Bootstrapping comparison on availability of parallel systems with non-identical components

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Abstract

Purpose – In order to develop a feasible and efficient method to acquire the long-run availability of a parallel system with distribution-free up and down times, the purpose of this paper is to perform the simulation comparisons on the interval estimations of system availability using four bootstrapping methods.

Design/methodology/approach – By using four bootstrap methods; standard bootstrap (SB) confidence interval, percentile bootstrap (PB) confidence interval, bias-corrected percentile bootstrap (BCPB) confidence interval, and bias-corrected and accelerated (BCa) confidence interval. A numerical simulation study is carried out in order to demonstrate performance of these proposed bootstrap confidence intervals. Especially, we investigate the accuracy of the four bootstrap confidence intervals by calculating the coverage percentage, the average length, and the relative coverage of confidence intervals.

Findings – Among the four bootstrap confidence intervals, the PB method has the largest relative coverage in most situations. That is, the PB method is the best one made by practitioners who want to obtain an efficient interval estimation of availability.

Originality/value – It is the first time that the relative coverage is introduced to evaluate the performance of estimation method, which is more efficient than the existing measures.

Keywords Computer bootstrapping, Parallel machines, Simulation

Paper type Research paper

Introduction

To assess the long-term performance of a repairable system, the steady-state availability of the system is often considered. Steady-state availability (henceforth availability) is defined as the ratio:

$$A = \frac{\mu_1}{\mu_1 + \mu_2} \tag{1}$$

where μ_1 and μ_2 denote the mean up and down times of system, respectively.

Availability is a very important measure in evaluating the long-term performance of a system, and numerous studies have sought to explore interval estimations for the availability of repairable systems assuming various specified up and down time distributions. Confidence intervals for availability are given in Thompson (1966), Gary and Lewis (1967), Masters and Lewis (1987), Elperin and Gertsbakh (1988), Mi (1991),

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Masters *et al.* (1992), Chandrasekhar *et al.* (1994), Ananda (1999, 2003), Yadavalli *et al.* (2002), Chandrasekhar *et al.* (2004), and Lim *et al.* (2004), among others. Recently, Ke and Chu (2007) considered the interval estimation of the steady-state availability for a repairable system consisting of one operating unit and one spare. But to the best of our knowledge there has been no research that explores interval estimation of availability for a parallel system with distribution-free up and down times. The contributions made by some important articles, concerning interval estimation of availability for a parallel system, are summarized in Table I.

All the Bayesian, M-, L-, G- and G-L methods (Table I) not only assumed exponential type distributions on up and down time of components in the parallel system, but are also assessed by coverage percentage. Their results are just theoretically perfect, but the applications are limited to some special systems. This motivates us to develop more useful interval estimation methods for the availability of a parallel system with distribution-free up and down times.

The parallel system is popularly used in practical system. So far very few researchers have studied the estimation of availability for a parallel system with distribution-free up and down times. In general, engineers urgently want to assess the availability of a system. Therefore, this would motivate that we study the availability of a parallel system using nonparametrically statistical methods. To propose an efficient estimation method to estimate the system availability, the performance of these methods is also evaluated by some statistical criterions.

The objective of this paper is to present bootstrapping interval estimation of availability A for a parallel system assuming distribution-free up and down times. In second section, we show that the natural estimator $\hat{A}$ of A is consistent and asymptotically normal (CAN). Further utilizing four bootstrapping interval estimation methods, we can construct four bootstrap confidence intervals for A. Subsequently, a numerical simulation study is conducted in third section to demonstrate performance of the four bootstrap confidence intervals for A. We assess and compare the accuracy of these four bootstrap interval estimation methods by virtue of relative coverage of confidence intervals. Finally, some conclusions are drawn.

Interval estimation of availability for a parallel system

Consider a parallel system having k independent renewable components with distribution-free up and down times. For the ith ($i = 1,2,\ldots,k$) component in the system, we let X_i and Y_i denote independent up and down times with means μ_{1i} and μ_{2i}, respectively. Then the availability of the parallel system in equilibrium is given by:

$$\begin{aligned} A &= 1 - P(\text{all components are down}) \\ &= 1 - \prod_{i=1}^{k} P(\text{the } i\text{th component is down}) \\ &= 1 - \prod_{i=1}^{k} \left(\frac{\mu_{2i}}{\mu_{1i} + \mu_{2i}} \right), \end{aligned} \tag{2}$$

We are interested in constructing confidence interval of A by utilizing bootstrap technique.

Assume that $X_{i1}, X_{i2}, \ldots, X_{in}$ is a random sample of X_i and $Y_{i1}, Y_{i2}, \ldots, Y_{in}$ is a random sample of Y_{i}. Let $\bar{X}_i$ and $\bar{Y}_i$ represent the sample means of the

Table I. Some special methods about interval estimation of availability for a parallel system

Authors	Distribution types of up and down times	Interval estimation method	Confidence interval type	Performance measure	Remarks
Case 1. Martz and Waller (1982)	Exponential	Bayesian method	LCL[a]	Coverage percentage	Although their contributions are theoretically perfect, these inference procedures and results are complex. Application is limited in practical use
Case 2. Elperin and Gertsbakh (1988)	Exponential	M-method and L-method	LCL	Coverage percentage	
Case 3. Ananda (1999)	Exponential	G-method	LCL	Coverage percentage	
Case 4. Ananda (2003)	Exponential up time, Lognormal down time	G-method and G-L method	LCL	Coverage percentage	
Case 5. This paper we construct a new performance measure, named *relative coverage*[b], to evaluate performance of estimation methods. This measure overcomes the shortcomings of classical coverage percentage[b]	Distribution-free	Four bootstrap interval estimation methods: SB, PB, BCPB, and BCa	Two-sided confidence interval	Relative coverage = Coverage percentage/average length	These inference procedures and results are simple and uncomplicated. The interval estimations considered in this paper can be easily and efficiently implemented. The performance measures of estimation interval methods based on relative coverage is reasonable[b]

