

行政院國家科學委員會補助專題研究計畫成果報告

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赴國外出差或研習心得報告一份

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有限溫度規範場論之行為及其宇宙學之應用 (子計劃一): 有限溫度粒子物理之研究(1/3-3/3)

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一、中文摘要

我們亦發展出弱場展開的技巧，處理磁場下之天文物理過程，包括背景磁場下之微中子-光子散射過程： $\bar{\nu} \rightarrow \nu \gamma$ 及 $\bar{\nu} \rightarrow \nu \gamma$ ；光子折射係數，微中子 Cherenkov 過程 $\nu \rightarrow \nu \gamma$ 等。對末態含帶電粒子的過程如 $\nu \rightarrow e^+ e^- \gamma$ ，我們運用解析性，將此過程之衰變寬度寫成光子極化函數在零動量處各階導數之反 Mellin 變換，這項方法將革新場論計算技巧。

關鍵詞：有效位能、電弱理論、微中子、磁場、光子、Mellin 變換。

Abstract

We have developed a weak-field expansion technique, which is applicable to astrophysical processes occurring in background magnetic fields, such as the photon-neutrino scatterings: $\bar{\nu} \rightarrow \nu \gamma$ and $\bar{\nu} \rightarrow \nu \gamma$, the photon index of refraction and the neutrino Cherenkov process $\nu \rightarrow \nu \gamma$. For processes with charged fermions in the final state, such as $\nu \rightarrow e^+ e^- \gamma$, we make use of analyticity such that the pair-production width is given by the inverse Mellin transform of the photon polarization function and its derivatives at the zero photon-energy. Such an approach

will be proven innovatory in the calculations of quantum field theory.

Keywords: effective potential, electroweak theory, neutrino, magnetic field, photon, Mellin transform.

二、緣由與目的

The relevance of neutrino-photon interactions in astrophysics and cosmology has been studied extensively [1]. For example, the plasmon decay $\nu \rightarrow \nu \gamma$ in horizontal-branch stars and red giants leads to a strong constraint on the neutrino magnetic-moment [2]. Similarly, the decay process $\nu \rightarrow e^+ e^- \gamma$ was also calculated [3], and its partial width has been constrained by various astrophysical observations [1]. It is natural to ask whether the two-photon processes such as the scatterings, $\bar{\nu} \rightarrow \nu \gamma \gamma$ and $\bar{\nu} \rightarrow \nu \gamma \gamma$ or the decay $\nu \rightarrow e^+ e^- \gamma \gamma$ are also relevant in astrophysics and cosmology. It turns out that, due to the chiral symmetry, the $O(G_F)$ -contributions to the amplitudes of the above processes are proportional to the mass of the neutrino [4]. Hence the resulting cross sections or decay rates are very suppressed. On the other hand, similar processes involving 3 photons such as

$\bar{\nu}_e$ or $\bar{\nu}_\mu$ are not suppressed by the same mechanism [5]. The large cross sections of these processes implied that $\bar{\nu}_e$ and its crossed processes are not suppressed in a background of strong magnetic field.

The previous work on $\bar{\nu}_e$ in a background magnetic field [6] is based on an effective Lagrangian for $\bar{\nu}_e$

[5] and replacing one of the external photon with the classical magnetic field. It is clear that such an approach is valid only in the limit that $E, E \ll m_e$. In our work, we compute magnetic-field effects to

$\bar{\nu}_e$, $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ with E and E larger than m_e but still considerably smaller than m_W . This generalization is motivated by the fact that the above processes may take place on stars with temperatures higher than m_e . In this case, the effective-Lagrangian approach is no longer appropriate.

