行政院國家科學委員會專題研究計畫 成果報告

序列複合買權法應用於履約保證下多期BOT基礎建設專案評價之研究

研究成果報告(精簡版)

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 100-2221-E-009-137-
執 行 期 間 ：100年08月01日至101年07月31日
執 行 單 位 ：國立交通大學土木工程學系（所）

計畫主持人：黃玉霖

計畫參與人員：博士班研究生-兼任助理人員：畢佳琪
博士班研究生-兼任助理人員：林岑縉

公 開 資 訊 ：本計畫可公開查詢

中　華　民　國　101年10月31日

中 文 摘 要 ： 利用 BOT 模式來推動基礎建設民營化，尤其是大規模基礎建設投資專案， 已成為全球營建市場一種成熟的趨勢。然而，在不確定的市場環境與無可預期的各種變化之下，這些大規模的 BOT 投資專案必須面對極大的投資風險，特別是針對特定市場量身制裁的長期專屬資產(dedicated assets)投資所形成沉默成本投資(sunk cost)。為了吸引私部門進行這些投資，各國政府提供各種特許權利誘因措施，以協助投資者進行 BOT 專案的風險控管，如：還款保證、最低營收保證、分期投資擴張規模的權利，以及特許期間屆滿前的放棄權利等。這些特許誘因的出現，意味著傳統的專案評價方法，如折現現金流模型(discounted cash flow model, DCF)，已不再適用於 BOT 專案評價。為了解決較複雜的 BOT 評價的問題，各國的研究人員已經發展出許多不同的實質選擇權評價模型(real-option valuation models)。

雖然實質選擇權的評價方法很盛行，但現有的 BOT 實質選擇權評價模型，尚未針對履約保證金對專案價值的影響，提出任何評估。特別是，BOT 專案價值在特許契約簽訂時是不確定的，必須依據契約執行期間所獲取的專案新訊息，不斷地進行重新評價。這將產生’套牢’(’hold-ups’)的可能性。亦即，即使某項建設經重新評價之後認定是不再值得投資的，政府仍然可能會利用已簽屬的合約條款向法院提出強制執行的請求，要求特許公司繼續進行該項不可行的投資。因此，為了迴避風險， BOT 特許契約通常會賦予特許公司，在契約執行期間提前放棄投資的權利。然而，政府也會藉由履約保證金的設定，確保投資者不會任意行使該項放棄權利。

在實質選擇權評價理論下，提前放棄投資的權利是具有潛在價值的；但是，履約保證金的設置將會降低此潛在價值。即使履約保證金可以避免任意行提前放棄的權利，這還是會帶來問題。更具體的說：如果 BOT 專案實質選擇權的評價不考量履約保證金的負面影響，評價結果將會高估專案投資價值，甚至誤導投資。

有鑑於此，本研究將建立序列複合買權評價模型(sequential compound call option valuation model)，推導多期 BOT 投資專案在履約保證之下的評價封閉解。本研究也將提供敏感性分析來探討履約保證金對於 BOT 專案價值的影響。同時，本研究將利用 MATLAB 程式來撰寫 SCCO 的專案評價軟體，選定實際的投資案例，進行專案評價的應用與分析。

中文關鍵詞： BOT、民營化、多期基礎建設投資、履約保證、序列複合買權、評價、敏感度分析

英 文 摘 要 ：

英文關鍵詞：

行政院國家科學委員會專題研究計畫成果報告

序列複合買權法應用於履約保證下多期BOT 基礎建設專案評價之研究

Sequential compound call option valuation of multistage BOT infrastructure projects under performance bonding

計畫編號：NSC 100－2221－E－009－137

執行期限：100年8月01日 至 101年7月31日

主 持 人：黃玉霖 交通大學土木系 教授

計畫參與人員：畢佳琪、林岑縉

中文摘要

利用 BOT 模式來推動基礎建設民營化，尤其是大規模基礎建設投資專案， 已成為全球營建市場一種成熟的趨勢。然而，在不確定的市場環境與無可預期的各種變化之下，這些大規模的 BOT 投資專案必須面對極大的投資風險，特別是針對特定市場量身制裁的長期專屬資產(dedicated assets)投資所形成沉默成本投資(sunk cost)。為了吸引私部門進行這些投資，各國政府提供各種特許權利誘因措施，以協助投資者進行 BOT 專案的風險控管，如：還款保證、最低營收保證、分期投資擴張規模的權利，以及特許期間屆滿前的放棄權利等。這些特許誘因的出現，意味著傳統的專案評價方法，如折現現金流模型(discounted cash flow model, DCF)，已不再適用於BOT 專案評價。為了解決較複雜的 BOT 評價的問題，各國的研究人員已經發展出許多不同的實質選擇權評價模型(real-option valuation models)。

雖然實質選擇權的評價方法很盛行，但現有的 BOT 實質選擇權評價模型，尚未針對履約保證金對專案價值的影響，提出任何評估。特別是，BOT 專案價值在特許契約簽訂時是不確定的，必須依據契約執行期間所獲取的專案新訊息，不斷地進行重新評價。這將產生“套牢” (“hold-ups”) 的可能性。亦即，即使某項建設經重新評價之後認定是不再值得投資的，政府仍然可能會利用已簽屬的合約條款向法院提出強制執行的請求，要求特許公司繼續進行該項不可行的投資。因此，為了迴避風險， BOT 特許契約通常會賦予特許公司，在契約執行期間提前放棄投資的權利。然而，政府也會藉由履約保證金的設定，確保投資者不會任意行使該項放棄權利。

在實質選擇權評價理論下，提前放棄投資的權利是具有潛在價值的；但是，履約保證金的設置將會降低此潛在價值。即使履約保證金可以避免任意行提前放棄的權利，這還是會帶來問題。更具體的說：如果 BOT 專案實質選擇權的評價不考量履約保證金的負面影響，評價結果將會高估專案投資價值，甚至誤導投資。

有鑑於此，本研究將建立序列複合買權評價模型(sequential compound call option valuation model)，推導多期 BOT 投資專案在履約保證之下的評價封閉解。本研究也將提供敏感性分析來探討履約保證金對於 BOT 專案價值的影響。同時，本研究將利用 MATLAB 程式來撰寫 SCCO 的專案評價軟體，選定實際的投資案例，進行專案評價的應用與分析。

關鍵詞：BOT、民營化、多期基礎建設投資、履約保證、序列複合買權、評價、敏感度分析

Abstract

The build-operate-transfer (BOT) model is a popular approach to infrastructure privatization, especially for large-scale infrastructure investments. However, large-scale BOT projects usually require long-term sunk investments in infrastructure facilities that are exposed to uncertain market conditions and unforeseen contingencies. To attract private-sector investments in BOT projects, host governments have offered a variety of concession arrangements for BOT risk management, such as loan repayment guarantee, minimum-revenue guarantee, the rights to expand incrementally, and the rights to abandon prematurely. The presence of these arrangements means that traditional valuation methods, such as the discounted cash flow model, are no longer satisfactory for BOT project valuation.

Researchers have developed various real-option models to treat the more complex BOT valuation issues.

Although the real-option approach is popular, existing BOT real-option valuation models have not incorporated the impact of performance bonds on project value. In particular, the value of BOT projects is uncertain at contract signing, and must be re-valuated when new project information is available during contract execution. This produces the possibility of a "hold-up:" the government may pursue court enforcement of literal contract terms, asking the concessionaire to invest in the underlying project even when the project is deemed infeasible by a re-valuation. To avoid the "hold-up" risk, BOT concession contracts often grant concessionaires voluntary abandonment rights during contract execution. However, host governments also require performance bonding by concessionaires as a security in order to ensure that voluntary termination is not exercised arbitrarily.

In real option theories, voluntary abandonment is potentially valuable, but imposing a bonding requirement creates a penalty term upon voluntary abandonment. Performance bonding reduces the value of voluntary abandonment rights even though the arbitrary exercise of the rights can be avoided. If the impact of performance bonds on BOT project value is not assessed in BOT real option valuation, the resulting project value would tend to be overstated.

Accordingly, this research will apply the theory of sequential compound call option (SCCO) to derive a closed form solution to the valuation of multistage BOT infrastructure projects under performance bonding. This research will also provide sensitivity analysis to examine the impact of performance bonding on BOT project value. A computer program will be written to support the numerical implementation of the closed solution using MATLAB, and a real-world BOT case will be chosen to demonstrate numerically the applicability of the proposed valuation model.

Keywords: Build-operate-transfer; multistage infrastructure investment, privatization, performance bond, sequential compound call option, valuation, sensitivity analysis

INTRODUCTION

For nearly three decades infrastructure privatization has gradually become a popular, well-established approach to the delivery of infrastructure services. For the early developments of this trend, Huang (1995) documented over eighty privatized infrastructure projects in the world. Huang (1995) focused on the institutional and regulatory designs of these projects. For more recent developments, Tam (1999) and Kumaraswamy and Morris (2002) investigated build-operate-transfer (BOT) infrastructure projects in Asia. Chen and Messner (2005) investigated BOT water supply projects in China. Kleiss and Imura (2006) investigated private finance initiative (PFI) in Japan. Winch (2000) investigated PFI public works projects in the United Kingdom. Koch and Buser (2006) investigated public-private partnership (PPP) governance in Denmark. Fischer, Jungbecker, and Alfen (2006) investigated PPP infrastructure developments in Germany. Vazquez and Allen (2004) investigated BOT highway projects in Central America and Mexico. Algarni, Arditi, and Polat (2007) investigated BOT infrastructure projects in the United States.

Between PFI and other types of infrastructure privatization approaches, the BOT model is popular for large-scale infrastructure developments.

Large-scale BOT projects usually require long-term sunk investments in infrastructure facilities that are exposed to uncertain market conditions and unforeseen contingencies (for example Grimsey and Lewis [2002]). To attract private-sector investments in BOT projects, host governments have offered a variety of concession arrangements for BOT risk management, such as loan repayment guarantee, minimum-revenue guarantee, the rights to expand incrementally, and the rights to abandon prematurely (for example Huang [1995] and Wibowo [2004]). The presence of these arrangements means that traditional valuation methods, such as the discounted cash flow model, are no longer satisfactory for BOT project valuation. Researchers have developed various real-option models to treat the more complex BOT valuation issues. For example, Rose (1998) evaluated interacting toll road investment options. Smit (2003) provided a real-option-based game theory model to evaluate airport expansions. Garvin and Cheah (2004) proposed a real-option pricing model for analyzing toll road investments. Wand and Min (2006) evaluated the interrelationships among power generation projects. Huang and Chou (2006) evaluated minimum-revenue guarantees. Damnjanovic, Duthie, and Waller (2008) evaluated the interconnectivity and flexibility of toll road expansions. Huang and Pi (2009) developed a European-style sequential compound call option (SCCO) model to evaluate multi-stage BOT projects involving dedicated asset investments. Huang and Pi (2011) extended the SCCO model to assess the impacts of competition and technological obsolescence on project value in privatized infrastructure markets.

In fact real-option models are powerful valuation tools not only for complex BOT concession arrangements. The real-option valuation approach has also been applied for information technology and other types of investment projects. For example, Panayi and Trigeorgis (1998) developed a real-option model for the valuation of multistage information technology projects. Yeo and Qiu (2002) discussed the valuation of investment flexibility of technology investment projects by the real-option approach. Chen, Zhang, and Lai (2009) developed an integrated real-option approach for the valuation of information technology projects. Eckhause, Hughes, and Gabriel (2009) developed a real-option approach for vendor selection in multi-stage R&D acquisitions.

Although the real-option approach is popular and powerful, existing BOT real-option valuation models have not incorporated the impact of performance bonds on project value. In particular, the value of BOT projects is uncertain at contract signing, and must be revaluated when new project information is available during contract execution. This produces the possibility of a “hold-up:” the government may pursue court enforcement of literal contract terms, asking the concessionaire to invest in the underlying project even when the project is deemed infeasible by a re-valuation. To avoid the “hold-up” risk, BOT concession contracts often grant concessionaires voluntary abandonment rights during contract execution. According to Klein (1996), voluntary termination can avoid “hold-ups.” However, host governments also require performance bonding by concessionaires as a security. As Vandegrift (1999) suggested, performance bonding can ensure that voluntary termination is not exercised arbitrarily.

