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Current manufacturing industries have increased their level of out-sourcing and relied more heavily on their supply chain as a source of competitive advantage. Supplier selection decisions have become an important component of production management. Those decisions have a significant impact on a firm's marketing competition, and suppliers may account for a large portion of the production cost. Production quality is one of the key factors in supplier evaluation. The manual of supplier certification includes a discussion of process capability analysis, which recommends a procedure for evaluating the most prevalent process capability index C_{pk}. However, the recommended procedure is applicable only when evaluating an individual supplier's performance. In this paper, we apply the bootstrap method to the supplier selection problem. We construct lower confidence intervals for the capability difference and ratio between two given suppliers. Performance comparisons are made among various bootstrap methods in terms of error probability and selection power. For convenience of applications, the sample sizes required for various designated selection power are also tabulated.

Keywords: Bootstrap resampling; Error probability; Lower confidence bound; Production yield; Supplier selection

1. Introduction

Manufacturers purchase components from suppliers or hire contract manufacturers to produce necessary parts, and they assemble these parts to deliver the finished products to customers. The major considerations when choosing a supplier or a contract manufacturer include quality, cost, goodwill, service, delivery, and so on. According to research conducted by Dickson (1966), quality and delivery are two of the most demanded items by component suppliers. Twenty five years after Dickson's research, Weber *et al.* (1991) still considered quality to be of 'extreme importance' and delivery to be of 'considerable importance'. According to Weber's research on the just-in-time (JIT) model, the importance of quality and delivery remains the same. Pearson and Ellram (1995) surveyed 210 members of the National Association

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of Purchasing Management (NAPM), who were randomly selected from the listings of electronic firms in the two-digit SIC code 38, and they indicated that quality is the most important criterion in the selection and evaluation of suppliers for both the small and large electronic firms that were surveyed. Moreover, according to the survey of current and potential outsourcing end-users by the Outsourcing Institute (2003), the top 10 factors in vendor selection are commitment to quality, price, reference/reputation, flexible contract terms, scope of resources, additional value-added capability, cultural match, existing relationship, location, and others. Quality is still the most important factor of all. Furthermore, Olhager and Selldin (2004) investigated supply chain management strategies and practices in a sample of 128 Swedish manufacturing firms and concluded that many aspects are important when companies choose supply chain partners, but quality is the single most important criterion. Kane (1986) stated that the quantification of the process mean and variation is central to understanding the quality of the units produced from a manufacturing process. Process capability indices (PCIs) can also be used to measure process potential at the initial stage of the production setting. These facts bring the issue of supplier selection based on PCIs into the main focus.

The first PCI appearing in the literature was the precision index C_p and it is defined as (see Juran 1974 and Kane 1986):

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma}, \tag{1}$$

where USL is the upper specification limit, LSL is the lower specification limit, and σ is the process standard deviation. The index C_p measures process precision (product quality consistency), and does not consider whether the process is centred. To measure the degree of process centring, Pearn *et al.* (1998) introduced the following accuracy index C_a:

$$C_a = 1 - \frac{|\mu - m|}{d}, \tag{2}$$

where μ is the process mean, $d = (\text{USL} - \text{LSL})/2$, and $m = (\text{USL} + \text{LSL})/2$. The index C_a measures the centring tendency, which alerts the user if the process mean deviates from its midpoint. The C_{pk} index considers process variation and the location of process mean,

$$C_{\text{pk}} = \min\{C_{\text{pu}}, C_{\text{pl}}\} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\} = \frac{d - |\mu - m|}{3\sigma}. \tag{3}$$

Obviously, we have $C_{\text{pk}} = C_p \times C_a$. Taguchi, on the other hand, emphasizes the product loss when one of its characteristics departs from the target value T. Hsiang and Taguchi (1985) introduced the index C_{pm}, which was also proposed independently by Chan *et al.* (1988). The index C_{pm} incorporates the variation of production items with the target value and the specification limits preset in the factory. It is defined as:

$$C_{\text{pm}} = \frac{\text{USL} - \text{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}}. \tag{4}$$

In practice, process mean μ and process variance σ^2 are usually unknown. Since sample data must be collected to calculate the index value, sampling errors are introduced into the capability assessments. Consequently, lower confidence bounds (LCBs) or capability testing must be performed using their sampling distributions. Many authors have promoted the use of various PCIs for evaluating a supplier's process capability. Examples include Boyles (1991), Pearn *et al.* (1992), Kushler and Hurley (1992), Kotz and Johnson (1993), Vännman and Kotz (1995), Vännman (1997), Kotz and Lovelace (1998), Pearn *et al.* (1998), Kotz and Johnson (2002), Spiring *et al.* (2003), Pearn and Shu (2003), Pearn *et al.* (2005), and references therein. However, one area that has received little attention in the literature is the comparison between two suppliers' PCIs. In a review of the problem of selecting the best manufacturing process based on PCIs, Tseng and Wu (1991) considered the problem for multiple available manufacturing processes based on the precision index C_p under a modified likelihood ratio (MLR) selection rule. Chou (1994) developed a test for comparing two one-sided processes and choosing a better supplier when the sample sizes are equal. Hubele *et al.* (2005) applied a Wald statistic for testing the equality of multiple C_{pu} or C_{pl} indices. Huang and Lee (1995) considered the supplier selection problem based on the index C_{pm}, and developed a mathematically complicated approximation method for selecting a subset of processes containing the best supplier from a given set of processes. The method essentially compares the average loss of a group of candidate processes and selects a subset of these processes with smaller process loss $\sigma^2 + (\mu - T)^2$, which, with certain level of confidence, contains the best process. Pearn *et al.* (2004) provided additional useful information regarding the sample size required for various designated selection power. A two-phase selection procedure was developed to select a better supplier. Chen and Chen (2004) offered four approximate confidence interval methods, one based on the statistical theory given in Boyles (1991) and three based on the bootstrap method, for selecting the better one of two suppliers. A comparison of the coverage percentage of the four methods was investigated by simulation. Although statistical tests have been developed to compare two C_p, C_{pm}, C_{pu}, and C_{pl} capability indices of normal processes, a statistical test for comparing two C_{pk} values has not been developed due to the complexity of the sampling distribution of $\hat{C}_{\text{pk2}} - \hat{C}_{\text{pk1}}$ or $\hat{C}_{\text{pk2}}/\hat{C}_{\text{pk1}}$. In this paper, we apply the bootstrap method to compare two processes based on C_{pk} in terms of error of probability and selecting power. The obtained confidence intervals provide information regarding actual process performance, which is useful in making reliable decisions for capability testing (H_0: $C_{\text{pk1}} \geq C_{\text{pk2}}$ *versus* H_1: $C_{\text{pk1}} < C_{\text{pk2}}$).