In general, the electroweak phenomena associated with an intensive background magnetic field are rather rich. Processes which are forbidden in vacuum, but nevertheless opened up by the magnetic fields include $\bar{\nu}_e$, $\bar{\nu}_e e^+ e^-$, $\bar{\nu}_\mu$, $\bar{\nu}_\mu e^+ e^-$ and so on. Due to the presence of magnetic field, the calculations of the above amplitudes are very complicated. We aim to obtain simple expressions for the above amplitudes for sub-critical background magnetic fields $B < B_c = m_e^2/e^2 = 4 \times 10^{13}$ G. Namely, we look for weak-field expansions for these amplitudes in powers of B/B_c . In fact, in a practical astrophysical analysis, such as simulating the attenuation of gamma-rays from pulsars, it is important to obtain simplified expressions for processes $\bar{\nu}_e$

and $\bar{\nu}_e e^+ e^-$ [7].

三、結果與討論

We have also successfully reproduced the charged fermion propagator under a homogeneous magnetic field [8]. The small B-field expansion of the fermion propagator is derived using the Landau-level representation of the propagator [9-11]. We also show that a nontrivial phase factor should be included in the derivation of the scattering amplitude. Our results on the scattering amplitudes of $\bar{\nu}_e$, $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ under a background magnetic field are valid throughout the kinematic region where both photon and neutrino energies are less than m_W . At $E, E \ll m_e$, our results reduce to those obtained from the effective-Lagrangian approach [6]. At general energies, for example, $0.1 m_e < E, E < 50 m_e$, we find the cross sections of the above $2 \rightarrow 2$ scatterings, with a background magnetic field $B = 0.1 B_c$, are comparable to those of $\bar{\nu}_e$, $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$ [5].

To apply our weak-field expansion technique, we also computed the photon index of refraction and the neutrino Cherenkov process $\bar{\nu}_e$ for $B < B_c$ [11]. The results are given by much simpler expressions compared to those obtained by previous works.

We also generalized the weak-field expansion technique to processes with charged fermions in the final state. Specifically, we investigate the process $\bar{\nu}_e e^+ e^-$. Although $\bar{\nu}_e e^+ e^-$ was shown to be non-perturbative in B (due to the non-

analytic behaviors of electron and positron wave functions at $B=0$), we have achieved a breakthrough in demonstrating that the *moments* of pair-production width are proportional to the derivatives of photon polarization function at the zero energy, which is perturbative in B . Hence the pair-production width can be easily obtained from the latter by the inverse Mellin transform [12]. Therefore we have extended the applicability of weak-field expansions to processes with charged-fermions in the final state. The work on magnetic effects to $\bar{\nu} e^+ + e^-$ is still continued. We aim to obtain the pair-production width valid for energies near the production threshold [13].

四、計畫成果自評

During the past three years of this research, we did not address much on the finite temperature issue as we intended to in our original proposal. However we did adhere to our greater goal of establishing a research program in particle astrophysics. The major achievements are summarized as follows.

The weak-field expansion method we have developed significantly simplifies the calculation of physical processes under a background magnetic field. This is demonstrated in our calculations of neutrino photon scatterings, photon index of refraction and neutrino Cherenkov process. Most significantly, we develop a new method of computing the photon absorption coefficients [12]. In the ordinary quantum field theory, the absorptive part of a Feynman diagram is often easier to calculate. With the absorptive part, the dispersive part of the Feynman diagram can be computed via the well-known Kramers-Kronig relation. However, due to the presence of the external field, both of the absorptive and dispersive parts of a Feynman diagram are difficult to calculate at arbitrary energy. It is gratifying that we now

know how to compute the absorptive part of a Feynman diagram from its dispersive part in the vicinity of zero energy. Knowing the absorptive part, the dispersive part at any given energy is calculable via the Kramers-Kronig relation.

We expect that this new approach is also useful for quantum field theory in the vacuum. This arises from the observation that the inverse Mellin transform and Kramers-Kronig relation can be easily combined into a single integral transform. Hence the dispersive part of a Feynman diagram at any given energy can be obtained from the dispersive part near the zero energy by a one-parameter integral transform. Similarly the more involved calculations in the finite temperature field theory can be hopefully simplified by this approach as well. This result was presented this August (by Guey-Lin) in DPF 2000 held in Ohio State University.