In real option theories, voluntary abandonment is a type of flexibility to avoid irreversible sunk investments under adverse market conditions (for example Dixit and Pinkyck [1994]). This type of flexibility is potentially valuable and can encourage private-sector investments in BOT projects (for example Huang and Pi [2009, 2011]). Nevertheless, imposing a bonding requirement creates a penalty term upon voluntary abandonment. This reduces the value of voluntary abandonment rights even though the arbitrary exercise of the rights can be avoided. If the impact of performance bonds on BOT project value is not assessed in BOT real option valuation, the resulting project value would tend to be overstated, and this may mislead investment decisions.

This study tries to propose a sequential compound call option pricing model with consideration the influence of performance bonding. The pricing model is constructed by following the typical lifecycle for multistage BOT investments shown in Figure1.

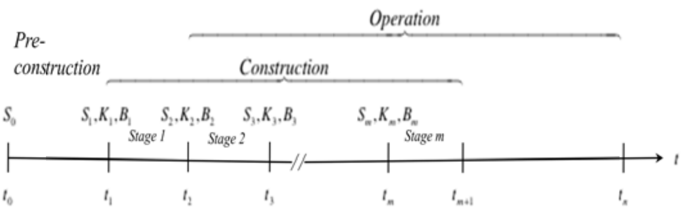


Figure 1. A typical lifecycle for multistage BOT investments.

The rest of this paper is organized as follows. The first section presents derives the pricing solution for n-fold SCCOs. The second section derives the partial derivatives of with respect to the parameter B to discusses the influence of the performance bond to the project value. The third section presents a real-world application to a three-stage BOT sanitary sewerage project. The fifth section concludes.

A CLOSED-FORM SOLUTION

This section derives a closed-form pricing model as a solution to a European Sequential compound call option (European SCCO) with the consideration of performance bond. In addition, the proposed solution is based on risk-neutral pricing approach.

First, the assumptions of the option pricing model followed the well known B-S-M model introduced by Black and Scholes (1973) and Merton (1973).

The idiosyncratic assets of a BOT project are at risk and assumed to follow a stochastic process of the form:

$$\frac{dS_t}{S_t} = (r - q)dt + \sigma dz_t^Q \qquad (1)$$

where S_t denotes the stochastic asset value and z_t^Q denotes the standard Brownian motion under risk-neutral environment Q. The parameter σ^2 is the deterministic variance of the return, r is the deterministic expected risk-free rate of return, and q is the deterministic dividend payout rate. To obtain a solution, assume that there is no-arbitrage in frictionless markets. Also, for a one-stage project, the investor would invest only if the time-t_1 value of the one-fold option $C_{\{1\},1}(S, t_1)$ is "in the money", i.e., $S_{\{1\},1} - K_{\{1\},1} \geq -B_{\{1\},1}$. By definition, $K_{\{1\},1}$ is the exercise price of the one-fold call option at time-t_1 and $B_{\{1\},1}$ is the amount of the performance bond. Then, the final payoff of a one-fold call option is given as

$$\begin{aligned} C_{\{1\},1}(S, t_1) &= \max\left(S_{\{1\},1} - K_{\{1\},1}, -B_{\{1\},1}\right) \\ &= \max\left(S_{\{1\},1} - \left(K_{\{1\},1} - B_{\{1\},1}\right), 0\right) - B_{\{1\},1} \end{aligned} \qquad (2)$$

Therefore, then risk-neutral pricing gives the initial payoff of a one-fold European call option with performance bond as:

$$C_{\{1\},1}(S,t_0) = e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{\max\left(S_{\{1\},1} - K_{\{1\},1}, -B_{\{1\},1}\right)\right\} \tag{3}$$

where E^Q denotes a conditional expectation operator.

In addition, for a 2-stage project, the investor would invest in the second stage at time t_1 only if the balance between $C_{\{2\},1}(S,t_1)$ and $K_{\{2\},1}$ is higher than the amount of the performance bond $B_{\{2\},1}$. Here the boundary condition can be known as $C_{\{2\},1}(S,t_1) - K_{\{2\},1} \geq -B_{\{2\},1}$, and the time-$t_1$ value of $C_{\{2\},2}(S,t_1)$ is given by

$$\begin{aligned} C_{\{2\},2}(S,t_1) &= \max\left(S_{\{2\},1} - K_{\{2\},1}, -B_{\{2\},1}\right) \\ &= \max\left(S_{\{2\},1} - \left(K_{\{2\},1} - B_{\{2\},1}\right), 0\right) - B_{\{2\},1} \end{aligned} \tag{4}$$

By induction, for the n-stage project, denote $C_{\{n\},n-(i-1)}(S,t_i)$ and $C_{\{n\},n-i}(S,t_i)$ respectively as the time-t_i value of the *n-(i-1)*th SCCO and the *(n-i)*th fold SCCO. Then the boundary condition for exercising the *n-(i-1)*th SCCO is given by

$$\begin{aligned} &C_{\{n\},n-(i-1)}(S,t_i) \\ &= \max\left(C_{\{n\},n-i}(S,t_i) - K_{\{n\},i}, -B_{\{n\},i}\right) \\ &= \max\left[C_{\{n\},n-i}(S,t_i) - \left(K_{\{n\},i} - B_{\{n\},i}\right), 0\right] - B_{\{n\},i} \end{aligned} \tag{5}$$

Applying the idiosyncratic asset value process in (1) and the boundary conditions in (5) gives the following theorem.

Theorem. *(The European SCCO Pricing Formula)*

$$\begin{aligned} &C_{\{n\},n}(S,t_0) \\ &= S_{\{n\},0} e^{\int_{t_0}^{t_{n+1}} q(u)du} N_n\{[g_{\{n\},i}]_{n\times 1};[\rho_{i,j}]_{n\times n}\} \\ &- \sum_{m=1}^{n} \left(K_{\{n\},m} - B_{\{n\},m}\right) e^{-\int_{t_0}^{t_m} r(u)du} N_m\{[h_{\{n\},i}]_{m\times 1};[\rho_{i,j}]_{m\times m}\} \\ &- \sum_{m=1}^{n} B_{\{n\},m} e^{-\int_{t_0}^{t_m} r(u)du} N_{m-1}\{[h_{\{n\},i}]_{(m-1)\times 1};[\rho_{i,j}]_{(m-1)\times(m-1)}\} \end{aligned} \tag{6}$$

where

$$g_{\{n\},i} = \frac{\ln(\frac{S_0}{\bar{S}_{i,\{n\}}}) + \int_{t_0}^{t_i}[r(u) - q(u) + \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_i}\sigma^2(u)du}}; \forall 1 \leq i \leq n$$

$$h_{\{n\},i} = \frac{\ln(\frac{S_0}{\bar{S}_{i,\{n\}}}) + \int_{t_0}^{t_i}[r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_i}\sigma^2(u)du}}; \forall 1 \leq i \leq n$$

Proof.

1-fold European call option

The final payoff of the option is written as equation (3). Base on the risk-neutral pricing method, provided by Cox and Ross (1976), Harrison and Kreps (1979) and Harrison and Pliska (1981); the fair price of a vanilla European call option with the consideration of the performance bond is equal to $e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{\max\left[S_{\{1\},1} - K_{\{1\},1}, -B_{\{1\},1}\right]\right\}$ under the measure Q.

Then

$$\begin{aligned} &C_{\{1\},0}(S,t_0) \\ &= e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{\max\left[S_{\{1\},1} - K_{\{1\},1}, -B_{\{1\},1}\right]\right\} \\ &= e^{-\int_{t_0}^{t_1} r(u)du}\left\{E^Q\left[\max[S_{\{1\},1} - (K_{\{1\},1} - B_{\{1\},1}), 0]\right] - B_{\{1\},1}\right\} \\ &= e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{(S_{\{1\},1} - (K_{\{1\},1} - B_{\{1\},1})) \cdot 1_{\{S_{\{1\},1} > (K_{\{1\},1} - B_{\{1\},1})\}}\right\} \\ &\quad - B_{\{1\},1} e^{-\int_{t_0}^{t_1} r(u)du} \\ &= e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{S_{\{1\},1} \cdot 1_{\{S_{\{1\},1} > (K_{\{1\},1} - B_{\{1\},1})\}}\right\} \\ &\quad - (K_{\{1\},1} - B_{\{1\},1}) e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{1_{\{S_{\{1\},1} > (K_{\{1\},1} - B_{\{1\},1}))\}}\right\} - B_{\{1\},1} e^{-\int_{t_0}^{t_1} r(u)du} \end{aligned} \tag{7}$$

According to equation (1), the solution of the stochastic differential equation (S.D.E.) is given as

$$S_{\{1\},1} = S_{\{1\},0} e^{\int_{t_0}^{t_1}[r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du + z^Q\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \tag{8}$$

Then we can use equation (8) to substitute the $S_{\{1\},1}$ in equation (3). The pricing formula becomes

$$\begin{aligned} &C_{\{1\},0}(S,t_0) \\ &= S_{\{1\},0} e^{-\int_{t_0}^{t_1} q(u)du} E^Q\left\{e^{-\int_{t_0}^{t_1}\frac{1}{2}\sigma^2(u)du + z^Q\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \cdot 1_{\{S_{\{1\},1} > (K_{\{1\},1} - B_{\{1\},1})\}}\right\} \\ &\quad - (K_{\{1\},1} - B_{\{1\},1}) e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{1_{\{S_{\{1\},1} > (K_{\{1\},1} - B_{\{1\},1}))\}}\right\} - B_{\{1\},1} e^{-\int_{t_0}^{t_1} r(u)du} \end{aligned} \tag{9}$$

Before deriving the pricing formula of a European call option with the consideration of the performance bond, we have to eliminate the uncertain term in the expectation operator E^Q. Therefore, we use Girsanov's Theory to change the probability measure, and the Radon-Nikodym derivative is defined as

$$\frac{dR}{dQ} = -\int_{t_0}^{t_1} \frac{1}{2}\sigma^2(u)du + z^Q \sqrt{\int_{t_0}^{t_1} \sigma^2(u)du} \qquad (10)$$

Base on the equation (10), the Brownian motion term before and after change of measure can be defined as

$$dz_t^Q = dz_t^R + \sqrt{\int_{t_0}^{t_1} \sigma^2(u)du} \qquad (11)$$

According to Girsanov's Theorem and equation (11), we can rewrite equation (1) as

$$\frac{dS}{S} = \left(r - q + \sigma^2\right)dt + \sigma dz_t^R \qquad (12)$$

where dz_t^R represent the standard Brownian motion under the measure *R*, underlying asset as numeraire.

