Fast Radix-q and Mixed-Radix Algorithms for Type-IV DCT

Han-Wen Hsu and Chi-Min Liu

***Abstract*—This letter proposes a fast radix-q algorithm to compute type-IV discrete cosine transform (DCT) of the length q^λ, where q is an odd positive integer. The proposed fast radix-q algorithm has merits in computational complexity, parallelism, and numerical stability over existing algorithms. Furthermore, the fast radix-q algorithm is used to develop the fast mixed-radix type-II/type-IV DCT algorithm for composite lengths.**

***Index Terms*—Fast mixed-radix DCT algorithm, fast radix-q DCT algorithm.**

I. Introduction

The type-IV discrete cosine transform (DCT-IV) is the fundamental module in the efficient computation of the lapped orthogonal transforms and cosine modulated filter banks known as modulated lapped transforms (MLTs) [1] or modified discrete cosine transforms (MDCTs) [2]. In the literature, there exist various fast radix-2 algorithms for type-II DCT (DCT-II) and type-III DCT (DCT-III) [3], [4]. The fast radix-q algorithms for the DCT-II/DCT-III computation have been also developed and extended to the fast mixed-radix algorithms for composite lengths [5], [6]. On the computation of DCT-IV, we can consider the four existing approaches [7], which convert DCT-IV into DCT-II or DCT-III, and then apply the associated fast algorithms through the four matrix product representations

$$\begin{aligned} C_N^{IV} &= LC_N^{II}D = D^{-1}C_N^{III}D_1L^{-1} = D^TC_N^{III}L^T \\ &= (L^{-1})^TD_1^TC_N^{II}(D^{-1})^T \end{aligned} \tag{1}$$

where diagonal matrices D and D_1 of order N are defined by $\operatorname{diag}\{2\cos(\pi(i+1/2)/2N)|i=0,1,\ldots,N-1\}$ and $\operatorname{diag}\{1/2,1,1,\ldots,1\}$, respectively, lower triangular matrix L is defined by the serial computation: $[y_0,y_1,\ldots,y_{N-1}]^T = L[x_0,x_1,\ldots,x_{N-1}]^T = [x_0/2, x_1-y_0, x_2-y_1,\ldots,x_{N-1}-y_{N-2}]^T$, and the DCT-II/DCT-III/DCT-IV matrices are defined by $\left(C_N^{II}\right)_{n,k} = \cos(\pi(n+1/2)k/N)/\left(C_N^{III}\right)_{n,k} = \cos(\pi n(k+1/2)/N)/\left(C_N^{IV}\right)_{n,k} = \cos(\pi(n+1/2)(k+1/2)/N)$, for $0 \leqq k, n \leqq N-1$. However, the four implementation methods indicated in (1) involve either serial computations or reciprocal cosine coefficients which result in large dynamic ranges. Therefore, these DCT-II/DCT-III-based fast algorithms have limitation in implementation. This letter proposes a fast radix-q algorithm for the DCT-IV computation with merits in parallelism, numerical stability, and computational complexity. Moreover, composite lengths have been used in several practical applications, such as the 12/36-point MDCT in MPEG-1/2 Layer-III (MP3) audio coding. The proposed radix-q algorithm is extended to the fast mixed-radix DCT-II/DCT-IV computation for composite lengths. Both the proposed radix-q and mixed-radix algorithms are examined on the merits in various transform lengths.

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The authors are with the Department of Computer Science, National Chiao Tung University, Hsinchu 300, Taiwan, R.O.C. (e-mail: hwhsu@csie.nctu.edu.tw; eldon.liu@videace.com).

