EAC(T, L)$ is a convex function of T.

From the correct expression of $(\partial^2/\partial T^2)EAC(T, L_i)$ in equation (11), we know that when $T \to \infty, (\partial^2/\partial T^2)EAC(T, L_i) < 0$. It is clear that, $EAC(T, L)$ is not a convex function of T. Their prediction for $EAC(T, L)$ being a convex function of T is then false. Now, we begin to develop our theorem for the normal distribution model. To simplify the expression, we assume that $\theta_i = A + C(L_i), \omega_1 = (Dh/2)$, $\omega_2 = (h\sigma/2)(k + (1-\beta)G(k))$ and

$$f_i(T) = \omega_1 T^2 + \omega_2 \frac{T^2}{\sqrt{T+L_i}} - \theta_i \text{ for } i = 0, 1, \ldots, n. \quad (12)$$

From equation (9), we realise that solving $(\partial/\partial T)EAC(T, L_i) = 0$ and $f_i(T) = 0$ are in fact equivalent. Since $(d/dT)f_i(T) = 2\omega_1 T + (\omega_2 T/\sqrt{(T+L_i)^3})((3T/2) + 2L_i) > 0$, we derive that $f_i(T)$ increases from $f_i(0) = -\theta_i < 0$ *to* $f_i(\infty) = \infty$, then $f_i(T) = 0$ has a unique positive solution, say $\hat{T_i}$. We know that $\hat{T_i}$ is the minimum solution for $EAC(T, L_i)$ without considering the constraints. There are two constraints for the periodic review length, so we consider the following two questions:

(1) Does $\hat{T_i}$ satisfy the second assumption as $L_i \le \hat{T_i}$?
(2) Does $\hat{T_i}$ satisfy the service level constraint $(\sigma G(k)/D\sqrt{\hat{T_i} + L_i}) \le \alpha$?

Therefore, we define

$$T_i^* = \max\left\{\hat{T_i}, L_i, \left(\frac{\sigma G(k)}{D\alpha}\right)^2 - L_i\right\}. \quad (13)$$

It implies from $T^2(\partial/\partial T)EAC(T, L_i) = f_i(T)$ that $(\partial/\partial T)EAC(T, L_i) > 0$ on $[T_i^*, \infty)$. So T_i^* is the minimum solution of $EAC(T, L_i)$ under these two constraints. Finally, suppose

$$EAC\left(T_{\text{opt}}, L_{\text{opt}}\right) = \min_{i=0,\ldots,n} EAC\left(T_i^*, L_i\right), \quad (14)$$

we prove that $(T_{\text{opt}}, L_{\text{opt}})$ is the optimal solution for this inventory model under these two constraints and the optimal solution will always exist. Hence, the prediction of Ouyang and Chuang (2000) that sometimes this inventory model has no feasible solution is false. Our improved algorithm is presented as follows:

(1) **Step 1**: For each L_i, $i = 0, 1, \ldots, n$, compute $\hat{T_i}$, from the root of function $f_i(T)$.

Table 1. Lead-time data.

Lead-time component, i	Normal duration, b_i (days)	Minimum duration, a_i (days)	$b_i - a_i$ (weeks)	Unit crashing cost, c_i (\$/week)
1	20	6	2	2.8
2	20	6	2	7
3	16	9	1	35

Table 2. Summary of the optimal solution from our proposed algorithm ($\alpha = 0.02$).

i	$T_i^{\wedge}$ (weeks)	L_i (weeks)	$(\sigma G(k)/D\alpha)^2 - L_i$	T_i^*	$EAC(T_i^*, L_i)$
0	9.7142	8	8.3142	9.7142	5005.12
1	9.7443	6	10.3142	10.3142	4984.02
2	9.8676	4	12.3142	12.3142	5095.64
3	10.2545	3	13.3142	13.3142	5291.56

(2) **Step 2**: Using equation (13), find T_i^* for $i = 0, 1, \ldots, n$.
(3) **Step 3**: From equation (14), find the optimal solution (T_{opt}, L_{opt}).