2. Process yield measure based on C_{pk}

2.1 *Fraction of nonconformities (NC)*

Process yield is traditionally defined as the percentage of the product units that pass the inspections. Units are inspected according to specification limits placed on various key product characteristics and sorted into two categories: passed (conforming) or rejected (non-conforming). Process yield has long been the most common and standard criteria used in the manufacturing industries for judging process performance. In the past, fraction nonconforming were calculated by counting the number of nonconforming items in a sample, then extrapolating the results. With the fraction nonconforming now commonly less than 0.01%, often expressed in parts per million (ppm), traditional methods for calculating the fraction nonconforming no longer work since all reasonably sized samples will probably have no defective items. Capability indices are alternatives for measuring fraction nonconforming.

Suppose that the proportion of conforming items is the primary concern then the most natural measure is the proportion itself called the yield, which we define as:

$$\text{Yield} = \int_{\text{LSL}}^{\text{USL}} \mathrm{d}F(x) = F(\text{USL}) - F(\text{LSL}) \tag{5}$$

where $F(x)$ is the cumulative distribution function (CDF) of the measured characteristic X. If the process characteristic X follows $N(\mu, \sigma^2)$, then the fraction of nonconformities NC is:

$$\text{NC} = 1 - \Phi\left(\frac{\text{USL} - \mu}{\sigma}\right) + \Phi\left(\frac{\mu - \text{LSL}}{\sigma}\right), \tag{6}$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution $N(0, 1)$.

2.2 *Yield assurance based on C_{pk}*

The index C_{pk} can be used to fill such a purpose for normally distributed processes. Given a fixed value of C_{pk}, we have $2\Phi(3C_{pk}) - 1 \leq \text{yield} \leq \Phi(3C_{pk})$. For $C_{pk} = 1.00$, one would expect that the fraction of defectives, is no more than 2700 ppm. The exact number of non-conformities can be expressed as a function of C_{pk} and C_a or C_{pk} and C_p together as follows:

$$\text{NC} = \Phi[-3C_{pk}] + \Phi[-3C_{pk}(2 - C_a)/C_a], \; \text{NC} = \Phi[-3C_{pk}] + \Phi[-3(2C_p - C_{pk})].$$

For most manufacturing factories, reducing the fraction of non-conformities is the primary concern and the guiding principle for quality improvement. Montgomery (2001) recommended some minimum capability requirements for processes running under certain designated quality conditions. In particular, $C_{pk} \geq 1.33$ is for existing processes, and $C_{pk} \geq 1.50$ is for new processes; $C_{pk} \geq 1.50$ is also for existing processes on safety, strength, or critical parameter, and $C_{pk} \geq 1.67$ is for new processes on safety, strength, or critical parameter. Finley (1992) also found that required C_{pk} values on all critical supplier processes are 1.33 or higher and C_{pk} values of 1.67 or higher are preferred. Many companies have recently adopted criteria for evaluating their processes that include more stringent process capability objectives. Motorola's Six Sigma program essentially requires the process capability to be at least 2.0 to accommodate the possible 1.5σ process shift (see Harry 1988), and no more than 3.4 ppm are defectives.

3. Selecting a better supplier by comparing two C_{pk}

We investigate the selection problem for cases with two candidate processes based on the C_{pk} index. Let π_i be the population assumed to be normally distributed with mean μ_i and variance σ_i^2, $i=1,2$, and $x_{i1}, x_{i2}, \ldots, x_{in_i}$ are the independent random samples from π_i, $i=1,2$. In most applications, if a new supplier no. 2 (S2) wants to compete for the orders by claiming that its capability is better than the existing supplier no. 1 (S1), then the new S2 must furnish convincing information justifying the claim with a prescribed level of confidence. Thus, the decision of supplier selection would be based on the hypothesis testing comparing the two C_{pk} values, H_0: $C_{\text{pk1}} \geq C_{\text{pk2}}$ *versus* H_1: $C_{\text{pk1}} < C_{\text{pk2}}$. If the test rejects the null hypothesis H_0: $C_{\text{pk1}} \geq C_{\text{pk2}}$, then one has sufficient information to conclude that the new S2 is superior to the original S1, and the decision of the replacement would be suggested. Equivalently, this test hypothesis problem can be rewritten as H_0: $C_{\text{pk2}} - C_{\text{pk1}} \leq 0$ *versus* H_1: $C_{\text{pk2}} - C_{\text{pk1}} > 0$ (difference testing), or H_0: $C_{\text{pk1}}/C_{\text{pk2}} \leq 1$ *versus* H_1: $C_{\text{pk2}}/C_{\text{pk1}} > 1$ (ratio testing). Thus, if the LCB for the difference between two PCIs $C_{\text{pk2}} - C_{\text{pk1}}$ is positive, then S2 has a better process capability than S1. Otherwise, we do not have sufficient information to conclude that the S2 has a better process capability than S1. In this case, we would believe that $C_{\text{pk1}} - C_{\text{pk2}} \leq 0$ is true, i.e. $C_{\text{pk1}} \geq C_{\text{pk2}}$. Similarly, if the LCB for the ratio between two PCIs $C_{\text{pk1}}/C_{\text{pk2}}$ is greater than 1, then S2 has a better process capability than S1. Otherwise, if the LCB of the ratio statistic is less than 1, then we conclude that S1 has a better process capability than S2.

The assessment of values requires knowledge of μ_i, and σ_i. From the definition of C_{pk} expressed in equation (3), the natural estimator $\hat{C}_{\text{pk}i}$ is obtained by replacing the process mean μ_i and the process standard deviation σ_i by their conventional estimators $\bar{x}_i$ and s_i, which may be obtained from a process that is demonstrably stable (under statistical control).

$$\hat{C}_{\text{pk}i} = \min\left\{\frac{\text{USL} - \bar{x}_i}{3s_i}, \frac{\bar{x}_i - \text{LSL}}{3s_i}\right\} = \frac{d - |\bar{x}_i - m|}{3s_i} = \left\{1 - \frac{|\bar{x}_i - m|}{d}\right\}\hat{C}_{\text{pi}}, \qquad (7)$$

where $\bar{x}_i = \sum_{j=1}^{n_i} x_{ij}/n_i$, $s_i = \left[\sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2/(n_i - 1)\right]^{1/2}$ and $\hat{C}_{\text{pi}} = d/3s_i$.