Since the underlying asset is log-normally distributed, we use the Ito's lemma to find the solution of equation (12). After some calculation, the dynamic price of the asset S under measure *R* is finally found and shown as follows

$$S_{\{1\},1} = S_{\{1\},0} e^{\int_{t_0}^{t_1}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du + z^R\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \qquad (13)$$

Next, we can put (13) back into to the pricing formula (9) and change the measure from *Q* to *R*, and recalculate the probability,

$$\begin{aligned}
&C_{\{1\},0}(S,t_0)\\
&= S_{\{1\},0} e^{-\int_{t_0}^{t_1} q(u)du} E^Q\left\{\frac{dR}{dQ}\cdot 1_{\{S_{\{1\},1} > (K_{\{1\},1} - B_{\{1\},1})\}}\right\}\\
&\quad -\left(K_{\{1\},1} - B_{\{1\},1}\right)e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{ 1_{\{S_{\{1\},1} > (K_{\{1\},1} - B_{\{1\},1}))\}}\right\} - B_{\{1\},1} e^{-\int_{t_0}^{t_1} r(u)du}\\
&= S_{\{1\},0} e^{-\int_{t_0}^{t_1} q(u)du} E^R\left\{ 1_{\{S_{\{1\},1} > (K_{\{1\},1} - B_{\{1\},1})\}}\right\}\\
&\quad -\left(K_1 - B_1\right)e^{-\int_{t_0}^{t_1} r(u)du} E^Q\left\{ 1_{\{S_{\{1\},1} > (K_{\{1\},1} - B_{\{1\},1})\}}\right\} - B_1 e^{-\int_{t_0}^{t_1} r(u)du}\\
&= S_{\{1\},0} e^{-\int_{t_0}^{t_1} q(u)du} P^R\left(\ln S_{\{1\},1} > \ln(K_{\{1\},1} - B_{\{1\},1})\right)\\
&\quad -\left(K_{\{1\},1} - B_{\{1\},1}\right)e^{-\int_{t_0}^{t_1} r(u)du} P^Q\left(\ln_{\{1\},1} > \ln(K_{\{1\},1} - B_{\{1\},1})\right) - B_{\{1\},1} e^{-\int_{t_0}^{t_1} r(u)du}
\end{aligned}$$

where

$$\begin{aligned}
&P^R\left(\ln S_{\{1\},1} > \ln(K_{\{1\},1} - B_{\{1\},1})\right)\\
&= P^R\left(\ln S_{\{1\},0} + \int_{t_0}^{t_1}[r(u) - q(u) + \frac{1}{2}\sigma^2(u)]du + z^R\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du} > \ln(K_{\{1\},1} - B_{\{1\},1})\right)\\
&= P^R\left(-z^R < \frac{\ln\left(\frac{S_{\{1\},0}}{K_{\{1\},1} - B_{\{1\},1}}\right) + \int_{t_0}^{t_1}[r(u) - q(u) + \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}\right) = P^R\left(-z^R < g_{\{1\},1}\right)
\end{aligned}$$

With the same pattern, we can have

$$P^Q\left(\ln S_{\{1\},1} > \ln(K_{\{1\},1} - B_{\{1\},1})\right) = P^R\left(-z^Q < h_{\{1\},1}\right)$$

where

$$h_{\{1\},1} = \frac{\ln\left(\frac{S_{\{1\},0}}{K_{\{1\},1} - B_{\{1\},1}}\right) + \int_{t_0}^{t_1}[r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$$

Therefore, the pricing model becomes to

$$\begin{aligned}
&C_{\{1\},0}(S,t_0)\\
&= S_{\{1\},0} e^{-\int_{t_0}^{t_1} q(u)du} P^R\left(-z^R < d_1\right)\\
&\quad -\left(K_{\{1\},1} - B_{\{1\},1}\right)e^{-\int_{t_0}^{t_1} r(u)du} P^Q\left(-z^Q < d_2\right) - B_{\{1\},1} e^{-\int_{t_0}^{t_1} r(u)du}\\
&= S_{\{1\},0} e^{-\int_{t_0}^{t_1} q(u)du} N(g_{\{1\},1})\\
&\quad -\left(K_{\{1\},1} - B_{\{1\},1}\right)e^{-\int_{t_0}^{t_1} r(u)du} N(h_{\{1\},1}) - B_{\{1\},1} e^{-\int_{t_0}^{t_1} r(u)du}
\end{aligned} \qquad (14)$$

Both z^R and z^Q are the standard Brownian Motion, given the fact that $z \sim N(0,1)$, and $1 - N(h_{\{1\},1}) = N(-h_{\{1\},1})$; $N(0) = 1$.

2-fold compound call option

For the 2-stage project, the investor would invest in the second stage at time t_1 only if $C_{\{2\},1}(S,t_1) \geq \left(K_{\{2\},1} - B_{\{2\},1}\right)$, and the time-$t_1$ value of

$C_{\{2\},2}(S,t_1)$ given as equation (4).

According to the boundary condition given by expression (4), the time-t_0 price of the two-fold SCCO can be found by the risk-neutral pricing approach elaborated by Lajeri-Chaherli (2002):

$$\begin{aligned} &C_{\{2\},2}(S,t_0) \\ &= e^{-\int_{t_0}^{t_1} r(u)du}\left\{E^Q\left[\max\left(0, C_{\{2\},1}(S,t_1)-\left(K_{\{2\},1}-B_{\{2\},1}\right)\right)\right]-B_{\{2\},1}\right\} \\ &= e^{-\int_{t_0}^{t_1} r(u)du}\int_{-\infty}^{\infty}\max\left\{0, C_{\{2\},1}(S,t_1)-\left(K_{\{2\},1}-B_{\{2\},1}\right)\right\}f(z)dz - B_{\{2\},1}e^{-\int_{t_0}^{t_1} r(u)du} \end{aligned} \tag{15}$$

Since $C_{\{2\},1}(S,t_1)$ is a one-fold call option, we can use the result of equation (14) and shift the initial time from t_0 to t_1, which is

$$\begin{aligned} &C_{\{2\},1}(S,t_1) \\ &= S_{\{2\},1}e^{-\int_{t_1}^{t_2} q(u)du}N(g_{\{1\},1,*1}) \\ &\quad -\left(K_{\{2\},2}-B_{\{2\},2}\right)e^{-\int_{t_1}^{t_2} r(u)du}N(h_{\{1\},1,*1}) - B_{\{2\},2}e^{-\int_{t_1}^{t_2} r(u)du} \end{aligned} .$$

Also, set $\bar{S}_{1,\{2\}}$ be the equivalent asset value found at the point where the underlying option finishes "at the money", i.e., $C_{\{2\},1}(S,t_1)=\left(K_{\{2\},1}-B_{\{2\},1}\right)$, and it can be found at the point where the underlying option finishes 'at the money'. Set $h_{\{2\},1}=\dfrac{\ln(\dfrac{S_{\{2\},0}}{\bar{S}_{1,\{2\}}})+\int_{t_0}^{t_1}[r(u)-q(u)-\frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$, and it means $S_{\{2\},1}\rangle\bar{S}_{1,\{2\}}$ if and only if $z\rangle -h_{\{2\},1}$. With the consideration of boundary condition and $S_{\{2\},1}=S_{\{2\},0}e^{\int_{t_0}^{t_1}[r(u)-q(u)-\frac{1}{2}\sigma^2(u)]du+z\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$, then the equation (15) can be rewritten as follows,

$$\begin{aligned} &C_{\{2\},2}(S,t_0) \\ &= e^{-\int_{t_0}^{t_1} r(u)du}\int_{-h_{\{2\},1}}^{\infty}\max\left\{\begin{matrix} S_{\{2\},1}e^{-\int_{t_1}^{t_2} q(u)du}N(g_{\{1\},1,*1}) \\ -\left(K_{\{2\},2}-B_{\{2\},2}\right)e^{-\int_{t_1}^{t_2} r(u)du}N(h_{\{1\},1,*1}) \\ -B_{\{2\},2}e^{-\int_{t_1}^{t_2} r(u)du}-\left(K_{\{2\},1}-B_{\{2\},1}\right) \end{matrix}\right\}f(z)dz - B_{\{2\},1}e^{-\int_{t_0}^{t_1} r(u)du} \\ &= e^{-\int_{t_0}^{t_1} r(u)du}\int_{-h_{\{2\},1}}^{\infty}e^{-\int_{t_1}^{t_2} q(u)du}S_{\{2\},0}e^{\int_{t_0}^{t_1}[r(u)-q(u)-\frac{1}{2}\sigma^2(u)]du+z\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}N(g_{\{1\},1,*1})f(z)dz \\ &\quad -\left(K_{\{2\},2}-B_{\{2\},2}\right)e^{-\int_{t_0}^{t_1} r(u)du}\int_{-h_{\{2\},1}}^{\infty}N(h_{\{1\},1,*1})f(z)dz - B_{\{2\},2}e^{-\int_{t_0}^{t_2} r(u)du}\int_{-h_{\{2\},1}}^{\infty}f(z)dz \\ &\quad -\left(K_{\{2\},1}-B_{\{2\},1}\right)e^{-\int_{t_0}^{t_1} r(u)du}\int_{-h_{\{2\},1}}^{\infty}f(z)dz - B_{\{2\},1}e^{-\int_{t_0}^{t_1} r(u)du} \\ &= S_{\{2\},0}e^{-\int_{t_0}^{t_2} q(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}[z-\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}]^2}N(g_{\{1\},1,*1})dz \\ &\quad -\left(K_{\{2\},2}-B_{\{2\},2}\right)e^{-\int_{t_0}^{t_2} r(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}N(h_{\{1\},1,*1})dze^{-\int_{t_0}^{t_2} r(u)du} \\ &\quad -\left(K_{\{2\},1}-B_{\{2\},1}\right)e^{-\int_{t_0}^{t_1} r(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz \\ &\quad -B_{\{2\},2}e^{-\int_{t_0}^{t_2} r(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz - B_{\{2\},1}e^{-\int_{t_0}^{t_1} r(u)du} \end{aligned}$$

Set

$$\overline{C_1}=S_{\{2\},0}e^{-\int_{t_0}^{t_2} q(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}[z-\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}]^2}N(g_{\{1\},1,*1})dz \;;$$

$$\begin{aligned} \overline{C_2} &= -\left(K_{\{2\},2}-B_{\{2\},2}\right)e^{-\int_{t_0}^{t_2} r(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}N(h_{\{1\},1,*1})dze^{-\int_{t_0}^{t_2} r(u)du} \;; \\ &\quad -\left(K_{\{2\},1}-B_{\{2\},1}\right)e^{-\int_{t_0}^{t_1} r(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{\frac{1}{2}z^2}dz \end{aligned}$$

$$\overline{C_3}=-B_{\{2\},2}e^{-\int_{t_0}^{t_2} r(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz - B_{\{2\},1}e^{-\int_{t_0}^{t_1} r(u)du} \;;$$ where

$$C_{\{2\},2}(S,t_0)=\sum_{i=1}^{3}\overline{C_i} .$$

Use bivariate normal distribution function to reform $\overline{C_1}$ and $\overline{C_2}$ where the factor $g_{\{1\},1,*1}$ and $h_{\{1\},1,*1}$ can be shift as

$$g_{\{1\},1,*1}=\frac{\ln(\frac{S_{\{2\},1}}{K_{\{2\},2}-B_{\{2\},2}})+\int_{t_1}^{t_2}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_1}^{t_2}\sigma^2(u)du}}$$

$$=\frac{\ln(\frac{S_{\{2\},0}}{K_{\{2\},2}-B_{\{2\},2}})+\int_{t_1}^{t_2}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_1}^{t_2}\sigma^2(u)du}}$$

$$+\frac{\int_{t_0}^{t_1}[r(u)-q(u)-\frac{1}{2}\sigma^2(u)]du+z\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}{\sqrt{\int_{t_1}^{t_2}\sigma^2(u)du}}$$

$$=\frac{\ln(\frac{S_{\{2\},0}}{K_{\{2\},2}-B_{\{2\},2}})+\int_{t_0}^{t_2}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du+z_2\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}{\sqrt{\int_{t_1}^{t_2}\sigma^2(u)du}}$$

$$=\frac{\frac{\ln(\frac{S_{\{2\},0}}{K_{\{2\},2}-B_{\{2\},2}})+\int_{t_0}^{t_2}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_2}\sigma^2(u)du}}+z_2\frac{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}{\sqrt{\int_{t_0}^{t_2}\sigma^2(u)du}}}{\frac{\sqrt{\int_{t_1}^{t_2}\sigma^2(u)du}}{\sqrt{\int_{t_0}^{t_2}\sigma^2(u)du}}}$$

where $z_2=z-\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}$ and $\frac{\sqrt{\int_{t_1}^{t_2}\sigma^2(u)du}}{\sqrt{\int_{t_0}^{t_2}\sigma^2(u)du}}=\sqrt{1-\rho_{1,2}{}^2}$.