Notes: [a]LCL denotes lower confidence limit. [b]On interval estimation, statisticians assess performance of estimation methods in terms of coverage percentage or average length of confidence interval. However, larger coverage percentage of confidence interval may be often due to larger standard deviation of interval estimation method. Alternatively, shorter confidence interval may often lead to smaller coverage percentage. To improve the shortcomings listed above, this paper proposes a new performance measure, named *relative coverage*, to evaluate performance of interval estimation methods. Relative coverage is defined as the ratio of coverage percentage to average length of confidence interval, and can be viewed as the amount of coverage percentage contained by per unit-length interval. The greater the relative coverage is, the better the estimation method is

Xs and Ys, respectively. According to the strong law of large numbers (Roussas, 1997, p. 196), $\bar{X}_i$ is a strongly consistent estimator of μ_{1i} and $\bar{Y}_i$ is a strongly consistent estimator of μ_{2i}. Hence a strongly consistent estimator of the availability A is:

$$\hat{A} = 1 - \prod_{i=1}^{k} \left(\frac{\bar{Y}_i}{\bar{X}_i + \bar{Y}_i} \right) \tag{3}$$

It may be noted that $\hat{A}$ is a differentiable function in $\bar{X}_i$ and $\bar{Y}_i$ ($i = 1, 2, \ldots, k$). Based upon the multivariate central limit theorem (Rao, 1973), we have:

$$(\hat{A} - A)/\sigma \xrightarrow{D} N(0, 1) \tag{4}$$

where σ is the theoretical standard deviation of $\hat{A}$, and $\xrightarrow{D}$ denotes convergence in distribution. It follows that $\hat{A}$ is a (strongly) CAN estimator of A. But the theoretical variance of $\hat{A}$ is complex and not easy to be computed for practical use. In real parallel systems, the distributions of up and down times are seldom known, the variance of $\hat{A}$ is more difficult to be derived and calculated. To estimate A efficiently, we will develop interval estimations of A by means of four bootstrap methods, and further evaluate performance of the four estimation methods in terms of relative coverage.

Efron (1979, 1982), the greatest statistician in the field of nonparametric resampling approach, originally developed and proposed the bootstrap, which is a resampling technique that can be effectively applied to estimate the sampling distribution of any statistic. Specifically, one can utilize the bootstrap approaches to approximate the sampling distribution of a statistic defined by a random sample from a population with unknown probability distribution. And due to the technological advances of PC and statistical software, today the bootstrap becomes the most powerful nonparametric estimation procedure. Especially on the topic of interval estimation, Efron and Gong (1983), Efron and Tibshirani (1986), and Efron (1987) presented four bootstrap confidence intervals: the standard bootstrap (SB) confidence interval, the percentile bootstrap (PB) confidence interval, the bias-corrected percentile bootstrap (BCPB) confidence interval, and the bias-corrected and accelerated (BCa) confidence interval.

In order to understand how the bootstrap confidence interval is developed, let $x_{i1}, x_{i2}, \ldots, x_{in}$ be a sample of n observations taken from the population X_i, and $y_{i1}, y_{i2}, \ldots, y_{in}$ be a sample of n observations drawn from the population Y_i, where $i = 1, 2, \ldots, k$. Then according to the bootstrap procedure, a simple random sample $x^*_{i1}, x^*_{i2}, \ldots, x^*_{in}$ can be taken from the empirical distribution of $x_{i1}, x_{i2}, \ldots, x_{in}$, called a bootstrap sample from $x_{i1}, x_{i2}, \ldots, x_{in}$. Similarly, we can draw a bootstrap sample $y^*_{i1}, y^*_{i2}, \ldots, y^*_{in}$ from $y_{i1}, y_{i2}, \ldots, y_{in}$. Based on equation (3), a CAN estimate of system availability $\hat{A}$ can be calculated from original samples as:

$$\hat{A} = 1 - \prod_{i=1}^{k} \left(\frac{\bar{y}_i}{\bar{x}_i + \bar{y}_i} \right), \tag{5}$$

where $\bar{x}_i$ and $\bar{y}_i$ are the sample means of $x_{i1}, x_{i2}, \ldots, x_{in}$ and $y_{i1}, y_{i2}, \ldots, y_{in}$, respectively. And another estimate of A computed from bootstrap samples is:

$$\hat{A}^* = 1 - \prod_{i=1}^{k} \left(\frac{\bar{y}^*_i}{\bar{x}^*_i + \bar{y}^*_i} \right), \tag{6}$$

where $\bar{x}_i^*$ and $\bar{y}_i^*$ are the sample means of $x_{i1}^*, x_{i2}^*, \ldots, x_{in}^*$ and $y_{i1}^*, y_{i2}^*, \ldots, y_{in}^*$, respectively. Also $\hat{A}^*$ is called a bootstrap estimate of A. The above resampling process can be repeated multiple times, for example, B times. The B bootstrap estimates $(\hat{A}_1^*, \hat{A}_2^*, \ldots, \hat{A}_B^*)$ can be computed from the bootstrap resamples. Averaging the B bootstrap estimates, we obtain that:

$$\hat{A}_B = \frac{1}{B}\sum_{j=1}^{B} \hat{A}_j^*, \tag{7}$$

is the bootstrap estimate of A. And the standard deviation of the CAN estimator $\hat{A}$ can be estimated by:

$$\mathrm{sd}(\hat{A}_B) = \left\{\frac{1}{B-1}\sum_{j=1}^{B}\left(\hat{A}_j^* - \hat{A}_B\right)^2\right\}^{1/2}. \tag{8}$$

For necessary backgrounds on bootstrap techniques, the interested readers can refer to Efron (1987, 1982, 1987), Efron and Gong (1983), Efron and Tibshirani (1986), Gunter (1991), Mooney and Duval (1993), or Young (1994). Next, we describe four types of bootstrap confidence intervals for A as follows.