Numerous methods for constructing approximate confidence intervals of C_{pk} have been proposed. Examples include Chou *et al.* (1990), Zhang *et al.* (1990), Franklin and Wasserman (1992a, b), Kushler and Hurley (1992), Nagata and Nagahata (1994), Tang *et al.* (1997), Hoffman (2001), and many others. Under the assumption of normality of the estimated particular $\hat{C}_{\text{pk}i}$ defined in equation (7), $\hat{C}_{\text{pi}}$ is distributed as $(n_i - 1)^{1/2} C_{\text{pi}}(\chi_{n_i-1}^{-1})$, and $n_i^{1/2}|\bar{x}_i - m|/\sigma_i$ is distributed as the folded normal distribution with parameter $n_i^{1/2}|\mu_i - m|/\sigma_i$ (see Leone *et al.* 1961 for details about this distribution). Thus, single $\hat{C}_{\text{pk}i}$ is a mixture of $\chi_{n_i-1}^{-1}$ and the folded normal distribution (Pearn *et al.* 1992). Furthermore, using the integration technique similar to that presented in Vännman (1997), an exact and explicit form of the CDF of the individual natural estimator $\hat{C}_{\text{pk}i}$ can be expressed as (see Pearn and Lin 2003):

$$F_{\hat{C}_{\text{pk}i}}(y) = 1 - \int_0^{b_i\sqrt{n_i}} G\left(\frac{(n_i - 1)(b_i\sqrt{n_i} - t)^2}{9n_i y^2}\right)\left[\phi(t + \xi_i\sqrt{n_i}) + \phi(t - \xi_i\sqrt{n_i})\right]\mathrm{d}t, \qquad (8)$$

for $y > 0$, where $b_i = d/\sigma_i$, $\xi_i = (\mu_i - m)/\sigma$, $G(\cdot)$ is the CDF of the chi-square distribution with degree of freedom $n_i - 1$, $\chi_{n_i-1}^2$, and $\phi(\cdot)$ is the probability density function (PDF) of the standard normal distribution $N(0, 1)$. Based on the CDF of $\hat{C}_{\text{pk}i}$, Pearn and Lin (2003) implemented the statistical theory of the hypotheses testing. Pearn and Shu (2003) further developed an efficient algorithm with the

Matlab computer program to find the reliable LCBs conveying critical information regarding the true process capability. However, their investigations are all developed for evaluating whether a single supplier's process conforms to a customer's requirements. Due to the complexities of the sampling distributions of $\hat{C}_{pk2} - \hat{C}_{pk1}$ or $\hat{C}_{pk2}/\hat{C}_{pk1}$, constructions of exact confidence intervals for $C_{pk2} - C_{pk1}$ or C_{pk2}/C_{pk1} are difficult.

3.1 *Bootstrap methodology*

The bootstrap, a data-based simulation technique for statistical inference introduced by Efron (1979, 1982), is a non-parametric, computationally intensive, but also effective, estimation method. It can be applied whenever the construction of confidence intervals for parameters using the standard statistical techniques becomes intractable. An overview of this topic in bootstrap confidence intervals can be found in Hall (1988), Efron and Tibshirani (1993). Moreover, traditionally, statistical research work has relied on the central limit theorem and normal approximations to obtain standard errors and confidence intervals. These techniques are valid only when the statistic, or some known transformation of the statistic, is asymptotically normally distributed. Unfortunately, many real world processes are not normally distributed and this departure from normality could potentially affect these estimates. The bootstrap approach is far more general. It does not rely on any distributional assumptions about the underlying population. The more ambiguous the information is to the researcher regarding the underlying population distribution, the more likely it is that the bootstrap may prove useful. Rather than using distribution frequency tables to compute approximate probability values, the nonparametric bootstrap method generates a unique sampling distribution based on the actual sample rather than the analytic methods. Due to the advantage of the bootstrap simulation technique, many studies of process capability analyses used the bootstrap approach to calculate confidence intervals for process capability indices, dating back at least to Franklin and Wasserman (1992). Also see Choi *et al.* (1996), Chen and Chen (2004), and the references therein. Most of them concluded that the performance of such bootstrap confidence limits is quite satisfactory in the majority of the cases. Therefore, we apply bootstrap re-sampling method to construct confidence intervals on $C_{pk2} - C_{pk1}$ and C_{pk2}/C_{pk1} for selecting a better supplier, which has never been done in the literature.

In the following, four bootstrap confidence limits are employed to determine the LCBs of difference and ratio statistics and the results are used to select the better supplier of the two candidates. For $n_1 = n_2 = n$, let two bootstrap samples of size n drawn with replacement from the two original samples be denoted by $\{x^*_{11}, x^*_{21}, \ldots, x^*_{1n}\}$ $\{x^*_{21}, x^*_{22}, \ldots, x^*_{2n}\}$. The bootstrap sample statistics $\bar{x}^*_1$, s^*_1, $\bar{x}^*_2$, and s^*_2 are computed, as well as $\hat{C}^*_{pk1}$, and $\hat{C}^*_{pk2}$. A random sample of n^n possible re-samples is drawn, the statistic is calculated for each of these, and the resulting empirical distribution is referred to as the bootstrap distribution of the statistic. Due to the overwhelming computation time, it is not of practical interest to choose n^n such samples. Empirical work (Eforn and Tibshirani 1986) indicated that a minimum of roughly 1000 bootstrap re-samples is usually sufficient to compute reasonably accurate confidence interval estimates for population parameters. For the purpose of accuracy, we consider $B = 5000$ bootstrap re-samples (rather than 1000). Thus, we take $B = 5000$ bootstrap estimates $\hat{\theta}^* = (\hat{C}^*_{\text{pk2}} - \hat{C}^*_{\text{pk1}})$ or $(\hat{C}^*_{\text{pk2}}/\hat{C}^*_{\text{pk1}})$ of $\theta = C_{\text{pk2}} - C_{\text{pk1}}$ or $C_{\text{pk2}}/C_{\text{pk1}}$, respectively, then order them from the smallest to the largest $\hat{\theta}^*_{(l)} = (\hat{C}^*_{\text{pk2}} - \hat{C}^*_{\text{pk1}})_{(l)}$ or $(\hat{C}^*_{\text{pk2}}/\hat{C}^*_{\text{pk1}})_{(l)}$ where $l = 1, 2, \ldots, B$.

Four types of bootstrap confidence intervals, including the standard bootstrap confidence interval (SB), the percentile bootstrap confidence interval (PB), the biased corrected percentile bootstrap confidence interval (BCPB), and the bootstrap-*t* (BT) method introduced by Efron (1981) and Efron and Tibshiraniwill (1986) are conducted in this paper. The generic notations $\hat{\theta}$ and $\hat{\theta}^*$ will be used to denote the estimator of θ and the associated ordered bootstrap estimate. Construction of a two-sided $100(1-2\alpha)\%$ confidence limit will be described. We note that a lower $100(1-\alpha)\%$ confidence limit can be obtained by using only the lower limit. The formulation details for the four types of confidence intervals are displayed as follows.