Finally, $g_{\{1\},1,*1}$ can be reform as $\frac{g_{\{2\},2}+\rho_{1,2}z_2}{\sqrt{1-\rho_{1,2}{}^2}}$. with the same deriving pattern, $h_{\{1\},1,*1}$ can be reform as $\frac{h_{\{2\},2}+\rho_{1,2}z}{\sqrt{1-\rho_{1,2}{}^2}}$.

$$\overline{C_1}=S_{\{2\},0}e^{-\int_{t_0}^{t_2}q(u)du}\int_{-g_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z_2^2}N(\frac{g_{\{2\},2}+\rho_{1,2}z_2}{\sqrt{1-\rho_{1,2}{}^2}})dz_2$$

$$=S_{\{2\},0}e^{-\int_{t_0}^{t_2}q(u)du}N_2(g_{\{2\},1},g_{\{2\},2};\rho_{1,2})$$

$$\overline{C_2}=-\left(K_{\{2\},2}-B_{\{2\},2}\right)e^{-\int_{t_0}^{t_2}r(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}N\left(\frac{h_{\{2\},2}+\rho_{1,2}z}{\sqrt{1-\rho_{1,2}{}^2}}\right)dz$$
$$-\left(K_{\{2\},1}-B_{\{2\},1}\right)e^{-\int_{t_0}^{t_1}r(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{\frac{1}{2}z^2}dz$$
$$=-\left(K_{\{2\},2}-B_{\{2\},2}\right)e^{-\int_{t_0}^{t_2}r(u)du}N_2\left(h_{\{2\},1},h_{\{2\},2};\rho_{1,2}\right)$$
$$-\left(K_{\{2\},1}-B_{\{2\},1}\right)e^{-\int_{t_0}^{t_1}r(u)du}N\left(h_{\{2\},1}\right)$$

and $\overline{C_3}$ is reformed by using traditional normal distribution function shown as follows

$$\overline{C_3}=-B_{\{2\},2}e^{-\int_{t_0}^{t_2}r(u)du}\int_{-h_{\{2\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz-B_{\{2\},1}e^{-\int_{t_0}^{t_1}r(u)du}$$
$$=-B_{\{2\},2}e^{-\int_{t_0}^{t_2}r(u)du}N\left(h_{\{2\},1}\right)-B_{\{2\},1}e^{-\int_{t_0}^{t_1}r(u)du}$$

Finally, the two-fold SCCO with consideration of performance bond is

$$\begin{aligned}&C_{\{2\},2}(S,t_0)\\&=S_{\{2\},0}e^{-\int_{t_0}^{t_2}q(u)du}N_2(g_{\{2\},1},g_{\{2\},2};\rho_{1,2})\\&-\left(K_{\{2\},2}-B_{\{2\},2}\right)e^{-\int_{t_0}^{t_2}r(u)du}N_2\left(h_{\{2\},1},h_{\{2\},2};\rho_{1,2}\right)\\&-\left(K_{\{2\},1}-B_{\{2\},1}\right)e^{-\int_{t_0}^{t_1}r(u)du}N\left(h_{\{2\},1}\right)\\&-B_{\{2\},2}e^{-\int_{t_0}^{t_2}r(u)du}N\left(h_{\{2\},1}\right)-B_{\{2\},1}e^{-\int_{t_0}^{t_1}r(u)du}\end{aligned}\qquad(16)$$

where

$$g_{\{2\},i}=\frac{\ln\left(\frac{S_{\{2\},0}}{S_{i,\{2\}}}\right)+\int_{t_0}^{t_i}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_i}\sigma^2(u)du}};$$

$$h_{\{2\},i}=\frac{\ln\left(\frac{S_{\{2\},0}}{S_{i,\{2\}}}\right)+\int_{t_0}^{t_i}[r(u)-q(u)-\frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_i}\sigma^2(u)du}}$$

Equation (16) can be rewritten as

$$\begin{aligned}&C_{\{2\},2}(S,t_0)\\&=S_{\{2\},0}e^{-\int_{t_0}^{t_2}q(u)du}N_2\{[g_{\{2\},i}]_{2\times1};[\rho_{i,j}]_{2\times2}\}\\&-\sum_{m=1}^{2}\left(K_{\{2\},m}-B_{\{2\},m}\right)e^{-\int_{t_0}^{t_m}r(u)du}N_m\{[h_{\{2\},i}]_{m\times1};[\rho_{i,j}]_{m\times m}\}\\&-\sum_{m=1}^{2}B_{\{2\},m}e^{-\int_{t_0}^{t_m}r(u)du}N_{m\text{-}1}\{[h_{\{2\},i}]_{(m\text{-}1)\times1};[\rho_{i,j}]_{(m\text{-}1)\times(m\text{-}1)}\}\end{aligned}\qquad(17)$$

n-fold compound call option

By induction, the n-fold SCCO pricing model shows as follows

$$\begin{aligned}&C_{\{n\},n}(S,t_0)\\&=S_{\{n\},0}e^{\int_{t_0}^{t_{n+1}}q(u)du}N_n\{[g_{\{n\},i}]_{n\times1};[\rho_{i,j}]_{n\times n}\}\\&-\sum_{m=1}^{n}\left(K_{\{n\},m}-B_{\{n\},m}\right)e^{-\int_{t_0}^{t_m}r(u)du}N_m\{[h_{\{n\},i}]_{m\times1};[\rho_{i,j}]_{m\times m}\}\\&-\sum_{m=1}^{n}B_{\{n\},m}e^{-\int_{t_0}^{t_m}r(u)du}N_{m-1}\{[h_{\{n\},i}]_{(m-1)\times1};[\rho_{i,j}]_{(m-1)\times(m-1)}\}\end{aligned}\qquad(18)$$

where

$$g_{\{n\},i} = \frac{\ln\left(\frac{S_{\{n\},0}}{S_{i,\{n\}}}\right) + \int_{t_0}^{t_i}[r(u) - q(u) + \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_i}\sigma^2(u)du}};$$

$$h_{\{n\},i} = \frac{\ln\left(\frac{S_{\{n\},0}}{S_{i,\{n\}}}\right) + \int_{t_0}^{t_i}[r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_i}\sigma^2(u)du}}$$

The correlation matrix is symmetric, i.e., $\rho_{i,j} = \rho_{j,i}$, and given by:

$$\rho_{i,j} = \begin{cases} 1\ ; & \forall i = j \\ \sqrt{\int_{t_0}^{t_i}\sigma^2(u)du \Big/ \int_{t_0}^{t_j}\sigma^2(u)du}\ ; & \forall 1 \le i < j \le n \end{cases}$$

If the above solution for the n-fold SCCO, then it is also true for an (n+1)-fold SCCO. To prove that, present value of the (n+1)-fold SCCO can be found by the sane risk-neutral approach as:

$$\begin{aligned} &C_{\{n+1\},n+1}(S,t_0) \\ &= S_{\{n+1\},0}e^{-\int_{t_0}^{t_{n+1}}q(u)du}N_{n+1}\left\{[g_{\{n+1\},i}]_{(n+1)\times 1};[\rho_{i,j}]_{(n+1)\times(n+1)}\right\} \\ &-\sum_{m=1}^{n+1}\left(K_{\{n+1\},m} - B_{\{n+1\},m}\right)e^{-\int_{t_0}^{t_m}r(u)du}N_m\left\{[h_{\{n+1\},i}]_{m\times 1};[\rho_{i,j}]_{m\times m}\right\} \\ &-\sum_{m=1}^{n+1}B_{\{n+1\},m}e^{-\int_{t_0}^{t_m}r(u)du}N_{m\text{-}1}\left\{[h_{\{n+1\},i}]_{(m\text{-}1)\times 1};[\rho_{i,j}]_{(m\text{-}1)\times(m\text{-}1)}\right\} \end{aligned}$$

where $g_{\{n+1\},i}$ and $h_{\{n+1\},i}$ are the *i*-th *g*, *h* values of the (n+1)-fold SCCO. The solution can be obtained directly from Equation (17) by adding one more fold layer. The following provides a more complete outline of this proof. First, first denote $C_{\{n+1\},n+1}(S,t_1)$ as the time-t_1 value of the (n+1)-fold SCCO. The value is given by the boundary condition

$$\begin{aligned} &C_{\{n+1\},n+1}(S,t_1) \\ &= \max[C_{\{n+1\},n}(S,t_1) - K_{\{n+1\},1}, -B_{\{n+1\},1}] \\ &= \max[0, C_{\{n+1\},n}(S,t_1) - \left(K_{\{n+1\},1} - B_{\{n+1\},1}\right)] - B_{\{n+1\},1} \end{aligned}$$

(19)

which states that the underlying n-fold SCCO $C_{\{n+1\},n}(S,t_1)$ will be exercised at time-t_1 if its time-t_1 value is greater than or equal to the difference of exercise price, $K_{\{n+1\},1}$, and performance bond, $B_{\{n+1\},1}$.

Under risk-neutral pricing, the time-t_0 value of the (n+1)-fold SCCO can be given by

$$\begin{aligned} &C_{\{n+1\},n+1}(S,t_0) \\ &= e^{-\int_{t_0}^{t_1}r(u)du}E^Q\left\{\ \max\left[0, C_{\{n+1\},n}(S,t_1) - \left(K_{\{n+1\},1} - B_{\{n+1\},1}\right)\right] - B_{\{n+1\},1}\right\} \end{aligned}$$

(20)

where E^Q is the conditional expectation operator with respect to a risk-neutral probability measure. Assume the underlying asset value process is a standard geometric Brownian motion in Equation (1). Then the time-t_1 value of the asset can be given by:

$$S_{\{n+1\},1} = S_{\{n+1\},0}e^{\int_{t_0}^{t_1}[r(u)-q(u)-\frac{1}{2}\sigma^2(u)]du + z^Q\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$$

(21)

where z is a standard normal random number with density function $f(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}$, $z \sim N(0,1)$.

In addition, the time-t_1 value of the underlying n-fold SCCO can be calculated by the value of the original n-fold SCCO with a shift of start time from t_0 to t_1. That is, on the basis of Equation (18),

$$\begin{aligned} &C_{\{n+1\},n}(S,t_1) \\ &= S_{\{n+1\},1}e^{-\int_{t_1}^{t_{n+1}}q(u)du}N_n\left\{[g_{\{n\},i,*1}]_{n\times 1};[\rho_{i,j,*1}]_{n\times n}\right\} \\ &-\sum_{m=1}^{n}\left(\begin{aligned}&\left(K_{\{n+1\},m+1} - B_{\{n+1\},m+1}\right)e^{-\int_{t_1}^{t_{m+1}}r(u)du} \times \\ &N_m\left\{[h_{\{n+1\},i,*1}]_{m\times 1};[\rho_{i,j,*1}]_{m\times m}\right\}\end{aligned}\right) \\ &-\sum_{m=1}^{n}\left(\begin{aligned}&B_{\{n+1\},m+1}e^{-\int_{t_1}^{t_{m+1}}r(u)du} \times \\ &N_{m-1}\left\{[h_{\{n+1\},i,*1}]_{(m-1)\times 1};[\rho_{i,j,*1}]_{(m-1)\times(m-1)}\right\}\end{aligned}\right) \end{aligned} \quad (22)$$

where *1 denotes the time shift. Then substituting equation (21) and equation (22) into equation (20) gets

$$C_{\{n+1\},n+1}(S,t_0)$$

$$= e^{-\int_{t_0}^{t_1} r(u)du} E^Q \left\{ \max[0, C_{\{n+1\},n}(S,t_1) - \left(K_{\{n+1\},1} - B_{\{n+1\},1}\right)] - B_{\{n+1\},1} \right\}$$