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II. Fast Radix-q Algorithm for DCT-IV Computation

We begin with the scaled DCT-IV (SDCT-IV) defined as

$$Y_k = \sqrt{2}\cdot\sum_{n=0}^{N-1} x_n\cos\Phi_{n,k}^N, \quad k=0,1,\ldots,N-1 \tag{2}$$

where $\Phi_{n,k}^N$ denotes $\pi(n+1/2)(k+1/2)/N$. For the case of the length 1, DCT-IV requires one multiplication, but SDCT-IV requires no multiplication. Let the sequence length N be a multiple of q that is an odd positive integer. Equation (2) can be partitioned into q superpositions by grouping the terms with the same indices under the module q as

$$\begin{aligned} Y_k =& \sqrt{2}\sum_{n=0}^{N/q-1} x_{qn+(q-1)/2}\cos\Phi_{n,k}^{N/q} \\ &+ \sqrt{2}\sum_{m=0}^{(q-3)/2}\sum_{n=0}^{N/q-1} x_{qn+m}\cos\Phi_{qn+m,k}^N \\ &+ \sqrt{2}\sum_{m=0}^{(q-3)/2}\sum_{n=0}^{N/q-1} x_{qn+q-1-m}\cos\Phi_{qn+q-1-m,k}^N. \end{aligned} \tag{3}$$

Combining the second and the third terms of (3) and using the trigonometric identity $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$, we obtain

$$Y_k = A_k + \sum_{m=0}^{(q-3)/2}\left(C_k^m\cos\Theta_{m,k}^N + S_k^m\sin\Theta_{m,k}^N\right) \tag{4}$$

where

$$A_k = \sqrt{2}\sum_{n=0}^{N/q-1} x_{qn+(q-1)/2}\cos\Phi_{n,k}^{N/q} \tag{5}$$

$$C_k^m = \sqrt{2}\sum_{n=0}^{N/q-1}(x_{qn+m}+x_{qn+q-1-m})\cos\Phi_{n,k}^{N/q} \tag{6}$$

$$S_k^m = \sqrt{2}\sum_{n=0}^{N/q-1}(x_{qn+m}-x_{qn+q-1-m})\sin\Phi_{n,k}^{N/q} \tag{7}$$

or,

$$S^m_{N/q-1-k} = \sqrt{2}\sum_{n=0}^{N/q-1}(-1)^n(x_{qn+m}-x_{qn+q-1-m}) \cdot \cos\Phi^{N/q}_{n,k} \tag{8}$$

$$\Theta^N_{m,k} = \frac{\pi}{N}\left(\frac{q-1}{2}-m\right)\left(k+\frac{1}{2}\right) \tag{9}$$

for $m=0,1,\ldots,(q-3)/2, \quad k=0,1,\ldots,N-1.$

Therefore, (4) consists of q length-N/q SDCTs-IV defined by (5)–(8). Furthermore, it can be shown that for any integer p

$$A_{2pN/q+k} = A_{2pN/q-1-k} = (-1)^p A_k \tag{10}$$

$$C^m_{2pN/q+k} = C^m_{2pN/q-1-k} = (-1)^p C^m_k \tag{11}$$

$$S^m_{2pN/q+k} = -S^m_{2pN/q-1-k} = (-1)^p S^m_k. \tag{12}$$

In order to save multiplications, utilizing properties (10)–(12), we form the two sequences U^p_k and V^p_k that are 1/2 of the sum and difference of $Y_{2pN/q+k}$ and $Y_{2pN/q-1-k}$ in the following:

$$U^p_k = (-1)^p A_k + \sum_{m=0}^{(q-3)/2}\left(C^m_k\cos\Theta^N_{m,k} + S^m_k\sin\Theta^N_{m,k}\right) \cdot \cos\frac{p(2m+1)\pi}{q}$$

for $p=0,1,\ldots,(q-1)/2, \quad k=0,1,\ldots,N/q-1 \quad (13)$

$$V^p_k = \sum_{m=0}^{(q-3)/2}\left(C^m_k\sin\Theta^N_{m,k} - S^m_k\cos\Theta^N_{m,k}\right) \cdot \sin\frac{p(2m+1)\pi}{q}$$

for $p=1,2,\ldots,(q-1)/2, \quad k=0,1,\ldots,N/q-1. \quad (14)$

Similar to the strategy in [5], for each k, $(q-1)/2$ multiplications are saved by moving the cosine coefficients to the outside of brackets in (13) and (14) for each p, respectively. However, the range of the angles $\Theta^N_{m,k}$ is from 0 to $\pi/2$, and the dynamic range of tangent values is large. To control the numerical stability, (13) and (14) are derived as