Standard bootstrap confidence interval

Because $\mathrm{sd}(\hat{A}_B)$ is a consistent estimate of σ (the standard deviation of $\hat{A}$), the Slutsky theorem (Hogg and Craig, 1995, p. 254) implies that:

$$(\hat{A} - A)/\mathrm{sd}(\hat{A}_B) \xrightarrow{D} N(0, 1). \tag{9}$$

Based upon the above asymptotic normality of $\hat{A}$, a 100(1-2α)% SB confidence interval for A is:

$$(\hat{A} - z_\alpha \mathrm{sd}(\hat{A}_B), \hat{A} + z_\alpha \mathrm{sd}(\hat{A}_B)), \tag{10}$$

where z_α is the upper αth quantile of the standard normal distribution.

Percentile bootstrap confidence interval

$\hat{A}_1^*, \hat{A}_2^*, \ldots, \hat{A}_B^*$ is called the bootstrap distribution of $\hat{A}$, and let $\hat{A}_1^*(1), \hat{A}_2^*(2), \ldots, \hat{A}_B^*(B)$ be the order statistic of $\hat{A}_1^*, \hat{A}_2^*, \ldots, \hat{A}_B^*$. Then utilizing the 100$\alpha$th and 100$(1-\alpha)$th percentage point of the bootstrap distribution, a 100$(1-2\alpha)$% PB confidence interval for A is obtained as:

$$(\hat{A}^*([B\alpha]),\ \hat{A}^*([B(1-\alpha)])), \tag{11}$$

where $[x]$ denotes the greatest integer less than or equal to x.

Bias-corrected percentile bootstrap confidence interval

The bootstrap distribution $\hat{A}_1^*, \hat{A}_2^*, \ldots, \hat{A}_B^*$ may be biased. Consequently, the third approach is designed to correct this potential bias of the bootstrap distribution. Set

$$p_0 = \sum_{j=1}^{B} I\left(\hat{A}_j^* < \hat{A}\right)/B$$

where $I(\cdot)$ is the indicator function. Define $\hat{z}_0 = \Phi^{-1}(p_0)$, where Φ^{-1} denotes the inverse function of the standard normal distribution function Φ. Then let $a_1 = \Phi(2\hat{z}_0 - z_\alpha)$ and $a_2 = \Phi(2\hat{z}_0 + z_\alpha)$. It follows that a $100(1-2\alpha)\%$ BCPB confidence interval for A is given by:

$$(\hat{A}^*([Ba_1]),\ \hat{A}^*([Ba_2])). \tag{12}$$

Bias-corrected and accelerated confidence interval

Besides for correcting the potential bias of the bootstrap distribution, we can accelerate convergence of the bootstrap distribution. Let $\tilde{X}_{i\ell}$ and $\tilde{Y}_{i\ell}$ denote the original samples $(x_{i1}, x_{i2}, \ldots, x_{in})$ and $(y_{i1}, y_{i2}, \ldots, y_{in})$ with the ℓth observations $x_{i\ell}$ and $y_{i\ell}$ deleted, also let $\tilde{A}_\ell$ be the CAN estimate of A calculated by using $\tilde{X}_{i\ell}$ and $\tilde{Y}_{i\ell}$ for $\ell = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, k$. Define:

$$\tilde{A} = \sum_{\ell=1}^{n} \tilde{A}_\ell / n,$$

and:

$$\hat{a} = \sum_{\ell=1}^{n} (\tilde{A} - \tilde{A}_\ell)^3 \Big/ \left\{ 6 \left[\sum_{\ell=1}^{n} (\tilde{A} - \tilde{A}_\ell)^2 \right]^{3/2} \right\}.$$

Here $\hat{z}_0$ and $\hat{a}$ are named bias-correction and acceleration, respectively. Thus, a $100(1-2\alpha)\%$ BCa confidence interval for A is constructed by:

$$(\hat{A}^*([B\alpha_1]),\ \hat{A}^*([B\alpha_2])). \tag{13}$$

where:

$$\alpha_1 = \Phi(\hat{z}_0 + (\hat{z}_0 - z_\alpha)/[1 - \hat{a}(\hat{z}_0 - z_\alpha)])$$

and:

$$\alpha_2 = \Phi(\hat{z}_0 + (\hat{z}_0 + z_\alpha)/[1 - \hat{a}(\hat{z}_0 + z_\alpha)]).$$

Simulation study

One principal goal of bootstrap methods is to establish good confidence interval. Efron and Tibshirani (1986) indicated that "good" means that the bootstrap confidence intervals should have relatively accurate coverage performance in all situations. On interval estimation, most statisticians assess performance of estimation methods in terms of coverage percentage or average length of confidence interval. But we find that larger coverage percentage of confidence interval may be often due to larger standard deviation of interval estimation method. On the other hand, shorter confidence interval may often lead to smaller coverage percentage. In order to improve the above two shortcomings, this paper proposes a new performance measure, named relative coverage, to evaluate performance of interval estimation methods. Relative coverage is defined as the ratio of coverage percentage to average length of confidence interval, and can be viewed as the amount of coverage percentage contained by per unit-length interval. The greater the relative coverage is, the better the estimation method is.

A numerical simulation study is conducted to evaluate performance of the four bootstrap estimation methods presented in second section. The four bootstrap confidence intervals are assessed in terms of their coverage percentages, average lengths, and relative coverages. In order to reach this goal, we not only set six different distributions for the up time, but also assume two various distributions for the down time. Moreover, we let the up time have a Gamma lifetime distribution GAM(a,b), where a and b are scale and shape parameters, respectively. Six different levels (5, 2), (10,1), (20,0.5), (2.5,2), (5,1), and (10,0.5) are assigned to (a,b). It is well known that the GAM (a, b) has increasing failure rate (IFR), constant failure rate (CFR), or decreasing failure rate (DFR) according to the shape parameter $b > 1$, $= 1$, or < 1, respectively. In this simulation study, we specify (i) GAM(5,2) and GAM(2.5,2), (ii) GAM(10,1) and GAM(5,1), and (iii) GAM(20,0.5) and GAM(10,0.5) to represent IFR, CFR, and DFR up times, respectively.