A. *Standard bootstrap (SB) method.* From the B bootstrap estimates $\hat{\theta}^*_{(l)}$, $l = 1, 2, \ldots, B$, the sample average and the sample standard deviation can be obtained as

$$\bar{\hat{\theta}}^* = \frac{1}{B}\sum_{l=1}^{B} \hat{\theta}^*_{(l)}, \quad S^*_{\theta} = \left(\frac{1}{B-1}\sum_{l=1}^{B} [\hat{\theta}^*_{(l)} - \bar{\hat{\theta}}^*]^2\right)^{1/2}.$$

The quantity S^*_{θ} is an estimator of the standard deviation of $\hat{\theta}$ if the distribution of $\hat{\theta}$ is approximately normal. Thus, the $100(1-2\alpha)\%$ SB confidence interval for θ can be constructed as $[\bar{\hat{\theta}}^* - z_\alpha S^*_{\theta},\ \bar{\hat{\theta}}^* + z_\alpha S^*_{\theta}]$, where $\hat{\theta}$ is the estimated θ for the original sample, and z_α is the upper α quantile of the standard normal distribution.

B. *Percentile bootstrap (PB) method.* From the ordered collection of $\hat{\theta}^*_{(l)}$, $l = 1, 2, \ldots, B$, the α percentage and $1-\alpha$ percentage points are used to obtain the $100(1-2\alpha)\%$ PB confidence interval for θ, $[\hat{\theta}^*(\alpha B), \hat{\theta}^*((1-\alpha)B)]$.

C. *Biased-corrected percentile bootstrap (BCPB) method.* While the percentile confidence interval is intuitively appealing, it is possible that due to sampling errors, the bootstrap distribution may be biased. In other words, it is possible that the bootstrap distributions obtained using only a sample of the complete bootstrap distribution may be shifted higher or lower than would be expected. A three-step procedure is suggested to correct for the possible bias (Efron 1982). First, using the ordered distribution of $\hat{\theta}^*$, we calculate the probability $p_0 = P[\hat{\theta}^* \le \hat{\theta}_0]$. Second, we compute the inverse of the CDF of a standard normal based upon p_0 as $z_0 = \Phi^{-1}(p_0)$, $p_L = \Phi(2z_0 - z_\alpha)$, and $p_U = \Phi(2z_0 - z_\alpha)$. Finally, we execute these steps to obtain the $100(1-2\alpha)\%$ BCPB confidence interval, $[\hat{\theta}^*(p_L B), \hat{\theta}^*(p_U B)]$.

D. *Percentile-t bootstrap (PT) method.* By using bootstrapping to approximate the distribution of a statistic of the form $(\hat{\theta} - \theta)/S_{\hat{\theta}}$, the bootstrap approximation in this case is obtained by taking bootstrap samples from the original data values, calculating the corresponding estimates $\hat{\theta}^*$ and their estimated standard error, and hence finding the bootstrapped T-values $T = (\hat{\theta}^* - \hat{\theta})/S^*_{\theta}$. The hope is then that the generated distribution will mimic the distribution of T. The $100(1-2\alpha)\%$ PT confidence interval for θ may constitute as $[\hat{\theta}^* - t^*_\alpha S^*_{\hat{\theta}}, \hat{\theta}^* - t^*_{1-\alpha} S^*_{\hat{\theta}}]$, where t^*_α and $t^*_{1-\alpha}$ are the upper α and $1-\alpha$ quantiles of the bootstrap *t*-distribution respectively, i.e. by finding the values that satisfy the two equations $P[(\hat{\theta}^* - \hat{\theta})/S_\theta^* > t_\alpha^*] = \alpha$ and $P[(\hat{\theta}^* - \hat{\theta})/S_\theta^* > t_{1-\alpha}^*] = 1 - \alpha$, for the generated bootstrap estimates.

4. Performance comparisons of four bootstrap methods

4.1 *Simulation layout setting*

When focusing on the capability of a process, there are two important characteristics, the process location relative to its specification limits and the process spread. The closer the process output is to the mid-point of the specification limits and the smaller the process spread, the more capable the process. Based on the relationship $C_{pk} = C_p \times C_a$, it is worth noting that there are several combinations of C_p and C_a for an equivalent C_{pk} value by trading-off between the degree of process centring and the magnitude of process variation. Table 1 displays various C_a values and the corresponding ranges of the departure magnitude of μ.

Figure 1 plots four processes with different combinations of (C_a, C_p) with $C_{pk} = 1.00$, i.e. $(C_a, C_p) = (0.25, 4)$ for process A, $(C_a, C_p) = (0.50, 2.00)$ for process B, $(C_a, C_p) = (0.75, 4/3)$ for process C, and $(C_a, C_p) = (1.00, 1.00)$ for process D (from left to right in plot). These four processes are equivalent according to C_{pk}

Table 1. C_a values and ranges of μ.

C_a value	Range of μ
$C_a = 1.00$	$\mu = m$
$0.75 < C_a < 1.00$	$0 < \lvert\mu - m\rvert < d/4$
$0.50 < C_a < 0.75$	$d/4 < \lvert\mu - m\rvert < d/2$
$0.25 < C_a < 0.50$	$d/2 < \lvert\mu - m\rvert < 3d/4$
$0.00 < C_a < 0.25$	$3d/4 < \lvert\mu - m\rvert < d$
$C_a = 0.00$	$\mu = \text{LSL}$ or $\mu = \text{USL}$
$C_a < 0.00$	$\mu < \text{LSL}$ or $\mu > \text{USL}$

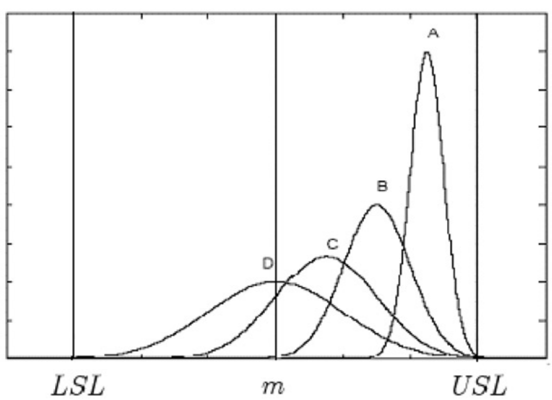

Figure 1. Four processes with $C_{pk} = 1.00$.

