行政院國家科學委員會專題研究計畫 期末報告

弦理論與凝態物理對偶

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 100-2112-M-009-007-
執 行 期 間 ：100 年 08 月 01 日至 101 年 10 月 31 日
執 行 單 位 ：國立交通大學電子物理學系（所）

計 畫 主 持 人：楊毅
共 同 主 持 人：李仁吉

報 告 附 件 ：出席國際會議研究心得報告及發表論文

公 開 資 訊 ：本計畫可公開查詢

中 華 民 國 101 年 12 月 27 日

中 文 摘 要 ： 在計畫執行期間，我與中國科學院數學研究所吳小寧教授合作研究黑洞相變問題。漸進 AdS 黑洞有非常豐富的相結構，不同類型的相變對應到與其對偶的場論中的不同相變。在場論中，相變常發生在強耦合情況下。研究場論中的相變通常需要複雜的非微擾方法研究。而與其對偶的引力理論是在弱耦合狀態下，一般的微擾方法，甚至古典極限就可以很好的用來研究它的相變。因此研究黑洞相變結構對於理解 AdS/CFT 對偶中的全像場論相變結構非常重要。我們研究了 Kerr/AdS 黑洞的相結構，仔細分析了 Kerr/AdS 黑洞中的三個不同的相變。我們的成果寫在論文[1]中。

在計畫執行期間，我還與國立台灣大學電子物理系教授李仁吉教授合作研究弦理論中的對稱性與高能弦散射振幅之間的關係。我們把我們之前在平直時空中計算任意質量弦在 Regge 極限下的高能散射振幅的方法進一步推廣，應用到緊致時空中[3]。在緊致時空與 Regge 極限下, 我們得到弦的高能散射振幅同樣可以表示成 Kummer 函數。並且通過一系列新得到的關於 Stirling 數的恒等式，我們同樣可以從 Regge 極限下的高能散射振幅得到在 Gross 極限下的高能散射振幅之間的線性比例常數。我們在[2]中嚴格證明了這一系列新得到的關於 Stirling 數的恒等式。更重要的是我們發現在緊致時空中的 Regge 散射在特定極限下表現出指數衰減的高能行為。我們在[4]中總結了我們近期得到的結果。

中文關鍵詞： 黑洞，相變，全像對偶；弦散射振幅，高能極限，Regge 表現，對稱性

英 文 摘 要 ：

英文關鍵詞：

報告內容

Phase structure of black holes

During the last few decades, black hole thermodynamics has been playing the role of a "thinking experiment" to understand quantum gravity. The discovery of Hawking radiation shows that the analogy between black hole mechanical laws and the laws of thermodynamics is physically meaningful. Based on this analogy, Davis pioneered to consider the phase transition of RN black holes. Hawking and Page later investigated the phase transition of Schwarzschild-AdS black holes. Following their path-breaking research, many works have been done along this direction and rich phase structures have been discovered. The later established AdS/CFT duality further inspired people to focus on the asymptotically anti-de Sitter (AdS) black holes. Critical phenomena were discovered in asymptotically AdS black holes.

In 1999, Chamblin et al. studied the phase structures of RN-AdS black hole. They identified a critical point in RN-AdS black hole by considering the divergence of heat capacity. Near this critical point, the behavior of isotherms are similar to that of van der Waals liquid/gas system. However, Wu showed that the critical exponents of RN-AdS black hole are different from that of the van der Waals case in 2000. A detailed investigation of the phase structure of Kerr-AdS black hole is needed to be compared to the previous results. Finding the critical phenomena will help us to achieve the ultimate goal of finding a microscopic description of the black hole phase structure.

The phase structure of Kerr-AdS black holes is also related to the holographic superconductors and their rotating extension. Sonner studied the superconducting phase transition on the boundary of Kerr-Newman-AdS black hole. The phase structure of the background field may affect some properties of the rotating holographic superconductor. And the knowledge of phase transitions of Kerr-AdS and RN-AdS black holes could be essential to fully understand the holographic superconductors.

We studied the phase structure of Kerr-AdS black hole. Rich phase structure

were discovered at three diverse critical temperatures, and this multi-critical phenomenon in Kerr-AdS black hole has not been carefully discussed in the previous literatures. We plotted the isotherm to describe the three critical temperatures. We detailed discussed the critical behavior of each isotherm near the three critical temperature T_{L}, T_{c1} and T_{c2}, respectively. We determined the asymptotic value of the angular momentum, which is important to understand the thermal stability of the Kerr-AdS black hole. At a certain temperature T_{c1}, we discovered van der Waals-like phase transition. Unlike the case of RN-AdS black hole, the critical exponents of Kerr-AdS black hole are found identical to the van der Waals liquid/gas system and the Weiss ferromagnet. It provides a strong evidence that Kerr-AdS black hole system belongs to the universality class which contains these two systems. We also discuss the scaling symmetry of the free energy near this critical point.