$$U^p_k = (-1)^p A_k + \sum_{m=0}^{(q-3)/2} T^m_k \cdot \left(\Lambda^m_k \cos\frac{p(2m+1)\pi}{q}\right)$$

for $p=0,1,\ldots,(q-1)/2, \quad k=0,1,\ldots,N/q-1 \quad (15)$

$$V^p_k = \sum_{m=0}^{(q-3)/2} H^m_k \cdot \left(\Lambda^m_k \sin\frac{p(2m+1)\pi}{q}\right)$$

for $p=1,2,\ldots,\frac{(q-1)}{2}, \quad k=0,1,\ldots,N/q-1 \quad (16)$

where we see (17) at the bottom of the page. In (17), the dynamic range of the tangent and cotangent values is controlled within the interval $[0,1]$. The final SDCT-IV outputs are obtained from

$$Y_k = U^0_k, \text{ for } k=0,1,\ldots,N/q-1 \tag{18}$$

$$Y_{2pN/q+k} = U^p_k + V^p_k; \quad Y_{2pN/q-1-k} = U^p_k - V^p_k$$

$$\text{for } p=1,2,\ldots,\frac{(q-1)}{2}, \quad k=0,1,\ldots,N/q-1. \tag{19}$$

Equation (18) is obtained from the symmetry around $k=-1/2$ of DCT-IV output. For a length-q^λ SDCT-IV, the decomposition must be repeated until the length of subsequences is one. To obtain the output of DCT-IV or inverse DCT-IV, N multiplications are required for the scaling operations. By absorbing the scaling factors δ, which are $1/\sqrt{2}$ and $\sqrt{2}/N$ for DCT-IV and inverse DCT-IV, respectively, into the inside of the summations of (15) and (16), the number of scaling operations can be reduced from N to N/q. To summarize, the proposed algorithm in (5)–(8) and (15)–(19) will be explained to have merits in complexity, numerical stability, and parallelism.

III. Fast Radix-3, -5, and -9 DCT-IV Algorithms

Based on the proposed fast algorithms, this section derives and tunes the fast algorithms for radix-3, radix-5, and radix-9 DCT-IV computation. For $k=0,1,\ldots,N/q-1$, the arithmetic costs for radix-q SDCT-IV computation are listed as follows:

1) $q(q-1)/2$ multiplications and $(q-1)^2/2$ additions in (15) and (16);
2) $q-1$ multiplications and $q-1$ additions in (17);
3) $q-1$ additions in (6) and (8) and $q-1$ additions in (19).

Totally, the numbers of multiplications and additions required by the radix-q algorithm for length N SDCT-IV computation are listed in Table I(a) and (b). For DCT-IV computation, additional N/q multiplications for scaling operations are required. Let $N=q^\lambda$, the arithmetic costs for DCT-IV are listed in Table I(c) and (d). In general, a lower computational cost than that induced from Table I(c) and (d) can be achieved by rearranging the operation factors. Also, the optimization of the initial case for small length-q SDCT-IV can reduce the overall complexity. In the following, the cases of $q=3$, 5, and 9 are examined in detail.

A. $N=3^\lambda$

As $q=3$ and $p=1$, one multiplication is saved for a trivial factor $\cos(\pi/3)=0.5$ in (15). Also, if N is odd and $k^*=(N/3-1)/2$, it implies $\Theta^N_{0,k}=\pi/6$ and

$$Y_{k^*} = \left(A_{k^*}+\frac{1}{2}S^0_{k^*}\right)+\frac{\sqrt{3}}{2}C^0_{k^*} \tag{20}$$

$$Y_{2N/3+k^*} = -\left(A_{k^*}+\frac{1}{2}S^0_{k^*}\right)+\frac{\sqrt{3}}{2}C^0_{k^*} \tag{21}$$