$$= e^{-\int_{t_0}^{t_1} r(u)du} \int_{-\infty}^{\infty} \max\left\{0, \left(\begin{array}{l} S_{\{n+1\},1} e^{-\int_{t_1}^{t_{n+1}} q(u)du} N_n \left\{ [g_{\{n\},i,*1}]_{n\times 1}; [\rho_{i,j,*1}]_{n\times n} \right\} \\ -\sum_{m=1}^{n} \left(K_{\{n+1\},m+1} - B_{\{n+1\},m+1}\right) \times \\ \quad e^{-\int_{t_1}^{t_{m+1}} r(u)du} N_m \left\{ [h_{\{n+1\},i,*1}]_{m\times 1}; [\rho_{i,j,*1}]_{m\times m} \right\} \\ -\sum_{m=1}^{n} B_{\{n+1\},m+1} e^{-\int_{t_1}^{t_{m+1}} r(u)du} \times \\ \quad N_{m-1} \left\{ [h_{\{n+1\},i,*1}]_{(m-1)\times 1}; [\rho_{i,j,*1}]_{(m-1)\times(m-1)} \right\} \\ -\left(K_{\{n+1\},1} - B_{\{n+1\},1}\right) \end{array} \right) \right\} f(z)dz$$

$$- B_{\{n+1\},1} e^{-\int_{t_0}^{t_1} r(u)du}$$

Now, let $\overline{S}_{1,n+1}$ be the time t1 equivalent value of the underlying asset such that at time-t_1, the value of the underlying n-fold SCCO is 'at the money', i.e. $C_{\{n+1\},n}(S,t_1) - \left(K_{\{n+1\},1} - B_{\{n+1\},1}\right) = 0$. Accordingly,

$$\ln\left(\frac{\overline{S}_{1,\{n+1\}}}{S_{\{n+1\},0}}\right) \ \langle \ \int_{t_0}^{t_1} [r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du + z\sqrt{\int_{t_0}^{t_1} \sigma^2(u)du}$$

and therefore

$$z \ \rangle \ -\frac{\ln\left(\frac{S_{\{n+1\},0}}{\overline{S}_{1,\{n+1\}}}\right) + \int_{t_0}^{t_1} [r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1} \sigma^2(u)du}}$$

Further let

$$h_{\{n+1\},1} = \frac{\ln\left(\frac{S_{\{n+1\},0}}{\overline{S}_{1,\{n+1\}}}\right) + \int_{t_0}^{t_1} [r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1} \sigma^2(u)du}}$$

Because $S_{\{n+1\},0} \rangle \overline{S}_{1,\{n+1\}}$, $z \rangle -h_{\{n+1\},1}$ Given above, the value of the (n+1)-fold can be solved by:

$$C_{\{n+1\},n+1}(S,t_0)$$

$$= e^{-\int_{t_0}^{t_1} r(u)du} \int_{-h_{\{n+1\},1}}^{\infty} \left(\begin{array}{r} S_{\{n+1\},0} e^{-\int_{t_1}^{t_{n+1}} q(u)du + \int_{t_0}^{t_1}[r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du + z\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \times \\ \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} N_n \left\{ [g_{\{n\},i,*1}]_{n\times 1}; [\rho_{i,j,*1}]_{n\times n} \right\} \end{array} \right) dz$$

$$- \sum_{m=1}^{n} \left(\begin{array}{r} \left(K_{\{n+1\},m+1} - B_{\{n+1\},m+1}\right) e^{-\int_{t_0}^{t_{m+1}} r(u)du} \times \\ \int_{-h_{\{n+1\},1}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} N_m \left\{ [h_{\{n+1\},i,*1}]_{m\times 1}; [\rho_{i,j,*1}]_{m\times m} \right\} \end{array} \right) dz$$

$$- \sum_{m=1}^{n} \left(\begin{array}{l} B_{\{n+1\},m+1} e^{-\int_{t_0}^{t_{m+1}} r(u)du} \times \\ \int_{-h_{\{n+1\},1}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} N_{m-1} \left\{ [h_{\{n+1\},i,*1}]_{(m-1)\times 1}; [\rho_{i,j,*1}]_{(m-1)\times(m-1)} \right\} \end{array} \right) dz$$

$$- \left(K_{\{n+1\},1} - B_{\{n+1\},1}\right) e^{-\int_{t_0}^{t_1} r(u)du} \int_{-h_{n+1,1}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz - B_{\{n+1\},1} e^{-\int_{t_0}^{t_1} r(u)du}$$

The solution can be found by deriving the pricing components separately. Denote the components as $\overline{C_1}$、$\overline{C_2}$、and $\overline{C_3}$ respectively, and start the derivation with the first component. Recall the asset price process in equation (21), and rewrite the value of $\overline{C_1}$ as:

$$\overline{C_1} = S_{\{n+1\},0} e^{\int_{t_0}^{t_{n+1}} q(u)du} \int_{-h_{\{n+1\},1}}^{\infty} e^{-\frac{1}{2}\left(z - \sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}\right)^2} N_n \left\{ [g_{\{n\},i,*1}]_{n\times 1}; [\rho_{i,j,*1}]_{n\times n} \right\} dz$$

To specify the parameters of the n-variate normal integration, let $z_2 = z - \sqrt{\int_{t_0}^{t_1} \sigma^2(u)du}$. Then

$$-h_{\{n+1\},1} - \sqrt{\int_{t_0}^{t_1} \sigma^2(u)du} = -(h_{\{n+1\},1} + \sqrt{\int_{t_0}^{t_1} \sigma^2(u)du}) = -g_{\{n+1\},1}$$

where the factor $g_{\{n+1\},1}$ is given by

$$g_{\{n+1\},1} = \frac{\ln\left(\frac{S_{\{n+1\},0}}{\overline{S}_{1,\{n+1\}}}\right) + \int_{t_0}^{t_1} [r(u) - q(u) + \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1} \sigma^2(u)du}}$$

Accordingly, the value of $\overline{C_1}$ is further rewritten as:

$$\overline{C_1} = S_{\{n+1\},0} e^{\int_{t_0}^{t_{n+1}} q(u)du} \int_{-g_{\{n+1\},1}}^{\infty} e^{-\frac{1}{2}(z_2)^2} N_n \left\{ [g_{\{n\},i,*1}]_{n\times 1}; [\rho_{i,j,*1}]_{n\times n} \right\} dz_2$$

The value of the factor $g_{\{n+1\},i}$ with the time shift can further be found by algebraic manipulations as

$$g_{n,i,*1} = \frac{\ln(\frac{S_{\{n+1\},1}}{\overline{S}_{i+1,\{n+1\}}}) + \int_{1}^{t_{i+1}} [r(u) - q(u) + \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_1}^{t_{i+1}} \sigma^2(u)du}} \quad ; \quad \forall 1 \le i \le n$$

$$= \frac{\ln(\frac{S_{\{n+1\},0}}{\overline{S}_{i+1,\{n+1\}}}) + \int_{t_1}^{t_{i+1}} [r(u) - q(u) + \frac{1}{2}\sigma^2(u)]du + \int_{t_0}^{t_1}[r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du + z\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}{\sqrt{\int_{t_1}^{t_{i+1}} \sigma^2(u)du}}$$

$$= \frac{g_{\{n+1\},i+1} + z_2\rho_{1,i+1}}{\sqrt{1-\rho_{1,i+1}{}^2}}$$

where the correlation coefficient is given by $\rho_{1,i+1} = \frac{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}{\sqrt{\int_{t_0}^{t_{i+1}}\sigma^2(u)du}}$ According to Theorem 1(a) of Lee et al. (2008), the value of the correlation coefficient with the time shift can be given by:

$$\rho_{i,j,*1} = Q_{\{n\},i,j} = \frac{Q_{\{n+1\},i+1,j+1} - Q_{\{n+1\},1,i+1}Q_{\{n+1\},1,j+1}}{\sqrt{1-\left(Q_{\{n+1\},1,i+1}\right)^2}\sqrt{1-\left(Q_{\{n+1\},1,j+1}\right)^2}}$$

where $Q_{\{n\},i,j}$ is a symmetric entry (i, j) of the n by n correlation matrix of the n-variate normal integral. Further algebraic manipulations get

$$Q_{\{n+1\},i+1,j+1}$$
$$= Q_{\{n\},i,j}\sqrt{1-\left(Q_{\{n+1\},1,i+1}\right)^2}\sqrt{1-\left(Q_{\{n+1\},1,j+1}\right)^2} + Q_{\{n+1\},1,i+1}Q_{\{n+1\},1,j+1}$$
$$= \frac{\sqrt{\int_{t_1}^{t_{i+1}}\sigma^2(u)du}}{\sqrt{\int_{t_1}^{t_{j+1}}\sigma^2(u)du}}\sqrt{1-\frac{\int_{t_0}^{t_1}\sigma^2(u)du}{\int_{t_0}^{t_{i+1}}\sigma^2(u)du}}\sqrt{1-\frac{\int_{t_0}^{t_1}\sigma^2(u)du}{\int_{t_0}^{t_{j+1}}\sigma^2(u)du}}$$
$$+ \sqrt{\frac{\int_{t_0}^{t_1}\sigma^2(u)du}{\int_{t_0}^{t_{i+1}}\sigma^2(u)du}}\sqrt{\frac{\int_{t_0}^{t_1}\sigma^2(u)du}{\int_{t_0}^{t_{j+1}}\sigma^2(u)du}}$$
$$= \frac{\int_{t_1}^{t_{i+1}}\sigma^2(u)du}{\sqrt{\int_{t_0}^{t_{i+1}}\sigma^2(u)du}\sqrt{\int_{t_0}^{t_{j+1}}\sigma^2(u)du}} + \frac{\int_{t_0}^{t_1}\sigma^2(u)du}{\sqrt{\int_{t_0}^{t_{i+1}}\sigma^2(u)du}\sqrt{\int_{t_0}^{t_{j+1}}\sigma^2(u)du}}$$
$$= \frac{\int_{t_0}^{t_{i+1}}\sigma^2(u)du}{\sqrt{\int_{t_0}^{t_{i+1}}\sigma^2(u)du}\sqrt{\int_{t_0}^{t_{j+1}}\sigma^2(u)du}} = \sqrt{\frac{\int_{t_0}^{t_{i+1}}\sigma^2(u)du}{\int_{t_0}^{t_{j+1}}\sigma^2(u)du}}$$

$$= \rho_{i+1,j+1} \quad ; \forall \; 1 < i < j \le n$$

Therefore, $[\rho_{i,j,*1}]_{n\times n} = [\rho_{i,j}]_{(n+1)\times(n+1)}$. Given above, the value of $\overline{C_1}$ is found as:

$$\overline{C_1} = S_{\{n+1\},0}e^{\int_{t_0}^{t_{n+1}}q(u)du}N_{n+1}\left\{[g_{\{n\},i}]_{(n+1)\times 1};[\rho_{i,j}]_{(n+1)\times(n+1)}\right\}$$

By a similar approach, the values of $\overline{C_2}$ and $\overline{C_3}$ can be found, respectively, as:

$$\overline{C_2} = -\sum_{m=1}^{n}\left(K_{\{n+1\},m+1} - B_{\{n+1\},m+1}\right)e^{-\int_{t_0}^{t_{m+1}}r(u)du} \times$$
$$\int_{-h_{\{n+1\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}N_m\left\{[h_{\{n+1\},i,*1}]_{m\times 1};[\rho_{i,j,*1}]_{m\times m}\right\}dz$$
$$-\left(K_{\{n+1\},1} - B_{\{n+1\},1}\right)e^{-\int_{t_0}^{t_1}r(u)du}\int_{-h_{n+1,1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}dz$$
$$= -\sum_{m=1}^{n+1}\left(K_{\{n+1\},m} - B_{\{n+1\},m}\right)e^{-\int_{t_0}^{t_m}r(u)du}N_m\left\{[h_{\{n+1\},i}]_{m\times 1};[\rho_{i,j}]_{m\times m}\right\}$$

$$\overline{C_3} = -\sum_{m=1}^{n}B_{\{n+1\},m+1}e^{-\int_{t_0}^{t_{m+1}}r(u)du} \times$$
$$\int_{-h_{\{n+1\},1}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}N_{m-1}\left\{[h_{\{n+1\},i,*1}]_{(m-1)\times 1};[\rho_{i,j,*1}]_{(m-1)\times(m-1)}\right\}dz$$
$$-B_{\{n+1\},1}e^{-\int_{t_0}^{t_1}r(u)du}$$
$$= -\sum_{m=1}^{n+1}B_{\{n+1\},m}e^{-\int_{t_1}^{t_m}r(u)du}N_{m-1}\left\{[h_{\{n+1\},i,*1}]_{(m-1)\times 1};[\rho_{i,j,*1}]_{(m-1)\times(m-1)}\right\}$$

The solution can be obtained directly from equation (18) by adding one more fold layer. Equation (18) is proven by induction.