String scattering amplitudes and symmetries in string theory

Recently high-energy fixed angle string scattering amplitudes were intensively investigated for string states at arbitrary mass levels. One of the motivation of this calculation has been to uncover the fundamental hidden stringy spacetime symmetry conjectured more than twenty years ago. It was conjectured in late 80s that there exist linear relations or symmetries among scattering amplitudes in the high energy fixed angle regime, or Gross regime (GR). In the recent calculations, an infinite number of linear relations among high-energy scattering amplitudes of different string states were derived and the complete ratios among the amplitudes at each fixed mass level can be determined. An important new ingredient of this string amplitude calculation was based on an old conjecture on the decoupling of zero-norm states (ZNS) in the spectrum, in particular, the identification of inter-particle symmetries induced by the inter-particle ZNS in the spectrum.

There are three fundamental characteristics of high-energy fixed angle string scattering amplitudes, which are not shared by the field theory scattering. These are the softer exponential fall-off behavior (in contrast to the hard power-law behavior of field theory scatterings), the infinite Regge-pole

structure of the form factor and the existence of infinite number of linear relations, or stringy symmetries.

Another fundamental regime of high-energy string scattering amplitudes is the Regge regime (RR). It has been found that the high-energy string scattering amplitudes in the GR and RR contain information complementary to each other. On the other hand, since the decoupling of ZNS applies to all kinematic regimes, one expects that the ratios obtained from the decoupling of ZNS in the GR are closely related to the decoupling of ZNS or scattering amplitudes in the RR. Moreover, it is conceivable that there exists some link between the patterns of the high-energy scattering amplitudes in the GR and RR. It was found that the number of high-energy scattering amplitudes for each fixed mass level in the RR is much more numerous than that of GR calculated previously. In contrast to the case of scattering amplitudes in the GR, there is no linear relation among scattering amplitudes in the RR. Moreover, it was discovered that the leading order amplitudes at each fixed mass level in the RR can be expressed in terms of the Kummer function of the second kind. More surprisingly, for those leading order string tree four-point high-energy amplitudes in the RR with the same type as those of GR, one can extract from them the ratios. The calculation was based on a set of identities which depend on an integer parameter. The calculation can be done for both the case of open string and the closed string as well. The proof of these identities for L = 0, 1 was previously given based on a set of signed Stirling number identities. However, the proof of these identities for arbitrary integer values L is still lacking, and it is crucial to complete the proof in order to link high-energy string scattering amplitudes in the RR and GR regimes as claimed above.

Following an old suggestion of Mende, we calculated high-energy fixed angle massive scattering amplitudes of closed bosonic string with some coordinates compactified on the torus. The calculation was extended to the compactified open string scatterings. An infinite number of linear relations among high-energy scattering amplitudes of different string states were obtained in the fixed angle or Gross kinematic regime (GR). The UV behavior in the GR shows the usual soft exponential fall-off behavior. These results are reminiscent

of the existence of an infinite number of massive ZNS in the compactified closed and open string spectrums constructed previously. In addition, it was discovered that, for some kinematic regime with super-highly winding modes at fixed angle, the so-called Mende kinematic regime (MR), these infinite linear relations break down and, simultaneously, the string amplitudes enhance to hard power-law behavior at high energies instead of the usual soft exponential fall-off behavior.

We calculate high-energy small angle or Regge string scattering amplitudes of open bosonic string with one coordinates compactified on the torus. The results can be generalized to more compactified coordinates. It is shown that there is no linear relations among Regge scattering amplitudes as expected. However, as in the case of noncompactified Regge string scattering amplitude calculation, we can deduce the infinite GR ratios in the fixed angle from these compactified Regge string scattering amplitudes. We stress that the GR ratios calculated in the present paper by this indirect method from the Regge calculation are for the most general high-energy vertex rather than only a subset of GR ratios obtained directly from the fixed angle calculation. In this calculation, we have used a set of master identities to extract the GR ratios from Regge scattering amplitudes. Mathematically, we completed proof of these identities for arbitrary real value L by using an identity of signless Stirling number of the first kind in combinatorial number theory. It is interesting to see that, physically, the identities for arbitrary real value L can only be realized in high-energy compactified string scatterings considered in this work. All other high-energy string scatterings calculated previously correspond to integer value of L only.