Moreover, the distribution of down time is assumed to be exponential with mean 5 (i.e. EXP(5)) or uniform on interval (1,9) (i.e. U(1,9)). The parallel system in simulation experiment is assumed to be $k = 2$ or 5 components, and we consider diverse combinations of distributions for up times and down times in simulation processes. The distributions of up and down times, and some configurations composed of different distributions in the simulated systems with k components, are described in Tables II-IV.

For each pair distributions of (X_i, Y_i) $(i = 1, 2, \ldots, k)$, a random sample $(x_{i1}, y_{i1}), (x_{i2}, y_{i2}), \ldots, (x_{in}, y_{in})$ of size n (= 15, 30, 60) is generated from (X_i, Y_i). Using equation (5), the CAN estimate $\hat{A}$ is calculated. Next, $B = 1{,}000$ bootstrap resamples $\{(x^*_{i1}, y^*_{i1}), (x^*_{i2}, y^*_{i2}), \ldots, (x^*_{in}, y^*_{in})\}$ are drawn from the original sample $\{(x_{i1}, y_{i1}), (x_{i2}, y_{i2}), \ldots, (x_{in}, y_{in})\}$ for $i = 1, 2, \ldots, k$. By virtue of equation (6), B bootstrap estimates $\hat{A}^*_1, \hat{A}^*_2, \ldots, \hat{A}^*_B$ are calculated from the bootstrap resamples. And utilizing equations (7) and (8), the estimated standard deviation of $\hat{A}$ is computed as sd($\hat{A}_B$). Finally, applying the four bootstrap methods (equations (10)-(13)) described in second section, we obtain the four bootstrap confidence intervals with confidence level 90 percent.

Subsequently, the above simulation process is replicated $N = 1{,}000$ times. We compute coverage percentage, average length, and relative coverage of the four bootstrap confidence intervals. Matlab® 7.0.4 code is easily implemented to accomplish all simulations. All simulation results are displayed in Table V and VI. The performance of the four bootstrap confidence intervals of availability A for a parallel system can be examined in terms of relative coverage recorded on Table V and VI. Examining these simulation results, we find that coverage percentages for the four bootstrap confidence intervals increase with sample size n, but average lengths decrease with n.

	Notation	Probability density function	Mean
Distribution of up time	GAM(5,2) – IFR	$f(x) = (1/25)xe^{-x/5}, \quad x > 0$	10
	GAM(10,1) – CFR	$f(x) = (1/10)e^{-x/10}, \quad x > 0$	10
	GAM(20,0.5) – DFR	$f(x) = (1/\sqrt{20\pi x})e^{-x/20}, \quad x > 0$	10
	GAM(2.5,2) – IFR	$f(x) = (1/6.25)xe^{-2x/5}, \quad x > 0$	5
	GAM(5,1) – CFR	$f(x) = (1/5)e^{-x/5}, \quad x > 0$	5
	GAM(10,0.5) – DFR	$f(x) = (1/\sqrt{10\pi x})e^{-x/10}, \quad x > 0$	5
Distribution of down time	EXP(5)	$g(y) = (1/5)e^{-y/5}, \quad y > 0$	5
	U(1,9)	$g(y) = 1/8,\ 1 < y < 9$	5

Table II. Different distributions of up and down times used in simulation study

Configuration case	Component distribution configuration X_1	X_2	Y_1	Y_2
A1	GAM(5,2)	GAM(5,2)	EXP(5)	EXP(5)
A2	GAM(5,2)	GAM(10,1)	EXP(5)	EXP(5)
A3	GAM(5,2)	GAM(20,0.5)	EXP(5)	EXP(5)
A4	GAM(10,1)	GAM(10,1)	EXP(5)	EXP(5)
A5	GAM(10,1)	GAM(20,0.5)	EXP(5)	EXP(5)
A6	GAM(20,0.5)	GAM(20,0.5)	EXP(5)	EXP(5)
A7	GAM(5,2)	GAM(5,2)	EXP(5)	U(1,9)
A8	GAM(5,2)	GAM(10,1)	EXP(5)	U(1,9)
A9	GAM(5,2)	GAM(20,0.5)	EXP(5)	U(1,9)
A10	GAM(10,1)	GAM(5,2)	EXP(5)	U(1,9)
A11	GAM(10,1)	GAM(10,1)	EXP(5)	U(1,9)
A12	GAM(10,1)	GAM(20,0.5)	EXP(5)	U(1,9)
A13	GAM(20,0.5)	GAM(5,2)	EXP(5)	U(1,9)
A14	GAM(20,0.5)	GAM(10,1)	EXP(5)	U(1,9)
A15	GAM(20,0.5)	GAM(20,0.5)	EXP(5)	U(1,9)
A16	GAM(5,2)	GAM(5,2)	U(1,9)	U(1,9)
A17	GAM(5,2)	GAM(10,1)	U(1,9)	U(1,9)
A18	GAM(5,2)	GAM(20,0.5)	U(1,9)	U(1,9)
A19	GAM(10,1)	GAM(10,1)	U(1,9)	U(1,9)
A20	GAM(10,1)	GAM(20,0.5)	U(1,9)	U(1,9)
A21	GAM(20,0.5)	GAM(20,0.5)	U(1,9)	U(1,9)

Table III. Configuration for component distributions of system in simulation study – two components system