(i.e. $C_{pk} = 1.00$ for all four processes) and all have yields exceeding 99.73%, but they differ substantially with respect to centring. Hence, in order to make a comparative study among four bootstrap confidence limits, a series of simulations are undertaken to investigate the error probability and selection power of difference and ratio testing statistics for the performance comparisons of the four bootstrap methods. The sets of parameter values for two manufacturing suppliers used in the simulation study are given in table 2. The selected parameters are chosen so as to investigate the performance of the methods for a wide range of index values and for on-target or off-target processes. For each combination, 5000 random samples are generated and, for each of these samples, the corresponding bootstrap confidence intervals are assessed in section 4.

4.2 *Error probability analysis*

The error probability is the proportion of times that the null hypothesis H_0: $C_{pk1} \geq C_{pk2}$ is rejected, when actually H_0: $C_{pk1} \geq C_{pk2}$ is true. That is, we will calculate the proportion of times that the LCB of $C_{pk2} - C_{pk1}$ is positive and the LCB of C_{pk1}/C_{pk2} is larger than 1. For each case given in table 2, a sample of size $n = 100$ was drawn with $B = 5000$ bootstrap re-samples, and the single simulation was then replicated $N = 3000$ times. Figures 2 and 3 show the error probability of those four bootstrap methods for the difference and the ratio statistics, respectively, with 16 combinations tabulated in table 2. Usually, it is required that the probability of the error selection be less than a maximum value α^*, generally referred to as the α^*-condition. The frequency of error selection is a binomial random variable with $N = 3000$ and $\alpha^* = 0.05$. Thus, a 99% confidence interval for the error probability is

$$\alpha^* \pm Z_{0.005} \times \sqrt{\frac{\alpha^*(1-\alpha^*)}{N}} = 0.05 \pm 2.576 \times \sqrt{\frac{(0.05 \times 0.95)}{3000}} = 0.05 \pm 0.0103.$$

Table 2. Parameter values for two manufacturing suppliers used in the simulation study under $C_{pk1} = C_{pk2} = 1.00$.

Cases	C_{pk1}	C_{p1}	C_{a1}	C_{pk2}	C_{p2}	C_{a2}
1	1	4	0.25	1	4	0.25
2	1	4	0.25	1	2	0.50
3	1	4	0.25	1	4/3	0.75
4	1	4	0.25	1	1	1.00
5	1	2	0.50	1	4	0.25
6	1	2	0.50	1	2	0.50
7	1	2	0.50	1	4/3	0.75
8	1	2	0.50	1	1	1.00
9	1	4/3	0.75	1	4	0.25
10	1	4/3	0.75	1	2	0.50
11	1	4/3	0.75	1	4/3	0.75
12	1	4/3	0.75	1	1	1.00
13	1	1	1.00	1	4	0.25
14	1	1	1.00	1	2	0.50
15	1	1	1.00	1	4/3	0.75
16	1	1	1.00	1	1	1.00

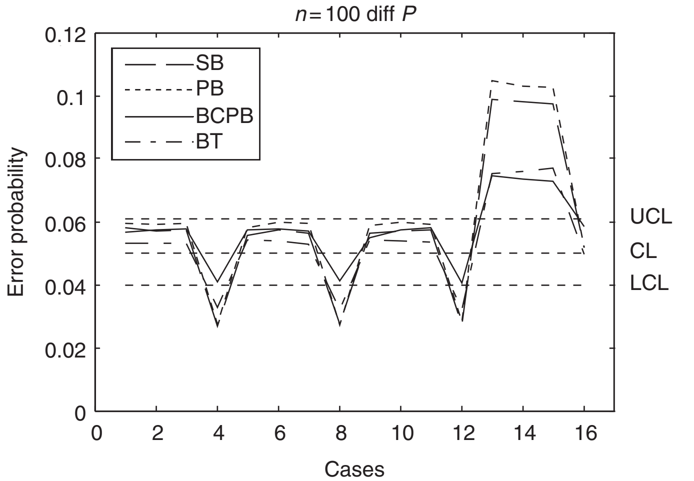

Figure 2. The error probability of four bootstrap methods for difference statistic under $C_{pk1} = C_{pk2} = 1.00$.

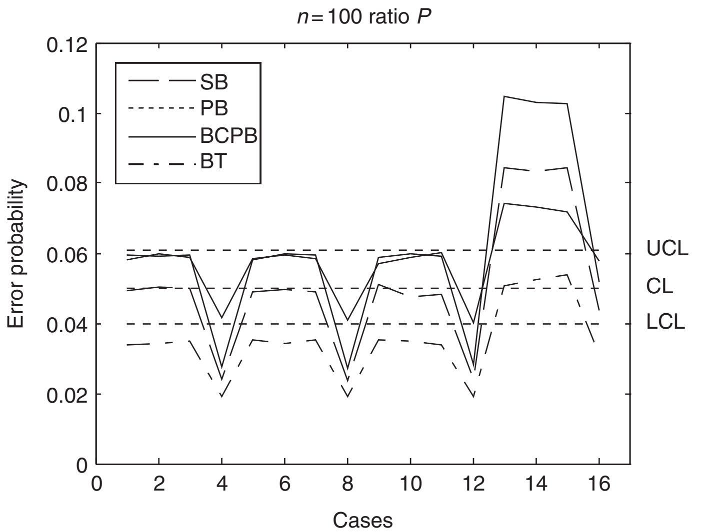

Figure 3. The error probability of four bootstrap methods for ratio statistic under $C_{pk1} = C_{pk2} = 1.00$.

That is, one could be 99% confident that a 'true 0.05% error probability' would have a proportion of range from 0.0397 to 0.0610.

In fact, for the difference statistic, there are six occurrences out of the 16 cases that are outside the interval (0.0397, 0.0610) for the SB, PB, and PT methods. In contrast with the BCPB method, three out of the 16 cases are beyond these limits. As for the ratio statistic, there are six occurrences out of the 16 cases that are outside the interval (0.0397, 0.0610) for the SB and PB methods. For the BT method, there are 13 occurrences out of the 16 cases outside the interval (0.0397, 0.0610). However, the BCPB method has only three out of the 16 cases beyond these limits. In addition, an average LCB and the standard deviation of the LCB are calculated based on the $N = 3000$ different trials. Table 3 also displays the average LCB and the standard deviation of the LCB for each of the four bootstrap confidence intervals.