INFLUENCE OF PERFORMANCE BOND ON PROJECT VALUE

To see the influence of the performance bond to the project value, the following derives the partial derivatives of C_0 with respect to the parameter B.

Proposition. *From the pricing formul*a (18),

$$\sum_{l=1}^{n}\frac{\partial C_{\{n\},n}(V,t_0)}{\partial B_{\{n\},l}} = \sum_{l=1}^{n}e^{-\int_{t_0}^{t_l}r(u)du}\begin{bmatrix}N_l\left\{\left[h_{\{n\},i}\right]_{l\times 1};\left[\rho_{\{n\},i,j}\right]_{l\times l}\right\} \\ -N_{l-1}\left\{\left[h_{\{n\},i}\right]_{(l-1)\times 1};\left[\rho_{\{n\},i,j}\right]_{(l-1)\times(l-1)}\right\}\end{bmatrix} < 0 \qquad (23)$$

Proof.

For n = 1, (9) reduces to a vanilla European call option, and therefore

$$\frac{\partial C_{\{1\},1}(S,t_0)}{\partial B_{\{1\},1}}$$

$$= \frac{\partial}{\partial B_{\{1\},1}}\left[\begin{array}{l} S_{\{1\},0}e^{-\int_{t_0}^{t_1} q(u)du} N_1(g_{\{1\},1}) - \\ (K_{\{1\},1}-B_{\{1\},1})e^{-\int_{t_0}^{t_1} r(u)du} N_1(h_{\{1\},1}) - B_{\{1\},1}e^{-\int_{t_0}^{t_1} r(u)du} N_0 \end{array}\right]$$

$$= S_{\{1\},0}e^{-\int_{t_0}^{t_1} q(u)du} \frac{\partial N_1(g_{\{1\},1})}{\partial B_{\{1\},1}}$$

$$-(K_{\{1\},1}-B_{\{1\},1})e^{-\int_{t_0}^{t_1} r(u)du} \frac{\partial N_1(h_{\{1\},1})}{\partial B_{\{1\},1}} + e^{-\int_{t_0}^{t_1} r(u)du} N_1(h_{\{1\},1}) - e^{-\int_{t_0}^{t_1} r(u)du}$$

By chain rule,

$$\frac{\partial N_1\{g_{\{1\},1}\}}{\partial B_{\{1\},1}} = \frac{\partial N_1\{g_{\{1\},1}\}}{\partial g_{\{1\},1}} \cdot \frac{\partial g_{\{1\},1}}{\partial B_{\{1\},1}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}g_{\{1\},1}^{\ 2}} \frac{1}{(K_{\{1\},1}-B_{\{1\},1})\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}};$$

$$\frac{\partial N_1\{h_{\{1\},1}\}}{\partial B_{\{1\},1}} = \frac{\partial N_1\{h_{\{1\},1}\}}{\partial h_{\{1\},1}} \cdot \frac{\partial h_{\{1\},1}}{\partial B_{\{1\},1}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}h_{\{1\},1}^{\ 2}} \frac{1}{(K_{\{1\},1}-B_{\{1\},1})\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}};$$

Since $\frac{\partial g_{\{1\},1}}{\partial B_{\{1\},1}} = \frac{\partial h_{\{1\},1}}{\partial B_{\{1\},1}}$ and replacing $g_{\{1\},1}$ by $h_{\{1\},1} + \sqrt{\int_{t_0}^{t_1} \sigma^2(u)du}$ and by some simple calculation, it follows that

$$\frac{\partial C_{\{n\},n}(S,t_0)}{\partial B_{\{1\},1}} = e^{-\int_{t_0}^{t_1} r(u)du}\left[N_1(h_{\{1\},1}) - 1\right] < 0 \tag{24}$$

The partial derivatives of the pricing formula $C_{\{n\},n}(S,t_0)$ with respect to the parameter B is written as $\partial C / \partial B$. Since

$$C_{\{n\},n}(S,t_0)$$

$$= S_{\{n\},0}e^{\int_{t_0}^{t_{n+1}} q(u)du} N_n\{[g_{\{n\},i}]_{n\times 1};[\rho_{i,j}]_{n\times n}\}$$

$$-\sum_{m=1}^{n} (K_{\{n\},m}-B_{\{n\},m})e^{-\int_{t_0}^{t_m} r(u)du} N_m\{[h_{\{n\},i}]_{m\times 1};[\rho_{i,j}]_{m\times m}\}$$

$$-\sum_{m=1}^{n} B_{\{n\},m}e^{-\int_{t_0}^{t_m} r(u)du} N_{m-1}\{[h_{\{n\},i}]_{(m-1)\times 1};[\rho_{i,j}]_{(m-1)\times(m-1)}\}$$

$$C_{\{n\},n}(S,t_0)$$

$$= S_{\{n\},0}e^{-\int_{t_0}^{t_n} q(u)du} N_n\{g_{\{n\},1}, g_{\{n\},2},\ldots, g_{\{n\},n};[\rho_{i,j}]_{n\times n}\}$$

$$-(K_{\{n\},n}-B_{\{n\},n})e^{-\int_{t_0}^{t_n} r(u)du} N_n\{h_{\{n\},1}, h_{\{n\},2},\ldots, h_{\{n\},n};[\rho_{i,j}]_{n\times n}\}$$

$$-(K_{\{n\},n-1}-B_{\{n\},n-1})e^{-\int_{t_0}^{t_{n-1}} r(u)du} N_{n-1}\{h_{\{n\},1}, h_{\{n\},2},\ldots, h_{\{n\},n-1};[\rho_{i,j}]_{(n-1)\times(n-1)}\}$$

$$-\ldots$$

$$-(K_{\{n\},2}-B_{\{n\},2})e^{-\int_{t_0}^{t_2} r(u)du} N_2\{h_{\{n\},1}, h_{\{n\},2};[\rho_{i,j}]_{2\times 2}\}$$

$$-(K_{\{n\},1}-B_{\{n\},1})e^{-\int_{t_0}^{t_1} r(u)du} N_1\{h_{\{n\},1}\}$$

$$-B_{\{n\},n}e^{-\int_{t_0}^{t_n} r(u)du} N_{n-1}\{h_{\{n\},1}, h_{\{n\},2},\ldots, h_{\{n\},n-1};[\rho_{i,j}]_{(n-1)\times(n-1)}\}$$

$$-B_{\{n\},n-1}e^{-\int_{t_0}^{t_{n-1}} r(u)du} N_{n-2}\{h_{\{n\},1}, h_{\{n\},2},\ldots, h_{\{n\},n-2};[\rho_{i,j}]_{(n-2)\times(n-2)}\}$$

$$-\ldots - B_{\{n\},2}e^{-\int_{t_0}^{t_2} r(u)du} N_1\{h_{\{n\},1}\} - B_{\{n\},1}e^{-\int_{t_0}^{t_1} r(u)du} N_0$$

We have different Bs for different fold numbers. Since the $B_{\{n\},i}$, i = 1, 2, …, n-1, does not exist in the multivariate normal functions, it follows that

$$\frac{\partial C_{\{n\},n}(S,t_0)}{\partial B_{\{n\},l}} = e^{-\int_{t_0}^{t_l} r(u)du}\left[\begin{array}{l} N_l\{[h_{\{n\},i}]_{l\times 1};[\rho_{\{n\},i,j}]_{l\times l}\} \\ -N_{l-1}\{[h_{\{n\},i}]_{(l-1)\times 1};[\rho_{\{n\},i,j}]_{(l-1)\times(l-1)}\}\end{array}\right] \tag{25}$$

For $i = n$，$\overline{S}_{i,\{n\}} = \overline{S}_{n,\{n\}} = K_{\{n\},n} - B_{\{n\},n}$。

$$\frac{\partial C_{\{n\},n}(S,t_0)}{\partial B_{\{n\},n}}$$

$$= S_{\{n\},0}e^{-\int_{t_0}^{t_n} q(u)du} \frac{\partial}{\partial B_{\{n\},n}} N_n\{[g_{\{n\},i}]_{n\times 1};[\rho_{i,j}]_{n\times n}\}$$

$$-(K_{\{n\},n}-B_{\{n\},n})e^{-\int_{t_0}^{t_n} r(u)du} \frac{\partial}{\partial B_{\{n\},n}} N_n\{[h_{\{n\},i}]_{n\times 1};[\rho_{i,j}]_{n\times n}\} \tag{26}$$

$$+e^{-\int_{t_0}^{t_n} r(u)du} N_n\{[h_{\{n\},i}]_{n\times 1};[\rho_{i,j}]_{n\times n}\}$$

$$-e^{-\int_{t_0}^{t_n} r(u)du} N_{n-1}\{[h_{\{n\},i}]_{(n-1)\times 1};[\rho_{i,j}]_{(n-1)\times n}\}$$

As a consequence, $B_{\{n\},n}$ exists in the multivariate integral $N_n\{g_{\{n\},1}, g_{\{n\},2},\ldots, g_{\{n\},n};[\rho_{i,j}]_{n\times n}\}$ and. $N_n\{h_{\{n\},1}, h_{\{n\},2},\ldots, h_{\{n\},n};[\rho_{i,j}]_{n\times n}\}$. Therefore, we have to use partial derivative of the multivariate normal integral formula which derived by Lee, Yeh, and Chen (2008) to do the sensitivity work. By Lee, Yeh, and Chen's Lemma 2,

$$
\begin{aligned}
&\frac{\partial N_n\left\{\left[g_{\{n\},i}\right]_{n\times 1};\left[\rho_{\{n\},i,j}\right]_{n\times n}\right\}}{\partial B_{\{n\},n}} \\
&=\sum_{v=1}^{n} f\left(g_{\{n\},v}\right)\left(\frac{\partial g_{\{n\},v}}{\partial B_{\{n\},n}}\right) N_{n-1}\left\{\begin{array}{l}\left(\left[\dfrac{g_{\{n\},i}-g_{\{n\},v}\rho_{\{n\},i,v}}{\sqrt{1-\left(\rho_{\{n\},i,v}\right)^2}}\right]_{n\times 1}\right)^{(-v,)};\\ \left(\left[\dfrac{\rho_{\{n\},i,j}-\rho_{\{n\},i,v}\rho_{\{n\},j,v}}{\sqrt{\left(1-\left(\rho_{\{n\},i,v}\right)^2\right)\left(1-\left(\rho_{\{n\},j,v}\right)^2\right)}}\right]_{n\times n}\right)^{(-v,-v)}\end{array}\right\} \\
&=\sum_{v=1}^{n} f\left(g_{\{n\},v}\right)\left(\frac{\partial g_{\{n\},v}}{\partial B_{\{n\},n}}\right) N_{v-1}\left\{\begin{array}{l}\left[\dfrac{g_{\{n\},i}-g_{\{n\},v}\rho_{\{n\},i,v}}{\sqrt{1-\left(\rho_{\{n\},i,v}\right)^2}}\right]_{(v-1)\times 1};\\ \left[\dfrac{\rho_{\{n\},i,j}-\rho_{\{n\},i,v}\rho_{\{n\},j,v}}{\sqrt{\left(1-\left(\rho_{\{n\},i,v}\right)^2\right)\left(1-\left(\rho_{\{n\},j,v}\right)^2\right)}}\right]_{(v-1)\times(v-1)}\end{array}\right\} \\
&\qquad\times N_{n-v}\left\{\left[g_{\{n\},i,\#v}\right]_{(n-v)\times 1};\left[\rho_{\{n\},i,j,*v}\right]_{(n-v)\times(n-v)}\right\}
\end{aligned}
\tag{27}
$$