Configuration case	Component distribution configuration X_1, X_2, X_3	X_4, X_5	Y_1, Y_2, Y_3	$Y_4\ Y,_5$
B1	GAM(2.5,2)	GAM(2.5,2)	EXP(5)	EXP(5)
B2	GAM(2.5,2)	GAM(5,1)	EXP(5)	EXP(5)
B3	GAM(2.5,2)	GAM(10,0.5)	EXP(5)	EXP(5)
B4	GAM(5,1)	GAM(5,1)	EXP(5)	EXP(5)
B5	GAM(5,1)	GAM(10,0.5)	EXP(5)	EXP(5)
B6	GAM(10,0.5)	GAM(10,0.5)	EXP(5)	EXP(5)
B7	GAM(2.5,2)	GAM(2.5,2)	EXP(5)	U(1,9)
B8	GAM(2.5,2)	GAM(5,1)	EXP(5)	U(1,9)
B9	GAM(2.5,2)	GAM(10,0.5)	EXP(5)	U(1,9)
B10	GAM(5,1)	GAM(2.5,2)	EXP(5)	U(1,9)
B11	GAM(5,1)	GAM(5,1)	EXP(5)	U(1,9)
B12	GAM(5,1)	GAM(10,0.5)	EXP(5)	U(1,9)
B13	GAM(10,0.5)	GAM(2.5,2)	EXP(5)	U(1,9)
B14	GAM(10,0.5)	GAM(5,1)	EXP(5)	U(1,9)
B15	GAM(10,0.5)	GAM(10,0.5)	EXP(5)	U(1,9)
B16	GAM(2.5,2)	GAM(2.5,2)	U(1,9)	U(1,9)
B17	GAM(2.5,2)	GAM(5,1)	U(1,9)	U(1,9)
B18	GAM(2.5,2)	GAM(10,0.5)	U(1,9)	U(1,9)
B19	GAM(5,1)	GAM(5,1)	U(1,9)	U(1,9)
B20	GAM(5,1)	GAM(10,0.5)	U(1,9)	U(1,9)
B21	GAM(10,0.5)	GAM(10,0.5)	U(1,9)	U(1,9)

Table IV. Configuration for component distributions of system in simulation study – five components system

Table V. Coverage percentage, average length, and relative coverage for four 90 percent bootstrap confidence intervals of availability ($A = 8/9$) for a parallel system with two components

Configuration of system distributions	Bootstrap methods	Coverage percentage n=15	n=30	n=60	Average length n=15	n=30	n=60	Relative coverage n=15	n=30	n=60
A1	SB	0.838	0.870	0.887	0.088447	0.065284	0.047359	9.474	13.326	18.729
	PB	0.857	0.878	0.894	0.088376	0.065274	0.047415	9.697[a]	13.451[a]	18.854[a]
	BCPB	0.857	0.881	0.892	0.090782	0.066350	0.047729	9.440	13.277	18.688
	BCa	0.856	0.884	0.891	0.092752	0.067269	0.048115	9.228	13.141	18.518
A2	SB	0.825	0.861	0.885	0.085381	0.063498	0.046631	9.662	13.559	18.978
	PB	0.838	0.871	0.890	0.085319	0.063564	0.046729	9.821[a]	13.702[a]	19.046[a]
	BCPB	0.837	0.874	0.889	0.087413	0.064491	0.047094	9.575	13.552	18.876
	BCa	0.844	0.882	0.898	0.089273	0.065434	0.047524	9.454	13.479	18.895
A3	SB	0.830	0.867	0.888	0.083901	0.063172	0.045781	9.892	13.724	19.396
	PB	0.837	0.877	0.889	0.083950	0.063213	0.045849	9.970[a]	13.873[a]	19.389
	BCPB	0.842	0.887	0.901	0.086034	0.064111	0.046199	9.786	13.835	19.502[a]
	BCa	0.847	0.884	0.898	0.087975	0.065180	0.046646	9.627	13.562	19.251
A4	SB	0.824	0.862	0.872	0.083226	0.062031	0.044956	9.900	13.896	19.396[a]
	PB	0.834	0.865	0.873	0.083233	0.062080	0.045036	10.020[a]	13.933[a]	19.384
	BCPB	0.849	0.859	0.878	0.085315	0.062993	0.045353	9.951	13.636	19.359
	BCa	0.851	0.867	0.885	0.087399	0.064031	0.045766	9.736	13.540	19.337
A5	SB	0.829	0.870	0.902	0.082155	0.061059	0.044662	10.090	14.248	20.196[a]
	PB	0.834	0.874	0.900	0.082201	0.061127	0.044727	10.145[a]	14.297[a]	20.122
	BCPB	0.847	0.880	0.904	0.084258	0.062025	0.045036	10.052	14.187	20.072
	BCa	0.846	0.883	0.907	0.086387	0.063061	0.045483	9.793	14.002	19.941
A6	SB	0.831	0.858	0.885	0.081704	0.060601	0.044048	10.170	14.158	20.091
	PB	0.843	0.858	0.886	0.081832	0.060682	0.044089	10.301[a]	14.139	20.095[a]
	BCPB	0.845	0.879	0.886	0.083656	0.061507	0.044375	10.100	14.291[a]	19.965
	BCa	0.857	0.887	0.886	0.085777	0.062625	0.044827	9.991	14.163	19.764
A7	SB	0.849	0.881	0.881	0.074082	0.054383	0.039124	11.460	16.199	22.518
	PB	0.855	0.887	0.885	0.074073	0.054504	0.039224	11.542[a]	16.274[a]	22.562[a]
	BCPB	0.856	0.890	0.878	0.074797	0.054675	0.039296	11.444	16.278	22.343
	BCa	0.857	0.891	0.877	0.075852	0.055127	0.039490	11.298	16.162	22.208

(continued)

Table V.