4.3 *Selection power analysis*

To compare the performance of those four bootstrap methods, further simulations of selection power analysis are conducted with sample sizes $n = 10(10)200$ for $C_{pk1} = 1.00$ and $C_{pk1} = 1.05(0.05)1.50$. The selection power computes the probability of rejecting the null hypothesis H_0: $C_{pk1} \geq C_{pk2}$ while actually H_1: $C_{pk1} < C_{pk2}$ is true. For the difference statistic, the selection power computes the proportion of times that the LCB of $C_{pk2} - C_{pk1}$ is positive in the simulation. Similarly, for the ratio statistic, the selection power computes the proportion of times that the LCB of C_{pk2}/C_{pk1} is larger than 1. Figures 4 and 5 display the power of the four bootstrap methods for the difference and ratio statistic with sample size $n = 10(10)200$, $C_{pk1} = 1.00$, $C_{pk1} = 1.50$, respectively.

According to figures 4 and 5, we find that the PB and BCPB methods have smaller required sample size with fixed selection power. By contrast, the SB and BT methods have larger required sample size with fixed selection power. In terms of error probability analysis described above and selection power analysis, the BCPB method has more correct error probability and better selection power with fixed sample size. Therefore, we recommend that the best of those four bootstrap methods in our approach is the BCPB method.

5. Supplier selection based on BCPB method

5.1 *Sample size determination with designated selection power*

In practice, if a new S2 wants to compete for the orders by claiming that its capability is better than the existing S1, the new S2 must furnish convincing information justifying the claim with a prescribed level of confidence. Thus, the sample size required for designated selection power must be determined to collect actual data from the factories. We investigate the BCPB method with $B = 5000$ bootstrap re-samples, and the single simulation was then replicated $N = 3000$ times. For users' convenience in applying our procedure in practice, we tabulate the sample size required for various designated selection power = 0.90, 0.95, 0.975, 0.99. The selection power computes the probability of rejecting the null hypothesis H_0: $C_{pk1} \geq C_{pk2}$ while actually H_1: $C_{pk1} < C_{pk2}$ is true. Tables 4 and 5 display the sample size required of the BCPB method for the difference with $C_{pk1} = 1.00$ and $C_{pk2} = 1.10(0.05)1.50$ and ratio statistic with $C_{pk2}/C_{pk1} = 1.10(0.05)1.50$. From tables 4 and 5, it can be seen that the larger the value of the difference $\delta = C_{pk2} - C_{pk1}$ between two suppliers, the smaller the sample size required for fixed selection power. For fixed δ and C_{pk1}, the sample size required increases as designated selection power increases. This phenomenon can be explained easily, since

Table 3. The simulation results of the error probability of four bootstrap methods for the difference statistic and ratio statistic with 16 combinations of (C_{a1}, C_{p1}) and (C_{a2}, C_{p2}) under $C_{pk1} = C_{pk2} = 1.00$.

USL = 3, LSL = −3, $d = 3$, $m = 0$											$n = 100$ (Difference statistic)			$n = 100$ (Ratio statistic)		
C_{pk1}	C_{p1}	C_{a1}	μ_1	σ_1	C_{pk2}	C_{p2}	C_{a2}	μ_2	σ_2	Bootstrap method	Error probability	Average LCB	Standard deviation of LCB	Error probability	Average LCB	Standard deviation of LCB
1	4	0.25	2.25	0.25	1	4	0.25	2.25	0.25	SB	0.0580	−0.1871	0.1186	0.0493	0.8280	0.0959
										PB	0.0593	−0.1868	0.1199	0.0593	0.8392	0.0970
										BCPB	0.0567	−0.1867	0.1184	0.0580	0.8393	0.0969
										BT	0.0533	−0.1867	0.1161	0.0340	0.8121	0.0946
1	4	0.25	2.25	0.25	1	2	0.5	1.5	0.5	SB	0.0570	−0.1871	0.1187	0.0503	0.8280	0.0960
										PB	0.0590	−0.1869	0.1199	0.0590	0.8392	0.0970
										BCPB	0.0573	−0.1868	0.1183	0.0597	0.8392	0.0968
										BT	0.0530	−0.1867	0.1161	0.0343	0.8121	0.0945
1	4	0.25	2.25	0.25	1	1.33	0.75	0.75	0.75	SB	0.0573	−0.1871	0.1187	0.0500	0.8280	0.0959
										PB	0.0593	−0.1869	0.1199	0.0593	0.8392	0.0970
										BCPB	0.0577	−0.1868	0.1184	0.0587	0.8392	0.0969
										BT	0.0533	−0.1867	0.1161	0.0350	0.8120	0.0945
1	4	0.25	2.25	0.25	1	1	1	0	1	SB	0.0270	−0.2190	0.1139	0.0240	0.7999	0.0892
										PB	0.0277	−0.2200	0.1151	0.0277	0.8098	0.0901
										BCPB	0.0410	−0.1973	0.1156	0.0417	0.8278	0.0932
										BT	0.0327	−0.2059	0.1123	0.0190	0.7963	0.0896
1	2	0.5	1.5	0.5	1	4	0.25	2.25	0.25	SB	0.0557	−0.1871	0.1186	0.0490	0.8280	0.0959
										PB	0.0580	−0.1869	0.1199	0.0580	0.8391	0.0970
										BCPB	0.0573	−0.1870	0.1185	0.0583	0.8390	0.0969
										BT	0.0543	−0.1866	0.1161	0.0353	0.8121	0.0946
1	2	0.5	1.5	0.5	1	2	0.5	1.5	0.5	SB	0.0573	−0.1871	0.1187	0.0497	0.8280	0.0960
										PB	0.0597	−0.1868	0.1199	0.0597	0.8392	0.0970
										BCPB	0.0577	−0.1869	0.1183	0.0593	0.8391	0.0969
										BT	0.0540	−0.1866	0.1161	0.0343	0.8121	0.0945