And,

$$
\begin{aligned}
&\frac{\partial N_n\left\{\left[h_{\{n\},i}\right]_{n\times 1};\left[\rho_{\{n\},i,j}\right]_{n\times n}\right\}}{\partial B_{\{n\},n}} \\
&=\sum_{v=1}^{n} f\left(h_{\{n\},v}\right)\left(\frac{\partial h_{\{n\},v}}{\partial B_{\{n\},n}}\right) N_{n-1}\left\{\begin{array}{l}\left(\left[\dfrac{h_{\{n\},i}-h_{\{n\},v}\rho_{\{n\},i,v}}{\sqrt{1-\left(\rho_{\{n\},i,v}\right)^2}}\right]_{n\times 1}\right)^{(-v,)};\\ \left(\left[\dfrac{\rho_{i,j}-\rho_{\{n\},i,v}\rho_{\{n\},j,v}}{\sqrt{\left(1-\left(\rho_{\{n\},i,v}\right)^2\right)\left(1-\left(\rho_{\{n\},j,v}\right)^2\right)}}\right]_{n\times n}\right)^{(-v,-v)}\end{array}\right\} \\
&=\sum_{v=1}^{n} f\left(h_{\{n\},v}\right)\left(\frac{\partial h_{\{n\},v}}{\partial B_{\{n\},n}}\right) N_{v-1}\left\{\begin{array}{l}\left[\dfrac{h_{\{n\},i}-h_{\{n\},v}\rho_{\{n\},i,v}}{\sqrt{1-\left(\rho_{\{n\},i,v}\right)^2}}\right]_{(v-1)\times 1};\\ \left[\dfrac{\rho_{\{n\},i,j}-\rho_{\{n\},i,v}\rho_{\{n\},j,v}}{\sqrt{\left(1-\left(\rho_{\{n\},i,v}\right)^2\right)\left(1-\left(\rho_{\{n\},j,v}\right)^2\right)}}\right]_{(v-1)\times(v-1)}\end{array}\right\} \\
&\qquad\times N_{n-v}\left\{\left[h_{\{n\},i,\#v}\right]_{(n-v)\times 1};\left[\rho_{\{n\},i,j,*v}\right]_{(n-v)\times(n-v)}\right\}
\end{aligned}
\tag{28}
$$

where the symbol *v indicates a time shift, such that the correlation matrix starts from time s, or

$$
\rho_{\{n\},i,j,*v}=\sqrt{\sum_{u=1+v}^{i}\sigma_u^2\tau_u \Big/ \sum_{u=1+v}^{j}\sigma_u^2\tau_u}\;;
$$

$$
g_{\{n\},i,\#v}=\frac{\left[\ln\left(\dfrac{\overline{S}_{v,\{n\}}}{\overline{S}_{i,\{n\}}}\right)+\displaystyle\sum_{u=1+v}^{i}\left(r_u-q_u+\frac{1}{2}\sigma_u^2\right)\tau_u\right]}{\sqrt{\displaystyle\sum_{u=1+v}^{i}\sigma_u^2\tau_u}},
$$

$$
h_{\{n\},i,\#v}=\frac{\left[\ln\left(\dfrac{\overline{S}_{v,\{n\}}}{\overline{S}_{i,\{n\}}}\right)+\displaystyle\sum_{u=1+v}^{i}\left(r_u-q_u-\frac{1}{2}\sigma_u^2\right)\tau_u\right]}{\sqrt{\displaystyle\sum_{u=1+v}^{i}\sigma_u^2\tau_u}},\quad \forall 1\le s\le i\le n\,;
$$

and $\dfrac{\partial g_{\{n\},i}}{\partial B_{\{n\},n}}=0$ $\forall 1 \le i < n$. Substituting (27) and (28) into (26) gives:

$$
\begin{aligned}
&\frac{\partial C_{\{n\},n}\left(S,t_0\right)}{\partial B_{\{n\},n}} \\
&=e^{-\int_{t_0}^{t_n} r(u)du} N_n\left\{\left[h_{\{n\},i}\right]_{n\times 1};\left[\rho_{i,j}\right]_{n\times n}\right\} \\
&\quad -e^{-\int_{t_0}^{t_n} r(u)du} N_{n-1}\left\{\left[h_{\{n\},i}\right]_{(n-1)\times 1};\left[\rho_{i,j}\right]_{(n-1)\times n}\right\} \\
&\quad +\hat{C}_{\partial B_{\{n\},n},1}-\hat{C}_{\partial B_{\{n\},n},2}
\end{aligned}
\tag{29}
$$

where

$$
\begin{aligned}
&\hat{C}_{\partial B_{\{n\},n},1} \\
&=S_{\{n\},0}e^{-\int_{t_0}^{t_n} q(u)du}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(g_{\{n\},n}\right)^2}\left(\frac{\partial g_{\{n\},n}}{\partial B_{\{n\},n}}\right) \\
&\quad\times N_{n-1}\left\{\begin{array}{l}\left[\dfrac{h_{\{n\},i}-h_{\{n\},n}\rho_{\{n\},n,i}}{\sqrt{1-\left(\rho_{\{n\},n,i}\right)^2}}\right]_{(n-1)\times 1};\\ \left[\dfrac{\rho_{\{n\},i,j}-\rho_{\{n\},n,i}\rho_{\{n\},n,j}}{\sqrt{\left(1-\left(\rho_{\{n\},n,i}\right)^2\right)\left(1-\left(\rho_{\{n\},n,j}\right)^2\right)}}\right]_{(n-1)\times(n-1)}\end{array}\right\}\times N_0
\end{aligned}
$$

and

$$
\begin{aligned}
&\hat{C}_{\partial B_{\{n\},n},2} \\
&=\left(K_{\{n\},n}-B_{\{n\},n}\right)e^{-\int_{t_0}^{t_n} r(u)du}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(h_{\{n\},n}\right)^2}\left(\frac{\partial h_{\{n\},n}}{\partial B_{\{n\},n}}\right) \\
&\quad\times N_{n-1}\left\{\begin{array}{l}\left[\dfrac{h_{\{n\},i}-h_{\{n\},n}\rho_{\{n\},n,i}}{\sqrt{1-\left(\rho_{\{n\},n,i}\right)^2}}\right]_{(n-1)\times 1};\\ \left[\dfrac{\rho_{\{n\},i,j}-\rho_{\{n\},n,i}\rho_{\{n\},n,j}}{\sqrt{\left(1-\left(\rho_{\{n\},n,i}\right)^2\right)\left(1-\left(\rho_{\{n\},n,j}\right)^2\right)}}\right]_{(n-1)\times(n-1)}\end{array}\right\}\times N_0
\end{aligned}
$$

Since

$$
\begin{aligned}
f\left(g_{\{n\},n}\right)&=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(g_{\{n\},n}\right)^2}=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left[\left(h_{\{n\},n}\right)^2+\sqrt{\int_{t_0}^{t_n}\sigma^2(u)du}\right]^2}, \\
&=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(h_{\{n\},n}\right)^2-\int_{t_0}^{t_n}\left(r(u)-q(u)\right)du}\left(\frac{K_{\{n\},n}-B_{n,\{n\}}}{S_{\{n\},0}}\right)
\end{aligned}
$$

$\dfrac{\partial g_{\{n\},n}}{\partial B_{\{n\},n}}=\dfrac{\partial h_{\{n\},n}}{\partial B_{\{n\},n}}$, and $\hat{C}_{\partial B_{\{n\},n},1}=\hat{C}_{\partial B_{\{n\},n},2}$, it follows that

$$\frac{\partial C_{\{n\},n}(S,t_0)}{\partial B_{\{n\},n}}$$
$$= e^{-\int_{t_0}^{t_n} r(u)du} N_n \left\{ \left[h_{\{n\},i} \right]_{n\times 1}; \left[\rho_{i,j} \right]_{n\times n} \right\}$$
$$- e^{-\int_{t_0}^{t_n} r(u)du} N_{n-1} \left\{ \left[h_{\{n\},i} \right]_{(n-1)\times 1}; \left[\rho_{i,j} \right]_{(n-1)\times n} \right\} .$$

Denote by B_l a subsequent performance bond. It can be verified by a similar method that for $\forall 1 < l \le n$,

$$\frac{\partial C_{\{n\},n}(S,t_0)}{\partial B_{\{n\},l}} = e^{-\int_{t_0}^{t_l} r(u)du} \left[\begin{array}{l} N_l \left\{ \left[h_{\{n\},i} \right]_{l\times 1}; \left[\rho_{\{n\},i,j} \right]_{l\times l} \right\} \\ - N_{l-1} \left\{ \left[h_{\{n\},i} \right]_{(l-1)\times 1}; \left[\rho_{\{n\},i,j} \right]_{(l-1)\times(l-1)} \right\} \end{array} \right] \quad (30)$$

Summing up (24) and (30) gives (23).☐

NUMERICAL IMPLEMENTATION

This section chooses a three-stage BOT sanitary sewerage project for the numerical implementation of the proposed valuation model. A MATLAB-based computer program is written to support the implementation.

Project profile

The three-stage BOT sanitary sewerage project is a large-scale BOT project with 35 concession years and a total construction cost of NT$1,356 million. The current population of the project's service area is 28,000. The forecasted population is 48,248 at the end of concession. To attract private investment, the investor is allowed to abandon voluntarily according to the conditions stipulated in the concession contract. The concessionaire could terminate the project prematurely, but a performance bond was required to guarantee that the concessionaire perform according to the concession contract. The initial value of the performance bond was NT$40.68 million, approximately 3 % of the total construction cost. The bond value was reduced to half of the initial amount after finishing 1-st stage construction work and remained the same to the end of the concession period.

Parameter estimates

This study collects market data from the Central Bank and the Taiwan Economic Journal Database (TEJD) to support parameter estimations.

The expected risk-free interest rate is 4.19%, which is estimated from monthly 10-year treasury-bond spot rates observed between January 1995 and September 2007. Use a portfolio of six public gas companies as a proxy to estimate volatilities and the five-year volatility is about 16%. This case study does not consider dividend payout, and thus assume q=0. The underlying asset values of the staged works are calculated by discounting the net cash flows of each stage by the base-case ROE, 6.49%, which is estimated by using the capital asset pricing model. According to the financial data of the project disclosed by the government, the project had an initial asset value S_0 of NT$1660 million at t_0 which were calculated from the discounted value of the project's earnings before interest, tax, and depreciation (EBITDA), without considering the effects of financing and taxation. Based on this discount rate, the time-t_1 discounted value of the construction cost is NT$1605 million. Table 1 summarizes the "base case" valuation parameters.

Table 1 The "base case" valuation parameters.

Variable	Value
Time-t_0 discounted value of the underlying project asset (S_0)	NT$ 1,660 million
Time-t_1 discounted value of the construction cost (K)	NT$ 1,605 million
Performance bond value (B)	NT$ 40.68 million
Risk-free interest rate (r)	4.19%
Asset return volatility (σ)	0.16
Dividend payout rate (q)	N.A.

Valuation outcomes

Without considering the effect of the performance bond (that is *B*=0), the base-case valuation parameters produce a project value of NT$ 143.63 million at t_0. However, this value is smaller than the net present value (NPV) of the project at t_0, which is NT$ 244.14 million calculated by the 6.49 % discounted rate.

When the effect of the performance bond is considered (that is B=3% of total construction cost), however, the project value is reduced from NT$ 143.63 million to NT$ 143.62 million. There is no much difference about the change of the project value whether considering the influence of the performance bond or not. That might due to the small base-case volatility. Moreover, if the investment risk raises and volatility is enlarged to 0.5, the project value would drop about 10% when the influence of performance bond is considered. Therefore, this result indicates that the effect of performance bonding on project value should be assessed carefully; otherwise, the project value will be overstated, and this may mislead investment decisions.