Configuration of system distributions	Bootstrap methods	Coverage percentage $n=15$	Coverage percentage $n=30$	Coverage percentage $n=60$	Average length $n=15$	Average length $n=30$	Average length $n=60$	Relative coverage $n=15$	Relative coverage $n=30$	Relative coverage $n=60$
A8	SB	0.853	0.896	0.886	0.071307	0.052643	0.037909	11.962	17.020	23.371[a]
	PB	0.861	0.900	0.888	0.071387	0.052740	0.038011	12.061	17.064	23.361
	BCPB	0.877	0.895	0.880	0.071925	0.052938	0.038030	12.193[a]	16.906[a]	23.139
	BCa	0.872	0.892	0.879	0.072985	0.053448	0.038283	11.947	16.689	22.960
A9	SB	0.840	0.858	0.880	0.069632	0.051129	0.037052	12.063	16.781	23.750
	PB	0.850	0.864	0.888	0.069808	0.051300	0.037190	12.176[a]	16.842[a]	23.877
	BCPB	0.855	0.865	0.890	0.070305	0.051475	0.037193	12.161	16.804	23.928[a]
	BCa	0.858	0.867	0.890	0.071315	0.051981	0.037399	12.031	16.679	23.797
A10	SB	0.840	0.882	0.884	0.071081	0.052530	0.037811	11.817	16.790	23.379
	PB	0.850	0.889	0.885	0.071197	0.052605	0.037904	11.938	16.899[a]	23.348
	BCPB	0.858	0.890	0.893	0.071737	0.052856	0.037964	11.960[a]	16.838	23.522[a]
	BCa	0.855	0.890	0.892	0.072820	0.053379	0.038201	11.741	16.673	23.349
A11	SB	0.835	0.883	0.881	0.068405	0.050273	0.036434	12.206	17.564	24.180[a]
	PB	0.843	0.888	0.881	0.068514	0.050320	0.036503	12.304[a]	17.647[a]	24.134
	BCPB	0.848	0.885	0.880	0.069022	0.050508	0.036528	12.286	17.521	24.090
	BCa	0.851	0.889	0.881	0.070129	0.051046	0.036761	12.134	17.415	23.965
A12	SB	0.844	0.867	0.880	0.066678	0.049340	0.035737	12.657	17.571	24.624
	PB	0.856	0.877	0.888	0.066853	0.049427	0.035846	12.804[a]	17.743	24.772[a]
	BCPB	0.851	0.884	0.882	0.067174	0.049561	0.035863	12.668	17.836[a]	24.593
	BCa	0.852	0.891	0.886	0.068320	0.050088	0.036105	12.470	17.788	24.539
A13	SB	0.833	0.885	0.883	0.070008	0.051416	0.037188	11.898	17.212	23.744
	PB	0.843	0.890	0.887	0.070103	0.051542	0.037287	12.025[a]	17.267	23.788[a]
	BCPB	0.849	0.895	0.885	0.070694	0.051738	0.037357	12.009	17.298[a]	23.690
	BCa	0.852	0.894	0.884	0.071749	0.052286	0.037586	11.874	17.098	23.519
A14	SB	0.855	0.874	0.896	0.067582	0.049367	0.036035	12.651	17.704	24.864
	PB	0.860	0.879	0.900	0.067744	0.049536	0.036113	12.694	17.744	24.921
	BCPB	0.866	0.884	0.903	0.068162	0.049677	0.036149	12.705[a]	17.795[a]	24.980[a]
	BCa	0.868	0.885	0.907	0.069286	0.050279	0.036396	12.527	17.601	24.920

(continued)

Table V.

Configuration of system distributions	Bootstrap methods	Coverage percentage			Average length			Relative coverage		
		$n=15$	$n=30$	$n=60$	$n=15$	$n=30$	$n=60$	$n=15$	$n=30$	$n=60$
A15	SB	0.832	0.878	0.882	0.065652	0.048728	0.035131	12.672	18.018	25.105[a]
	PB	0.835	0.879	0.884	0.065790	0.048905	0.035224	12.691[a]	17.973	25.096
	BCPB	0.837	0.886	0.880	0.066131	0.049052	0.035247	12.656	18.062[a]	24.966
	BCa	0.845	0.882	0.885	0.067246	0.049672	0.035479	12.565	17.756	24.944
A16	SB	0.867	0.881	0.890	0.055827	0.039523	0.028469	15.530	22.290	31.262[a]
	PB	0.872	0.887	0.883	0.055778	0.039583	0.028533	15.633[a]	22.408[a]	30.946
	BCPB	0.868	0.882	0.882	0.055824	0.039553	0.028473	15.548	22.299	30.976
	BCa	0.872	0.882	0.885	0.055794	0.039543	0.028463	15.628	22.304	31.093
A17	SB	0.874	0.892	0.898	0.051831	0.037382	0.026583	16.862	23.861	33.781[a]
	PB	0.882	0.898	0.894	0.051919	0.037474	0.026634	16.987	23.963[a]	33.566
	BCPB	0.884	0.896	0.893	0.051958	0.037430	0.026590	17.013[a]	23.937	33.584
	BCa	0.883	0.897	0.894	0.052017	0.037436	0.026582	16.975	23.961	33.632
A18	SB	0.858	0.891	0.893	0.050129	0.035919	0.025672	17.115	24.805	34.785
	PB	0.866	0.891	0.897	0.050212	0.035972	0.025738	17.246	24.768	34.851[a]
	BCPB	0.868	0.897	0.892	0.050286	0.035973	0.025682	17.261	24.935[a]	34.732
	BCa	0.876	0.896	0.894	0.050319	0.035956	0.025682	17.409[a]	24.919	34.809
A19	SB	0.889	0.877	0.902	0.048467	0.034744	0.024799	18.342	25.241	36.372[a]
	PB	0.893	0.879	0.900	0.048562	0.034827	0.024866	18.388	25.239	36.194
	BCPB	0.896	0.879	0.899	0.048583	0.034803	0.024828	18.442[a]	25.256	36.208
	BCa	0.891	0.882	0.900	0.048622	0.034808	0.024822	18.324	25.339[a]	36.258
A20	SB	0.893	0.890	0.910	0.046417	0.033362	0.023748	19.238	26.676	38.319[a]
	PB	0.893	0.894	0.910	0.046506	0.033430	0.023809	19.201	26.742[a]	38.220
	BCPB	0.894	0.886	0.909	0.046558	0.033433	0.023760	19.201	26.500	38.257
	BCa	0.898	0.888	0.908	0.046598	0.033439	0.023761	19.271[a]	26.555	38.213
A21	SB	0.874	0.876	0.883	0.044169	0.031740	0.022582	19.787	27.599	39.102[a]
	PB	0.879	0.884	0.877	0.044257	0.031821	0.022652	19.861	27.780	38.716
	BCPB	0.884	0.885	0.873	0.044279	0.031789	0.022603	19.964	27.839	38.622
	BCa	0.887	0.889	0.876	0.044353	0.031805	0.022606	19.998[a]	27.951[a]	38.750