1	2	0.5	1.5	0.5	1	1.33	0.75	0.75	0.75	SB	0.0563	−0.1871	0.1187	0.0490	0.8280	0.0960
										PB	0.0593	−0.1869	0.1200	0.0593	0.8392	0.0970
										BCPB	0.0570	−0.1867	0.1185	0.0583	0.8392	0.0970
										BT	0.0527	−0.1866	0.1160	0.0353	0.8122	0.0945
1	2	0.5	1.5	0.5	1	1	1	0	1	SB	0.0277	−0.2190	0.1140	0.0237	0.7999	0.0892
										PB	0.0273	−0.2200	0.1153	0.0273	0.8099	0.0903
										BCPB	0.0413	−0.1973	0.1156	0.0410	0.8278	0.0932
										BT	0.0320	−0.2059	0.1123	0.0190	0.7962	0.0895
1	1.33	0.75	0.75	0.75	1	4	0.25	2.25	0.25	SB	0.0563	−0.1870	0.1187	0.0510	0.8281	0.0960
										PB	0.0587	−0.1868	0.1199	0.0587	0.8392	0.0970
										BCPB	0.0550	−0.1868	0.1182	0.0570	0.8391	0.0967
										BT	0.0543	−0.1866	0.1162	0.0353	0.8122	0.0945
1	1.33	0.75	0.75	0.75	1	2	0.5	1.5	0.5	SB	0.0570	−0.1871	0.1187	0.0477	0.8280	0.0960
										PB	0.0597	−0.1869	0.1200	0.0597	0.8392	0.0971
										BCPB	0.0573	−0.1867	0.1185	0.0587	0.8392	0.0969
										BT	0.0540	−0.1867	0.1162	0.0350	0.8121	0.0945
1	1.33	0.75	0.75	0.75	1	1.33	0.75	0.75	0.75	SB	0.0573	−0.1871	0.1186	0.0483	0.8280	0.0959
										PB	0.0590	−0.1868	0.1199	0.0590	0.8392	0.0970
										BCPB	0.0580	−0.1867	0.1184	0.0603	0.8392	0.0969
										BT	0.0537	−0.1866	0.1160	0.0340	0.8121	0.0944
1	1.33	0.75	0.75	0.75	1	1	1	0	1	SB	0.0273	−0.2191	0.1139	0.0240	0.7999	0.0892
										PB	0.0283	−0.2201	0.1152	0.0283	0.8098	0.0902
										BCPB	0.0407	−0.1974	0.1156	0.0403	0.8276	0.0932
										BT	0.0320	−0.2059	0.1122	0.0190	0.7963	0.0894
1	1	1	0	1	1	4	0.25	2.25	0.25	SB	0.0987	−0.1427	0.1111	0.0843	0.8620	0.0966
										PB	0.1047	−0.1412	0.1123	0.1047	0.8737	0.0976
										BCPB	0.0747	−0.1635	0.1129	0.0743	0.8549	0.0959
										BT	0.0753	−0.1551	0.1092	0.0507	0.8341	0.0939

(*continued*)

Table 3. Continued.

USL = 3, LSL = −3, d = 3, m = 0											n = 100 (Difference statistic)			n = 100 (Ratio statistic)		
C_{pk1}	C_{p1}	C_{a1}	μ_1	σ_1	C_{pk2}	C_{p2}	C_{a2}	μ_2	σ_2	Bootstrap method	Error probability	Average LCB	Standard deviation of LCB	Error probability	Average LCB	Standard deviation of LCB
1	1	1	0	1	1	2	0.5	1.5	0.5	SB	0.0980	−0.1427	0.1112	0.0833	0.8620	0.0967
										PB	0.1030	−0.1412	0.1124	0.1030	0.8737	0.0977
										BCPB	0.0737	−0.1635	0.1128	0.0733	0.8548	0.0958
										BT	0.0760	−0.1551	0.1094	0.0523	0.8342	0.0942
1	1	1	0	1	1	1.33	0.75	0.75	0.75	SB	0.0973	−0.1426	0.1111	0.0843	0.8620	0.0965
										PB	0.1027	−0.1412	0.1122	0.1027	0.8737	0.0975
										BCPB	0.0727	−0.1635	0.1128	0.0717	0.8548	0.0958
										BT	0.0770	−0.1551	0.1092	0.0540	0.8342	0.0939
1	1	1	0	1	1	1	1	0	1	SB	0.0517	−0.1743	0.1063	0.0437	0.8328	0.0897
										PB	0.0517	−0.1741	0.1074	0.0517	0.8431	0.0907
										BCPB	0.0583	−0.1745	0.1104	0.0577	0.8430	0.0924
										BT	0.0497	−0.1742	0.1055	0.0310	0.8179	0.0891

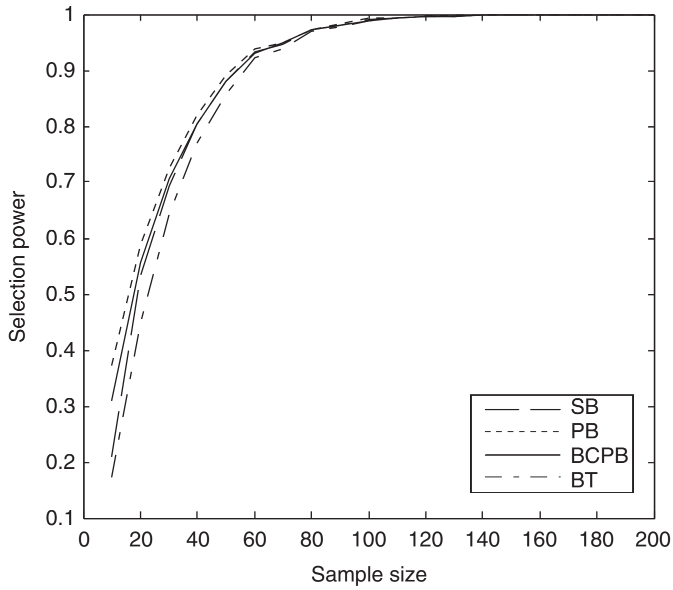

Figure 4. The selection power of the four bootstrap methods for the difference statistic with sample size $n = 10(10)200$.

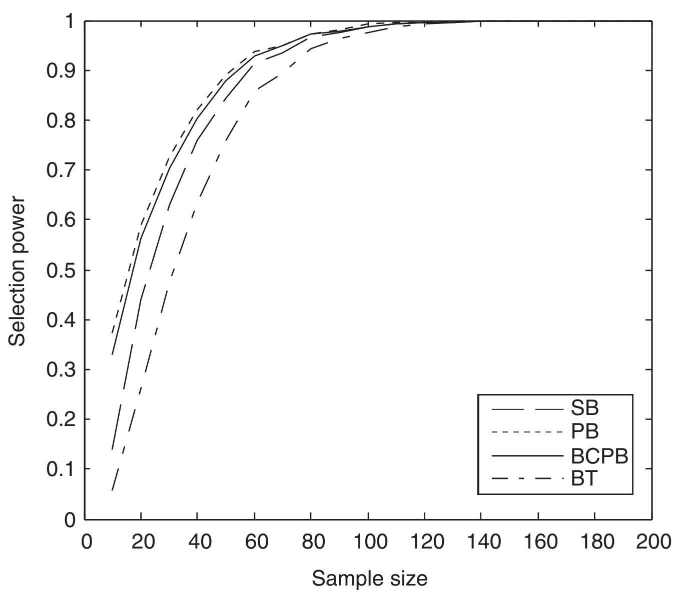

Figure 5. The selection power of the four bootstrap methods for the ratio statistic with sample size $n = 10(10)200$.