$$Y_{2N/3-1-k^*} = S^0_{k^*} - A_{k^*}. \tag{22}$$

Equations (20)–(22) require one multiplication and four additions; thus, three multiplications and two additions are saved. Hence, from Table I(a) and (b), the nontrivial arithmetic costs

$$(\Lambda^m_k, T^m_k, H^m_k) = \begin{cases}\left(\cos\Theta^N_{m,k}, C^m_k + S^m_k\tan\Theta^N_{m,k}, C^m_k\tan\Theta^N_{m,k} - S^m_k\right), & \text{if } \Theta^N_{m,k} \le \frac{\pi}{4}\\ \left(\sin\Theta^N_{m,k}, C^m_k\cot\Theta^N_{m,k} + S^m_k, C^m_k - S^m_k\cot\Theta^N_{m,k}\right), & \text{if } \Theta^N_{m,k} > \frac{\pi}{4}\end{cases} \tag{17}$$

TABLE I
ARITHMETIC COSTS LIST

$M_{S\text{-}IV}(N) = qM_{S\text{-}IV}(N/q)+(q-1)(q+2)/2\cdot N/q$	(a)	$A_{S\text{-}IV}(N) = qA_{S\text{-}IV}(N/q)+(q-1)(q+5)/2\cdot N/q$	(b)
$M_{IV}(N)=(q-1)(q+2)/(2q)\cdot N\log_q N+N/q$	(c)	$A_{IV}(N)=(q-1)(q+5)/(2q)\cdot N\log_q N$	(d)
$M_{S\text{-}IV}(N)=3M_{S\text{-}IV}(N/3)+4N/3-3,\ N>1$	(e)	$A_{S\text{-}IV}(N)=3A_{S\text{-}IV}(N/3)+8N/3-2,\ N>1$	(f)
$M_{IV}(N)=M_{S\text{-}IV}(N)+N/3+1,\ N>1$	(g)	$M_{S\text{-}IV}(N)=5M_{S\text{-}IV}(N/5)+11N/5,\ N>1$	(h)
$A_{S\text{-}IV}(N)=5A_{S\text{-}IV}(N/5)+21N/5,\ N>1$	(i)	$M_{IV}(N)=M_{S\text{-}IV}(N)+N/5,\ N>1$	(j)
$M_{S\text{-}IV}(N)=9M_{S\text{-}IV}(N/9)+20N/9-3,\ N>9$	(k)	$A_{S\text{-}IV}(N)=9A_{S\text{-}IV}(N/9)+53N/9+1,\ N>9$	(l)
$M_{IV}(N)=M_{S\text{-}IV}(N)+N/9+1,\ N>9$	(m)		
$M_{II}(N)=M_{II}(N/2)+M_{IV}(N/2)$	(n)	$M_{IV}(N)=2M_{II}(N/2)+3N/2$	(o)
$A_{II}(N)=A_{II}(N/2)+A_{IV}(N/2)+N$	(p)	$A_{IV}(N)=2A_{II}(N/2)+5N/2-2$	(q)
$M_{II}(N)=2M_{II}(N/2)+N/2,\ N>q$	(r)	$A_{II}(N)=2A_{II}(N/2)+3N/2-1,\ N>q$	(s)

TABLE II
ARITHMETIC COMPLEXITY COMPARISON FOR DCT-IV OF $N = q^\lambda$

	The proposed algorithm		DCT-II-based algorithm	
q	$M_{IV}(N),\ N>q$	$M_{IV}(q)$	$M_{IV}(N),\ N>q$	$M_{IV}(q)$
3	$4/3N\log_3 N-7N/6+5/2$	3	$4/3\cdot N\log_3 N-17N/18+3/2$	4
5	$11/5N\log_5 N$	11	$11/5\cdot N\log_5 N-7N/10+3/2$	9
7	$27/7\cdot N\log_7 N+N/7$	28	$27/7\cdot N\log_7 N-N/2+3/2$	25
9	$20/9\cdot N\log_9 N-59N/72+11/8$	16	$23/9\cdot N\log_9 N-7N/8+15/8$	17
q	$A_{IV}(N),\ N>q$	$A_{IV}(q)$	$A_{IV}(N),\ N>q$	$A_{IV}(q)$
3	$8/3\cdot N\log_3 N-N+1$	6	$8/3\cdot N\log_3 N-7N/9+1$	6
5	$21/5\cdot N\log_5 N$	21	$21/5\cdot N\log_5 N-N+1$	17
7	$36/7\cdot N\log_7 N$	36	$36/7\cdot N\log_7 N-N+1$	30
9	$53/9\cdot N\log_9 N-103N/72-1/8$	40	$50/9\cdot N\log_9 N-N+1$	42