Note: [a]Denotes the largest relative coverage among the four bootstrap confidence intervals

Table VI.
Coverage percentage, average length, and relative coverage for four 90 percent bootstrap confidence intervals of availability ($A = 31/32$) for a parallel system with five components

Configuration of system distributions	Bootstrap methods	Coverage percentage $n=15$	$n=30$	$n=60$	Average length $n=15$	$n=30$	$n=60$	Relative coverage $n=15$	$n=30$	$n=60$
B1	SB	0.824	0.866	0.904	0.030928	0.023239	0.016995	26.642	37.265	53.181
	PB	0.829	0.872	0.907	0.030591	0.023127	0.016972	27.099[a]	37.705[a]	53.440[a]
	BCPB	0.879	0.893	0.911	0.033935	0.024471	0.017482	25.902	36.492	52.112
	BCa	0.877	0.896	0.913	0.034395	0.024719	0.017574	25.498	36.247	51.951
B2	SB	0.821	0.859	0.878	0.030035	0.022343	0.016364	27.334	38.446	53.655
	PB	0.831	0.867	0.882	0.029769	0.022256	0.016340	27.914[a]	38.955[a]	53.967[a]
	BCPB	0.870	0.882	0.894	0.032989	0.023531	0.016838	26.372	37.482	53.092
	BCa	0.865	0.885	0.894	0.033475	0.023777	0.016948	25.840	37.220	52.950
B3	SB	0.819	0.848	0.882	0.028872	0.022014	0.015999	28.366	38.520	55.127
	PB	0.825	0.859	0.888	0.028615	0.021938	0.015998	28.831[a]	39.155[a]	55.507[a]
	BCPB	0.868	0.881	0.902	0.031702	0.023224	0.016507	27.379	37.935	54.644
	BCa	0.871	0.883	0.902	0.032256	0.023503	0.016623	27.002	37.569	54.261
B4	SB	0.775	0.853	0.862	0.027669	0.021150	0.015620	28.009	40.330	55.184
	PB	0.787	0.860	0.868	0.027477	0.021081	0.015622	28.642[a]	40.794[a]	55.564[a]
	BCPB	0.839	0.895	0.884	0.030476	0.022335	0.016090	27.530	40.071	54.940
	BCa	0.847	0.897	0.886	0.031110	0.022666	0.016217	27.225	39.574	54.635
B5	SB	0.790	0.838	0.889	0.026876	0.020930	0.015426	29.394	40.037[a]	57.628
	PB	0.794	0.835	0.891	0.026700	0.020887	0.015433	29.737[a]	38.977	57.731[a]
	BCPB	0.858	0.873	0.905	0.029635	0.022154	0.015907	28.952	39.405	56.892
	BCa	0.860	0.875	0.905	0.030253	0.022482	0.016054	28.426	38.920	56.373
B6	SB	0.768	0.841	0.853	0.026097	0.020245	0.014886	29.428[a]	41.540[a]	57.300
	PB	0.762	0.833	0.847	0.025921	0.020207	0.014881	29.396	41.222	56.918
	BCPB	0.835	0.887	0.882	0.028801	0.021459	0.015353	28.992	41.334	57.446[a]
	BCa	0.846	0.887	0.886	0.029535	0.021807	0.015508	28.643	40.675	57.121
B7	SB	0.839	0.852	0.883	0.028230	0.020898	0.015037	29.720	40.770	58.722
	PB	0.854	0.860	0.885	0.028026	0.020840	0.015026	30.471[a]	41.266[a]	58.897[a]
	BCPB	0.863	0.874	0.893	0.029754	0.021523	0.015288	29.004	40.607	58.410
	BCa	0.863	0.878	0.893	0.030006	0.021655	0.015332	28.761	40.545	58.244

(*continued*)

Table VI.