Sensitivity analysis

In general, the value of sequential multi-fold CCOs increases with underlying asset value and volatility, and decreases with risk-free rate. Figure 2 shows the project values calculated by different financial tools. In this case, the project under performance bonding with small volatility has irregular outcomes because the 2nd-fold of the SCCO options wouldn't be exercised.

From the result of derivation in section 3, the project value decreases with an increase of the performance bonding. The sensitivity with respect to the performance bonding (B) is further graphed in Figure 3.

Moreover, the consideration of different performance bonding decisions gives different results of project value. Figure 4 shows the investment value of consistent performance bonding project worth less than the project with decreasing performance bonding decisions set by contract.

Figure 2. Sensitivity analysis with respect to sigma.

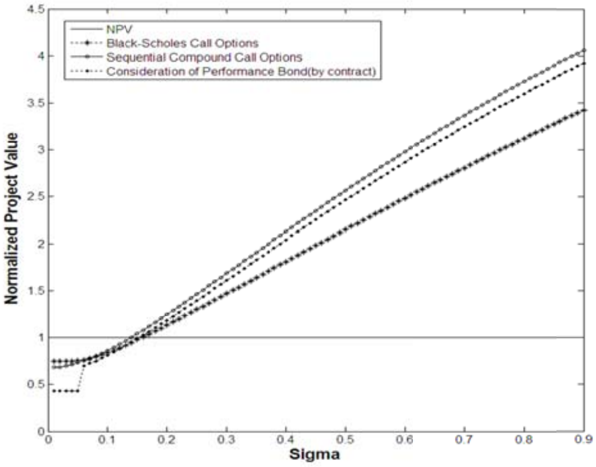


Figure 3. Sensitivity analysis with respect to B.

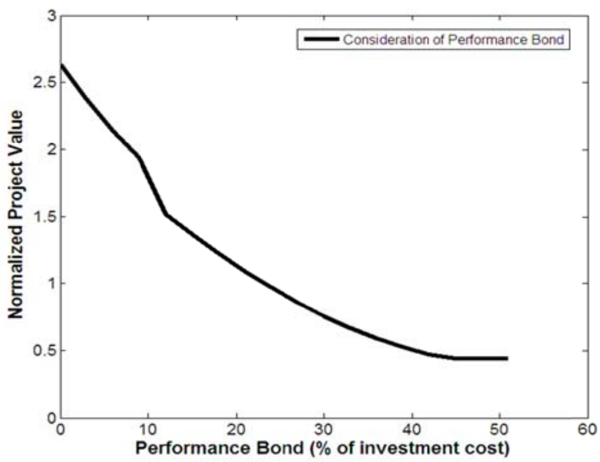
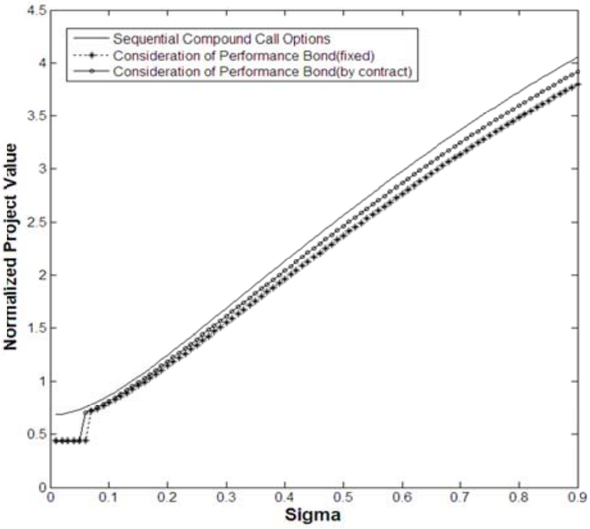


Figure 4. Sensitivity analysis with respect to different performance bonding decisions.



CONCLUSION

Real-option theory is popular in managing complex valuation problems in BOT undertakings. However, previous BOT real-option valuation models have not incorporated performance bonds in valuation. This tends to overstate BOT project values when the projects in question involve voluntary abandonment rights. This paper derives a valuation model to contend with this issue. Sensitivity analysis shows that the value of flexibility created by the option to abandon decreases when the value of the bond is increased. Moreover, different setting of performance bonds could leads to different investment decisions. Therefore, considering the influence of performance bonds during valuation is necessary.

REFERENCES

Algarni, A.M., Arditi, D., Polat, G., 2007. Build–operate–transfer in infrastructure projects in the United States. J. Constr. Eng. and Manage., **133**(10), 728–735.

Black, F., Scholes, M., 1973. The pricing of options and corporate liabilities. J. Polit. Econ, **81**(3), 637–57.

Chen, C., Messner, J., 2005. An investigation of Chinese BOT projects in water supply: A comparative perspective. Constr. Manage. and Econ., **23**(9), 913–925.

Chen, T., Zhang, J., and Lai, K.K., 2009. An integrated real options evaluating model for information technology projects under multiple risks. Int. J. Project Manage., **27**(8), 776–786.

Cox, J.C., Ross, S.A., 1976. The valuation of options for alternative stochastic processes. J. Financ. Econ., **3**(1–2), 145–166.

Damnjanovic I., Duthie J., Waller S. T., 2008. Valuation of strategic network flexibility in development of toll road projects, Constr. Manage. Econ., **26**(9), 979–990.

Dixit, A.K., Pindyck, R.S., 1994. Investment Under Uncertainty, Princeton University Press, Princeton, New Jersey.

Eckhause, J.M., Hughes, D.R., and Gabriel, S.A., 2009. Evaluating real options for mitigating technical risk in public sector R&D acquisition. Int. J. of Project Manage., **27**(4), 365–377.

Fischer, K., Jungbecker, A., Alfen, H.W., 2006. The emergence of PPP task forces and their influence on project delivery in Germany. Int. J. Project Manage., **24**(7), 539–547.

Garvin, M.J., Cheah, C.Y.L., 2004. Valuation techniques for infrastructure investment decisions. Constr. Manage. Econ., **22**(4), 373–383.

Grimsey, D., Lewis, M.K., 2002. Evaluating the risks of public–private partnerships for infrastructure projects. Int. J. of Project Manage., **20**(2), 107–118.

Harrison, M. J., Kreps, D.M., 1979. Martingales and arbitrage in multiperiod securities market. J. Econ. theory, **20**(3), 381–408.

Harrison, M. J., Pliska, S.R., 1981. Martingales and stochastic integrals in the theory of continuous trading. Stoch. Prpc. Appl., **11**(3), 215–260.

Huang, Y.L., 1995. *Project and Policy Analysis of Build-Operate-Transfer Infrastructure Development*, PhD Thesis, UC Berkeley, Berkeley, CA.

Huang, Y.L., Chou, S.P., 2006. Valuation of the minimum revenue guarantee and the option to abandon in BOT infrastructure projects. Constr. Manage Econ., **24**(4), 379–389.

Huang, Y.L., Pi, C.C., 2009. Valuation of multistage BOT projects involving dedicated asset investments: A sequential compound option approach. Constr. Manage. Econ., **27**(7), 653–666.

Huang, Y.L., Pi, C.C., 2010. Competition, dedicated assets, and technological obsolescence in multistage infrastructure investments: A sequential compound option valuation. IEEE Trans. Eng. Manage., Forthcoming.

Klein, B., 1996. Why 'hold-ups' occur: The 'self-enforcing range' of contractual relationships. Econ. Inq., **34**(3), 444–463.

Kleiss, T., Imura, H., 2006. The Japanese private finance initiative and its application in the municipal solid waste management sector. Int. J. Project Manage., **24**(7), 614–621.

Koch, C., Buser, M., 2006. Emerging metagovernance as an institutional framework for public private partnership networks in Denmark. Int. J. Project Manage., **24**(7), 548–556.

Kumaraswamy, M.M., Morris, D.A., 2002. Build–operate–transfer type procurement in Asian megaprojects. J. Constr. Eng. and Manage., **128**(2), 93–102.

Merton, R.C., 1973. Theory of rational option pricing. Bell J. Econ. and Manage. Sci., **4**(1), 141–83.

Panayi S, Trigeorgis L., 1998. Multi-stage real options: The cases of information technology infrastructure and international bank expansion. Quart. Rev. Econ. Finance, **38**(Special issue), 675–92.

Rose, S., 1998. Valuation of interacting real options in a tollroad infrastructure project. Q. Rev. Econ. Financ., **38**(3), 711–723.

Shreve, S.E., 2004. Stochastic Calculus for Finance II: Continuous-Time Models, Second ed., Springer, New York.

Smit, H.T.J., 2003. Infrastructure investment as a real options game: The case of European airport expansion. Financ. Manage., **32**(4), 27–58.

Tam, C.M., 1999. Build–operate–transfer model for infrastructure developments in Asia: Reasons for successes and failures. Int. J. Project Manage., **17**(6), 377–382.

Vandegrift, D., 1999, Asset specificity, long-term contracts, and the good faith requirement. Eastern Eco. J., 24(4), 475–493.

Vazquez, F., Allen, S., 2004. Private sector participation in the delivery of highway infrastructure in Central America and Mexico. Constr. Manage. and Econ. **22**(7), 745–754.

Wand, C.H., Min, K.J., 2006. Electric power generation planning for interrelated projects: A real option approach, IEEE Trans. Eng. Manage., **53**(2), 312–322.

Wibowo, A., 2004. Valuing guarantees in a BOT infrastructure project. Eng., Constr. and Archi. Manage., **11**(6), 395–403.

Winch, G.M., 2000. Institutional reform in British construction: Partnering and private finance. Building Re. and Infor., **28**(2), 141–155.

Yeo, K.T. and Qiu, F., 2003. The value of management flexibility—A real option approach to investment evaluation. Int. J. Project Manage., **21**(4), 243–250

國科會補助計畫衍生研發成果推廣資料表

日期:2012/10/31

<table>
<tr><td rowspan="3">國科會補助計畫</td><td>計畫名稱: 序列複合買權法應用於履約保證下多期BOT基礎建設專案評價之研究</td></tr>
<tr><td>計畫主持人: 黃玉霖</td></tr>
<tr><td>計畫編號: 100-2221-E-009-137-　　　　學門領域: 營建管理</td></tr>
<tr><td colspan="2">無研發成果推廣資料</td></tr>
</table>

100 年度專題研究計畫研究成果彙整表

計畫主持人：黃玉霖	計畫編號：100-2221-E-009-137-

計畫名稱：序列複合買權法應用於履約保證下多期 BOT 基礎建設專案評價之研究

成果項目			量化：實際已達成數（被接受或已發表）	量化：預期總達成數(含實際已達成數)	量化：本計畫實際貢獻百分比	單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估 ■達成目標 □未達成目標（請說明，以 100 字為限） □實驗失敗 □因故實驗中斷 □其他原因 說明：
2. 研究成果在學術期刊發表或申請專利等情形： 論文：□已發表 □未發表之文稿 ■撰寫中 □無 專利：□已獲得 □申請中 ■無 技轉：□已技轉 □洽談中 ■無 其他：（以 100 字為限）
3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限） 學術成就方面，利用考量履約保證金影響的序列複合買權評價模型(Sequential Compound Call Options Pricing Model)，深入探討履約保證金對於多期複雜專案投資價值的影響。同時，藉由選擇權評價模型之建構及敏感度分析之公式推導的過程，了解建構數理統計模型之必需及相關議題。 技術創新層面在於有別於傳統折現現金流量(Discount Cash Flow, DCF)的投資評估分析模式，利用考量投資彈性與環境風險的財務選擇權(Financial Options)進行專案投資價值的探討與分析。再者，因序列複合買權評價模型在應用上涉及複雜的數學計算，透過Matlab 程式的撰寫，協助序列複合買權評價模型在實務應用上的複雜數學計算。