required for SDCT-IV computation are listed in Table I(e) and (f), where the initial values are $M_{S\text{-}IV}(1) = A_{S\text{-}IV}(1) = 0$. The corresponding DCT-IV multiplicative complexity is given in Table I(g), where the number of scaling multiplications is two for (20) and (22) instead of one.

B. $N = 5^\lambda$

As $q = 5$, applying $\cos 4\pi/5 + 1/2 = -\cos 2\pi/5$ to (15) for $p = 1$ and 2 gives

$$U_k^1 = -A_k + \frac{1}{2}B_k^0 + \left(B_k^0 - B_k^1\right)\cos\frac{2\pi}{5} \tag{23}$$

$$U_k^2 = A_k - \frac{1}{2}B_k^1 + \left(B_k^0 - B_k^1\right)\cos\frac{2\pi}{5} \tag{24}$$

where $B_k^m = C_k^m \cos\Theta_{m,k}^N + S_k^m \sin\Theta_{m,k}^N$ for $m = 0,\ 1$. Equations (23) and (24) require one multiplication and five additions instead of four multiplications and four additions; thus, three multiplications are saved but one more addition is required. Hence, according to Table I(a) and (b), the nontrivial arithmetic costs required for SDCT-IV are listed in Table I(h) and (i). The corresponding DCT-IV multiplicative complexity is given by Table I(j), where the scaling factor δ can be absorbed into B_k^m as $C_k^m \cdot \delta\cos\Theta_{m,k}^N + S_k^m \cdot \delta\sin\Theta_{m,k}^N$. For the initial case $N = 5$, using $\sin\pi/10 = \cos 2\pi/5$ and $\cos\pi/10 = \sin 2\pi/5$ in (16) gives

$$V_0^1 = a\cdot\sin^2\frac{\pi}{5} - b\cdot\sin^2\frac{\pi}{5} \tag{25}$$

$$V_0^2 = (a-b)\cdot\sin\frac{\pi}{5}\sin\frac{2\pi}{5} \tag{26}$$

where $a = C_0^0 - S_0^0\cot\frac{\pi}{5}$ and $b = C_0^1\cot\frac{2\pi}{5} - S_0^1$.

C. $N = 9^\lambda$

For $q = 9$, applying the relations $\cos\pi/9 = \cos 4\pi/9 + \cos 2\pi/9$ and $\sin\pi/9 = \sin 4\pi/9 - \sin 2\pi/9$ to (15) and (16) for each p gives

$$\begin{cases} U_k^0 = a_k + b_k \\ U_k^3 = -a_k + b_k/2 \\ U_k^2 = c_k + d_k\cos\frac{\pi}{9} - e_k\cos\frac{4\pi}{9} \\ U_k^4 = c_k - d_k\cos\frac{2\pi}{9} + e_k\cos\frac{\pi}{9} \\ U_k^1 = -c_k + d_k\cos\frac{4\pi}{9} + e_k\cos\frac{2\pi}{9} \end{cases} \tag{27}$$

$$\begin{cases} V_k^1 = E_k^1 + f_k\sin\frac{\pi}{9} + g_k\sin\frac{2\pi}{9} \\ V_k^2 = E_k^1 + f_k\sin\frac{2\pi}{9} - g_k\sin\frac{4\pi}{9} \\ V_k^4 = -E_k^1 + f_k\sin\frac{4\pi}{9} - g_k\sin\frac{\pi}{9} \\ V_k^3 = \frac{\sqrt{3}}{2}(E_k^0 - E_k^2 + E_k^3) \end{cases} \tag{28}$$