Table 4. Sample size required of BCPB method for the difference statistics under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $C_{pk1} = 1.00$, $C_{pk2} = 1.10(0.05)1.50$.

C_{pk1} C_{pk2}	1 1.1	1 1.15	1 1.2	1 1.25	1 1.3	1 1.35	1 1.4	1 1.45	1 1.5
90.0%	1045	478	285	185	133	103	79	65	55
95.0%	1328	601	357	233	168	130	103	81	72
97.5%	1567	757	432	283	204	156	127	98	84
99.0%	1972	875	497	356	233	191	156	124	104

Table 5. Sample size required of BCPB method for the ratio statistics under $\alpha = 0.05$, with power = 0.90, 0.95, 0.975, 0.99, $C_{pk1} = 1.00$, $C_{pk2} = 1.10(0.05)1.50$.

C_{pk1} C_{pk2}	1 1.1	1 1.15	1 1.2	1 1.25	1 1.3	1 1.35	1 1.4	1 1.45	1 1.5
90.0%	1045	475	289	191	139	101	84	63	55
95.0%	1340	625	358	239	170	126	107	84	73
97.5%	1600	738	424	286	203	161	122	94	84
99.0%	1975	895	549	391	268	211	162	124	105

the smaller the difference and the larger the designated selection power, the more collected sample is required to account for the smaller uncertainty in the estimation.

5.2 *Selecting the better supplier*

In order to satisfy the user's need and distinguish which supplier has better process capability, the minimum required C_{pk} values for the two candidate processes and the minimal difference δ are determined, then the sample size required with designated selection power need to be sampled. Thus, based on the BCPB method if the LCB of $\hat{C}_{pk2} - \hat{C}_{pk1}$ is positive or the LCB of $\hat{C}_{pk2}/\hat{C}_{pk1}$ is larger than 1, then we conclude that the S2 is better than the S1. Otherwise, we would believe that the existing S1 is better than the new S2 since we don't have sufficient information to reject the null hypothesis H_0: $C_{pk1} \geq C_{pk2}$.

6. Application example: PCB supplier selection

Printed circuit boards (PCBs) are widely used in the microelectronic manufacturing industry, making computers and peripherals, digital phones, fax machines, channel switch devices, remote controls, and many others. Factories producing various PCBs and related products generally are classified as 'the PCB industry' because the core components inside those products are the PCBs. The PCB manufacturing process mainly consists of a series of chemical related operations, and the chemical operations determine the functions of a PCB. PCBs are laminates. This means that they are made from two or more sheets of material stuck together, often copper and fibreglass. Unwanted areas of the copper are etched away to form conductive lands or tracks, which replace the wires carrying the electric currents in other forms of construction.

Some parts of the side with copper tracks are coated with solder resist (usually green in colour) to prevent solder sticking to those areas where it is not required. This avoids unwanted solder bridges between tracks. The solder resist is an important operation in the post-process for PCB manufacturing, which is chemically unrelated. The effects of the solder resist are to protect the metal-ingredients inside the circuits from oxidizing, and also to protect the board itself from exterior damaging when embedding specific electronic components for various applications. The uniformity smooth surface of the PCB is an essential quality characteristic considered in all PCB quality control schemes. The operation of the solder-resist is the key to surface coating in the PCB manufacturing industry. The simple method to judge whether the PCBs satisfy the uniformity flat requirement after the solder resist, is to measure its thickness. It particularly checks the uneven parts including the caves and towers of a PCB. By measuring the thickness, one can obtain the degrees of the uniformity for a PCB's surface, which is used for PCBs capability measures on thickness.

The example investigated is taken from a company, located in Tao-Yuan Industrial Park in Taiwan. The company has two competing suppliers manufacturing multi-layer PCBs for the company's orders. The company would like to determine which supplier provides better PCBs. The nominal-the-better characteristic thickness is the key measurement for the comparison. For a particular model of PCBs, the USL, LSL, and the target value of a PCB's thickness are set to 28.5 μm, 13.5 μm, and 21.0 μm, respectively.

6.1 *Data analysis and supplier selection*

For the supplier selection problem, we begin by setting two factors, (1) the minimum requirement of the C_{pk} value, and (2) δ, the minimal difference of C_{pk} between these two suppliers, and then we can decide the required sample size for preset selection power. In this example, the upper specification limit is 28.5, the lower specification limit is 13.5, and the target value is 21.0. The minimum requirement for the PCB product is 1.00 and $\delta = 0.30$, with selection power 0.95. Then, we have to take 168 samples for the difference statistics and 170 samples for the ratio statistics (by checking tables 4 and 5). In this study, we take 170 samples for S1 and S2 respectively.

The histogram and the normal probability plot of the 170 samples for S1 and S2 are used to check whether the sample data is normal. The statistic W of Shapiro–Wilk test is found to be 0.9967 with p-value > 0.1 for S1, and 0.9965 with p-value > 0.1 for S2. Thus, we conclude that the sample data for the two suppliers can be regarded as taken from near normal processes. We calculate the sample means, sample standard deviations, and the sample estimators $\hat{C}_{pk}$ for S1 and S2, as summarized in table 6. Based on the selection procedure, we execute the Matlab program and determine the LCB for the difference between the two processes $\hat{C}_{pk2} - \hat{C}_{pk1}$ to be 0.1821 and the LCB for the ratio $\hat{C}_{pk2}/\hat{C}_{pk1}$ to be 1.1479. Therefore, we conclude that S2 is a better supplier than S1.

Table 6. The calculated sample statistics for two suppliers.

Population	$\overline{X}$	S	$\hat{C}_{pk}$
I	20.8950	2.1598	1.1413
II	20.9711	1.6820	1.4806

7. Conclusions

Supplier's performance variability is a key issue that needs to be considered in the evaluation process. It provides the buyer with effective alternative choices within suppliers. Process capability indices are useful management tools that provide common quantitative measures on manufacturing capability and production quality. The manual of supplier certification includes a discussion of process capability analysis, which recommends a procedure for evaluating the most prevalent process capability index C_{pk}. In this paper, we implemented the bootstrap re-sampling approach and developed a practical procedure for practitioners to use in making supplier selection decisions between two given suppliers. Performance of the various selection methods is investigated in terms of the error probability and the selecting power by using a simulation technique. For user's convenience in applying our procedure, we provide the sample size required with designated selection power. To make the proposed method practical for in-plant applications, a real example of PCB manufacturing processes is presented to demonstrate the applicability of the proposed method.

The study of making reliable supplier decisions in comparing $i \geq 2$ available production yields of manufacturing processes, the performance of the bootstrap approach methods, and the sample size determination for various designed selection power under different distributional assumptions that usually arise in applications, would be an interesting issue for further research.

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