More importantly, we discover an exponential fall-off behavior of high-energy compactified open string scatterings in a kinematic regime with highly winding modes at small angle. The existence of this regime was conjectured in our previous works. However, no Regge scatterings were calculated there and thus the results for the small angle scatterings extracted from the fixed angle calculation were not completed and fully reliable. The discovery of the soft exponential fall-off behavior in this kinematic regime with small angle in compactified string scatterings is in complementary with a kenematic regime

discovered previously, which shows the unusual power-law behavior in the high-energy fixed angle compactified string scatterings.

計劃期間發表論文：

[1] Y.D. Tsai, X.N. Wu and Y.Y., "Phase structure of the Kerr-AdS Black Hole", arXiv: Phys.Rev. D85, 044005 (2012)

[2] J.C. Lee, C.H. Yan and Y.Y., "High-energy String Scattering Amplitudes and Signless Stirling Number Identity", SIGMA 8 (2012), 045.

[3] S. He, J.C. Lee and Y.Y., "Exponential fall-off Behavior of Regge Scatterings in Compactified Open String Theory", Prog.Theor.Phys. 128, No. 5 (2012).

[4] J.C. Lee and Y.Y., "String Scattering Amplitudes in High Energy Limits", arXiv:1112.6077.

參加國際研討會

[1] Yi Yang, " Phase structure of the Kerr-AdS Black Hole ", poster presented at Strings2012, July 23-28, 2012, München, Germany.

[2] Yi Yang " Phase Structure in a Soft-Wall Holographic QCD Model ", poster presented at YITP From Gravity to Strong Coupling Physics workshop, Oct. 15-19, 2012, Kyoto, Japan.

Report on Strings2012

Yi Yang

I attended the conference of Strings2012 in München, Germany, during July 23th – 28th, 2012. Strings2012 is an annual international conferences on string theory and high energy physics and it is the most important international conference on string theory. The Strings conferences are held in difference countries every year. Almost 400 physicists attended the conference this year. More information can be found at http://strings2012.mpp.mpg.de.

The Strings2012 conference was held in Ludwig-Maximilians-Universität München in Germany. During the six days, 43 conference lectures were given as well as two public lectures. These lectures covered the recent development about string theory during the last year (the videos and slides of the lectures can be found at http://wwwth.mpp.mpg.de/members/strings/strings2012/strings_files/program/talks.html). Strings2012 is a great opportunity to learn the most important and the most recent development in string theory.

The lectures cover a wide range of string theory and string related topics which include particle physics and cosmology, etc. In their lectures, most of the speakers mainly focus on the following several topics:

- String duality and mirror symmetry
- String phenomenology
- String scattering amplitudes
- Higher spin theory
- LHC physics
- String cosmology
- Holographic models

These are the topics on which most of string physicists have been working during the last year. In Strings2012, I made a poster presentation about my current work on building a holographic model of strong coupled field theory by AdS/CFT duality. By studying the phase structure of the Einstein-Maxwell-dilaton background, we are able to explore the phase structure of the dual strong coupled field theory, such as QCD and superconductor theory.

During the conference, I had a lot of useful discussion with string physicists from all over the world. Through the discussion, I had better understanding about many important works related to string theory and learned many new interesting topics which people are working on recently. And of course, I also got to know many new friends in string society, which certainly will increase the opportunity for me to have extensive communication and collaboration with them in the future.

After all, Strings2012 is a very successful conference. I have learned many things and got to know many string people. It is very helpful for our future research. I hope to have more chances to attend the conference like this. I sincerely appreciate National Science Council (NSC) for financially supporting me to attend Strings2012.

國科會補助計畫衍生研發成果推廣資料表

日期:2012/12/26

國科會補助計畫	計畫名稱: 弦理論與凝態物理對偶	
	計畫主持人: 楊毅	
	計畫編號: 100-2112-M-009-007-	學門領域: 重力、高能、粒子及核物理－理論

無研發成果推廣資料

100 年度專題研究計畫研究成果彙整表

計畫主持人：楊毅	計畫編號：100-2112-M-009-007-
計畫名稱：弦理論與凝態物理對偶	

成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	1	1	100%	人次	
		博士生	1	1	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	3	3	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	1	1	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	1	1	100%	人次	
		博士生	0	0	100%		
		博士後研究員	1	1	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以 100 字為限）

□實驗失敗

□因故實驗中斷

□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：■已發表 □未發表之文稿 □撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：(以 100 字為限)

執行本計劃期間，我們共完成 3 篇學術論文，均已已經發表在國際 SCI 期刊上。另有一篇會議論文。

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）(以 500 字為限)

我們研究了漸進 AdS 黑洞的相變結構，分析了不同類型的相變行為。研究結果對於用 AdS/CFT 對偶方法來研究包括超導，量子色動力學等強耦合場論有一定的指導意義。

我們還研究了不同高能極限下緊致時空中的弦散射振幅，研究結果使我們進一步理解弦理論中的隱藏對稱性。