where for $k = 0, 1, \ldots, N/9 - 1$, we define

$$a_k = A_k + B_k^1;\ b_k = B_k^0 + B_k^2 + B_k^3;\ c = A_k - \frac{1}{2}B_k^1;$$
$$d_k = B_k^0 - B_k^2;\ e_k = B_k^0 - B_k^3;\ f_k = E_k^0 + E_k^2;$$
$$g_k = E_k^2 + E_k^3;\ E_k^1 = H_k^1\cdot\Lambda_k^1\sin\frac{\pi}{3};$$
$$E_k^m = C_k^m\sin\Theta_{m,k}^N - S_k^m\cos\Theta_{m,k}^N,\ m = 0, 2, 3;$$
$$B_k^m = C_k^m\cos\Theta_{m,k}^N + S_k^m\sin\Theta_{m,k}^N,\ m = 0, 1, 2, 3.$$

By the strategy adopted in [5, App. C], computing (27) and (28) requires only 20 multiplications and 37 additions for each k. Furthermore, if N is odd, it implies that $\Theta_{1,k}^N = \pi/6$ as $k^* = (N/9 - 1)/2$ and $m = 1$, and thus

$$B_{k^*}^1 = \frac{\sqrt{3}}{2}C_{k^*}^1 + \frac{1}{2}S_{k^*}^1 \tag{29}$$

$$E_{k^*}^1 = \frac{1}{2}\left(\frac{\sqrt{3}}{2}C_{k^*}^1\right) - \frac{1}{4}\left(2S_{k^*}^1 + S_{k^*}^1\right) \tag{30}$$

which require only one multiplication and three additions. Hence, three multiplications are saved, but one addition is wasted. On the other hand, 16 additions used in (6)–(8), and (19) should be counted. For the scaling operations of DCT-IV, it requires one more multiplication as (31) and (32) given by

$$\delta B_{k^*}^1 = \delta\frac{\sqrt{3}}{2}C_{k^*}^1 + \frac{1}{2}\delta S_{k^*}^1 \tag{31}$$

$$\delta E_{k^*}^1 = \frac{1}{2}\left(\delta\frac{\sqrt{3}}{2}C_{k^*}^1\right) - \frac{1}{4}\left(2\delta S_{k^*}^1 + \delta S_{k^*}^1\right). \tag{32}$$

In summary, the nontrivial arithmetic costs required for the SDCT-IV are listed in Table I(k) and (l). The initial cases are $M_{S\text{-}IV}(9) = 12$ and $A_{S\text{-}IV}(9) = 40$ that are derived from the radix-3 algorithm. Thus, the corresponding DCT-IV multiplicative complexity is given by Table I(m) with $M_{IV}(9) = 16$ that are derived from the radix-3 algorithm.

IV. COMPARISON ON PARALLELISM, NUMERICAL STABILITY, AND ARITHMETIC COMPLEXITY

A. Parallelism and Numerical Stability

Each DCT-II-based algorithm for DCT-IV computation illustrated in (1) involves either serial computations or reciprocal cosine coefficients. However, the proposed radix-q algorithm avoids reciprocal cosine coefficients, especially due to the mechanism in (17), and has good numerical stability.

Moreover, if the latency of hardware implementation is considered, the length of the critical path of the DCT-II-based algorithm involving the serial computation is N because of the recursive computation for matrix L. The unit of the length is one multiplication or addition operation. For the proposed radix-q algorithm, the length of the critical path is $ceiling\{\log_2[(q-3)/2]\}$ because of the summation in (15). This result shows the critical path of the proposed radix-q algorithm is significantly shorter than that of the DCT-II-based algorithm involving the serial computation.

B. Arithmetic Complexity for $N = q^\lambda$

By (1), any DCT-IV can be computed through a fast DCT-II algorithm with additional N multiplications and $N - 1$ divisions. Table II compares the arithmetic complexity of the proposed DCT-IV algorithm and the DCT-II-based algorithm, where the matrix decomposition is $C_N^{IV} = LC_N^{II}D$, and the fast DCT-II algorithm of the length $N = q^\lambda$ is adopted from [5]. The proposed algorithm not only is free from the serial computation and numerical instability but also achieves a lower arithmetic complexity than the DCT-II-based algorithm for $q = 3$ and 9.