In addition, we have published 6 other papers [14-19] on the topics related to quantum cosmology, Kaluza-Klein theory, monopole in curved space and inflationary models. We also have a number of preprints under journal review.

五、參考文獻

- [1] For a comprehensive review, see G. G. Raffelt, *Stars as Laboratories for Fundamental Physics*, (University of Chicago Press, Chicago, 1996).
- [2] G. G. Raffet, *APJ*. 365, 559 (1990).
- [3] P. B. Pal and L. Wolfenstein, *Phys. Rev. D* 25, 766 (1982).
- [4] M. Gell-Mann, *Phys. Rev. Lett.* 6, 70 (1961).
- [5] D. A. Dicus and W. W. Repko, *Phys. Rev. Lett.* 79, 569 (1997).
- [6] R. Shaisultanov, *Phys. Rev. Lett.* 80, 1586 (1998).
- [7] M. G. Baring and A. K. Harding, *APJ* 507, L55 (1998).

- [8] J. Schwinger, Phys. Rev. 82, 664 (1951).
- [9] A. Chodos, K. Everding, and D. A. Owen, Phys. Rev. D 42,2881 (1990).
- [10] T.-K. Chyi, C.-W. Hwang, W. F. Kao, G.-L. Lin, K.-W. Ng, and J.-J. Tseng, Phys. Lett. B466, 274 (1999).
- [11] T.-K. Chyi, C.-W. Hwang, W. F. Kao, G.-L. Lin, K.-W. Ng, and J.-J. Tseng, hep-th/9912134, Phys. Rev. D62 (2000) 105014.
- [12] W.-F. Kao, G.-L. Lin and J.-J. Tseng, hep-ph/0008240, to appear in Phys. Lett. B.
- [13] W.-F. Kao, G.-L. Lin and J.-J. Tseng, work in progress.
- [14] W.F. Kao, Kaluza-Klein Induced Gravity Inflation, Phys. Rev. D62 (2000) 084009, hep-th/0003153;
- [15] W.F. Kao, Inflationary Universe in Higher Derivative Induced Gravity, Phys. Rev. D62 (2000) 087301, hep-th/0003206;
- [16] W.F. Kao, Higher Derivative Weyl Gravity, Phys. Rev. D61(2000) 047501;
- [17] W.F. Kao, Magnetic Monopole In Induced Einstein-Yang-Mills Models, Phys. Rev. D61(2000)044004;
- [18] W.F. Kao, Shih-Yuin Lin, On Metric Dependence of Vilkovisky-DeWitt Effective Potential, MPL A13 (1998) 2111-16
- [19] W.B.Dai, W.F.Kao, Shang-Yung Wang, Magnetically Charged Black Holes In Induced Gravity, CJP, 36(1998)693-704
- Crossed Processes in a Background Magnetic Field, T.-K. Chyi, C.-W. Hwang, W.-F. Kao, G.-L. Lin, K.-W. Ng and J.-J. Tseng, Phys. Lett. B 466, 274-280 (1999), NSC-88-2112-M-009-002.
2. The Weak Field Expansion for Processes in a Homogeneous Background Magnetic Field, T.-K. Chyi, C.-W. Hwang, W.-F. Kao, G.-L. Lin, K.-W. Ng and J.-J. Tseng, hep-th/9912134, to appear in Phys. Rev. D, NSC-89-2112-M-009-035.
3. A new look at the pair production width in a strong magnetic field, W.-F. Kao, G.-L. Lin and Jie-Jun Tseng, hep-ph/0008240, to appear in Phys. Lett. B, NSC-89-2112-M-009-035.
4. Also references [14-19] listed above.

六、本計畫支助之論文成果

(1)、期刊論文：

1. Neutrino-Photon Scattering and its