5. Numerical example

To demonstrate the improvement of our algorithm, we consider an example with the following data: $D = 500$ units/year, $A = \$400$ per order, $h = \$40$ per item per year, $\sigma = 7$ units/week, $\beta = 1$, $q = 0.2$ (in this situation, the safety factor $k = 0.845$) and lead-time with three components. The data is shown in table 1. According to our proposed algorithm, the optimal length of a period $T^* = 10.3142$ weeks, optimal lead-time $L^* = 6$ weeks and the minimum total expected annual cost $EAC(T^*, L^*) = \$4984.02$. The results are listed in table 2.

The results found by Ouyang and Chuang are computed in table 3. They have $EAC(T_1, L_1) = \min_{i=0,\ldots,3} EAC(T_i, L_i)$. However, ($T_1$, L_1) do not satisfy the service level constraint. They obtained $EAC(T_0, L_0) = \text{next } \min_{i=0,\ldots,3} EAC(T_i, L_i)$ where (T_0, L_0) meets the service level constraint. Ouyang and Chuang finally came up with the optimal periodic review length $T^* = 9.7142$ weeks, optimal lead-time $L^* = 8$ weeks and the minimum total expected annual cost $EAC(T^*, L^*) = \$5005.12$. Corresponding to their results, our proposed algorithm saved \$21.1.

When $\alpha = 0.01$ from table 2, we know that the critical point for the first partial derivative of the total expect annual cost does not satisfy the service level constraint. Hence, Ouyang and Chuang (2000) could not find solutions using their algorithm. They implied that when $\alpha = 0.01$, there is no feasible solution for this inventory model. However, from table 4, the results from implementing proposed algorithm: the optimal length for a period $T^* = 57.2569$ weeks, the optimal lead-time $L^* = 8$ weeks and the minimum total expected annual cost $EAC(T^*, L^*) = \$13285.53$ can be effectively found.

Table 3. Summary of the optimal solution from Ouyang and Chuang's algorithm.

i	L_i (weeks)	$C(L_i)$	T_i (weeks)	$EAC(T_i, L_i)$	$\sigma G(k)/D\sqrt{T_i + L_i}$
0	8	0	9.7142	5005.12	0.0192
1	6	5.6	9.7443	4977.18	0.0204
2	4	19.6	9.8676	4989.89	0.0217
3	3	54.6	10.2545	5138.65	0.0222

Table 4. Summary of the optimal solution using our proposed algorithm ($\alpha = 0.01$).

i	$T_i^{\wedge}$ (weeks)	L_i (weeks)	$(\sigma G(k)/D\alpha)^2 - L_i$	T_i^*	$EAC(T_i^*, L_i)$
0	9.7142	8	57.2569	57.2569	13285.53
1	9.7443	6	59.2569	59.2569	13662.80
2	9.8676	4	61.2569	61.2569	14047.68
3	10.2545	3	62.2569	62.2569	14263.50

6. Conclusion

Enterprises face the harsh challenge of short product life cycles and delivery time. The challenge forces managers to put much effort into lead-time management. For instance, building upon this conviction and under tremendous pressure from foreign brand firms (e.g. Dell, Apple, Nokia and so forth) and Integrated Device Manufacturers (IDM) (e.g. Intel, Motorola, TI, NEC and so forth), Taiwan's leading electronic OEM and ODM factories have successfully developed 95-3, 98-3 and 10-2 delivery models. 95-3 model represents delivering 95% of total orders to those enterprise clients within three days, 98-3 model moves up the delivery volume to 98% within three days, and 10-2 model even moves forward one big step and tries to deliver 100% of total orders within two days. The accomplishment and applications of these models enable Taiwan's OEM and ODM factories to efficiently lower the lead-time, establish competitive advantages and constantly gain orders from those international enterprise giants. In a word, flexible and solid control capability of lead-time and delivery requirement from customers is undoubtedly not only one of the most potent means of acquiring market power in the present ferocious business arena but also in the meantime the most effective way of obtaining the minimum total expected annual cost.

In the above discussions, we pointed out the questionable algorithm in the paper of Ouyang and Chuang (2000). Their approach is too complicated and lacks theoretical solidity so that the consistency and feasibility from a mathematical programming point of view and further practicability are dubitable. We provide a simplified and theoretically-rigorous algorithm to greatly improve their weaknesses and shortcomings. Our approach promises the existence and uniqueness of optimal solution.

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