C. Fast Mixed-Radix DCT-II /DCT-IV Algorithm

For composite lengths, i.e., $N = 2^{\lambda_0} \cdot q_1^{\lambda_1} \cdot q_2^{\lambda_2} \cdot \cdots \cdot q_n^{\lambda_n}$, for odd integers $0 < q_1 < q_2 < \cdots < q_n$ and any nonnegative integers $\lambda^0, \lambda^1, \ldots, \lambda^n$, DCT-IV can be computed through the fast DCT-II algorithm given by (1). However, the proposed radix-q algorithm can be flexibly combined with the existing fast DCT-II/DCT-IV algorithms for composite lengths to achieve the better performance.

1) Radix-2 DCT-II/DCT-IV Algorithm: Let the length N be even. The illustrated radix-2 DCT-II/DCT-IV algorithms consists of Wang's and Britanak's algorithms that are expressed as [8, eq. (50)] and [6, eq. (16)]. The radix-2 DCT-II/DCT-IV decomposes length-N DCT-II and DCT-IV into a length-$N/2$ DCT-II and length-$N/2$ DCT-IV or two length-$N/2$ DCTs-II without serial computations and reciprocal cosine coefficients. The complexity expressions are listed in Table I(n)–(q). By combining the proposed radix-q DCT-IV algorithm with the existing radix-2 DCT-II/DCT-IV algorithm and the existing radix-q DCT-II algorithm [5], the proposed mixed-radix algorithm for DCT-II/DCT-IV computation can achieve the merits in parallelism, complexity, and numerical stability.

Fig. 1 compares the number of multiplications per input sample for DCT-IV with $N = q \times 2^\lambda$ and shows the efficiency of sequence lengths other than a power of two.

2) Comparison of $N = q \times 2^\lambda$: The fast computation of DCT-II with composite length $q \times 2^\lambda$ can be realized through the radix-2 [3] and radix-q [5] algorithms. The associated arithmetical costs are listed in Table I(r) and (s). Table III lists the arithmetic complexity reduction of DCT-II and DCT-IV when $q = 3$ and 9 by comparing the proposed mixed-radix method and the DCT-II-based method. The result shows the introduction of the proposed radix-q algorithm for DCT-IV computation improves not only the computation of DCT-IV but also that of DCT-II for the two cases due to the more efficient computation of length-3 and length-9 DCTs-IV.

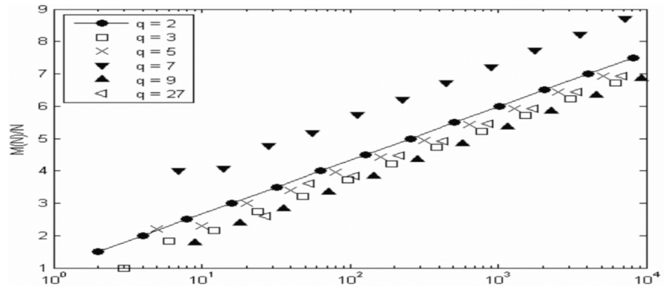

Fig. 1. Multiplicative cost of DCT-IV by the proposed method for $N = q \times 2^\lambda$.

TABLE III
ARITHMETIC COMPLEXITY REDUCTION

q=3	DCT-IV		DCT-II		q=9	DCT-IV		DCT-II	
N	×	+	×	+	N	×	+	×	+
12	2	0	1	0	18	0	0	1	2
24	2	0	3	0	36	2	4	1	2
48	6	0	5	0	72	2	4	3	6
96	10	0	11	0	144	6	12	5	10
192	22	0	21	0	288	10	20	11	22
384	42	0	43	0	576	22	44	21	42
768	86	0	85	0	1152	42	84	43	86
1536	170	0	171	0	2304	86	172	85	170

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