Configuration of system distributions	Bootstrap methods	Coverage percentage			Average length			Relative coverage		
		n=15	n=30	n=60	n=15	n=30	n=60	n=15	n=30	n=60
B8	SB	0.829	0.862	0.876	0.027173	0.019962	0.014585	30.507	43.182	60.062
	PB	0.836	0.871	0.883	0.027004	0.019926	0.014591	30.958[a]	43.711[a]	60.517[a]
	BCPB	0.855	0.891	0.879	0.028668	0.020606	0.014826	29.824	43.239	59.285
	BCa	0.855	0.890	0.882	0.028993	0.020760	0.014889	29.489	42.871	59.237
B9	SB	0.862	0.860	0.871	0.026376	0.019555	0.014136	32.681	43.978	61.616
	PB	0.875	0.866	0.879	0.026253	0.019521	0.014139	33.329[a]	44.363[a]	62.167[a]
	BCPB	0.878	0.873	0.885	0.027785	0.020157	0.014361	31.599	43.309	61.626
	BCa	0.875	0.874	0.888	0.028126	0.020325	0.014421	31.109	43.000	61.577
B10	SB	0.846	0.861	0.872	0.026758	0.019652	0.014214	31.617	43.812	61.349
	PB	0.853	0.860	0.878	0.026595	0.019617	0.014228	32.074[a]	43.839[a]	61.707
	BCPB	0.881	0.885	0.896	0.028402	0.020344	0.014464	31.019	43.502	61.946[a]
	BCa	0.878	0.887	0.894	0.028782	0.020503	0.014525	30.505	43.262	61.547
B11	SB	0.853	0.855	0.859	0.025321	0.018650	0.013496	33.687	45.843	63.649
	PB	0.860	0.863	0.867	0.025212	0.018638	0.013499	34.110[a]	46.302[a]	64.226[a]
	BCPB	0.881	0.884	0.874	0.026782	0.019280	0.013729	32.894	45.850	63.662
	BCa	0.883	0.881	0.879	0.027190	0.019481	0.013805	32.474	45.222	63.674
B12	SB	0.819	0.869	0.878	0.024097	0.018262	0.013223	33.987	47.585	66.400
	PB	0.822	0.869	0.888	0.024032	0.018266	0.013227	34.204	47.573	67.134[a]
	BCPB	0.874	0.898	0.893	0.025492	0.018837	0.013456	34.285[a]	47.672[a]	66.362
	BCa	0.871	0.904	0.893	0.025870	0.019046	0.013542	33.668	47.465	65.944
B13	SB	0.837	0.876	0.878	0.025515	0.019108	0.013693	32.803	45.844	64.120[a]
	PB	0.842	0.880	0.877	0.025393	0.019078	0.013684	33.158[a]	46.127[a]	64.087
	BCPB	0.864	0.890	0.888	0.027080	0.019782	0.013942	31.905	44.989	63.692
	BCa	0.874	0.891	0.890	0.027514	0.019974	0.014014	31.765	44.609	63.508
B14	SB	0.831	0.840	0.895	0.024101	0.017873	0.013085	34.479	46.999	68.397
	PB	0.838	0.841	0.897	0.024017	0.017871	0.013109	34.892[a]	47.058[a]	68.428[a]
	BCPB	0.878	0.864	0.903	0.025531	0.018504	0.013328	34.389	46.693	67.754
	BCa	0.882	0.871	0.903	0.025969	0.018727	0.013417	33.963	46.511	67.301

(*continued*)

Configuration of system distributions	Bootstrap methods	Coverage percentage n=15	n=30	n=60	Average length n=15	n=30	n=60	Relative coverage n=15	n=30	n=60
B15	SB	0.813	0.866	0.868	0.023262	0.017583	0.012753	34.949	49.251	68.062
	PB	0.818	0.871	0.874	0.023222	0.017595	0.012757	35.225[a]	49.502[a]	68.511[a]
	BCPB	0.858	0.879	0.872	0.024658	0.018164	0.012971	34.795	48.391	67.229
	BCa	0.862	0.884	0.875	0.025240	0.018423	0.013061	34.152	47.983	66.990
B16	SB	0.888	0.890	0.905	0.023221	0.016394	0.011657	38.241	54.287	77.638[a]
	PB	0.884	0.888	0.902	0.023083	0.016370	0.011659	38.295	54.246	77.367
	BCPB	0.884	0.888	0.900	0.023249	0.016405	0.011657	38.022	54.130	77.208
	BCa	0.892	0.889	0.899	0.023133	0.016361	0.011635	38.559[a]	54.337[a]	77.264
B17	SB	0.864	0.901	0.892	0.021207	0.015189	0.010823	40.741	59.317	82.413
	PB	0.868	0.897	0.892	0.021124	0.015179	0.010833	41.090	59.094	82.342
	BCPB	0.878	0.902	0.902	0.021366	0.015247	0.010853	41.093[a]	59.157	83.109
	BCa	0.872	0.903	0.904	0.021290	0.015215	0.010837	40.958	59.350[a]	83.420[a]
B18	SB	0.866	0.885	0.910	0.020303	0.014561	0.010354	42.653	60.776	87.892
	PB	0.866	0.896	0.911	0.020239	0.014563	0.010359	42.787	61.525[a]	87.941
	BCPB	0.879	0.898	0.914	0.020495	0.014632	0.010376	42.888	61.371	88.085
	BCa	0.887	0.896	0.913	0.020434	0.014602	0.010359	43.408[a]	61.359	88.136[a]
B19	SB	0.879	0.882	0.903	0.018492	0.013279	0.009485	47.534	66.421	95.202[a]
	PB	0.880	0.889	0.902	0.018464	0.013296	0.009498	47.659[a]	66.861[a]	94.968
	BCPB	0.891	0.888	0.897	0.018776	0.013382	0.009523	47.454	66.356	94.189
	BCa	0.885	0.888	0.897	0.018746	0.013371	0.009514	47.210	66.411	94.281
B20	SB	0.868	0.882	0.907	0.017468	0.012645	0.009048	49.691	69.753	100.244[a]
	PB	0.872	0.888	0.906	0.017441	0.012659	0.009060	49.996	70.150	100.000
	BCPB	0.890	0.902	0.906	0.017769	0.012759	0.009081	50.087[a]	70.694[a]	99.773
	BCa	0.887	0.899	0.907	0.017753	0.012749	0.009077	49.964	70.516	99.921
B21	SB	0.883	0.895	0.899	0.015989	0.011601	0.008243	55.225[a]	77.146	109.061[a]
	PB	0.882	0.897	0.899	0.016003	0.011619	0.008257	55.114	77.199[a]	108.878
	BCPB	0.900	0.896	0.902	0.016302	0.011716	0.008283	55.206	76.478	108.892
	BCa	0.900	0.901	0.901	0.016303	0.011717	0.008282	55.205	76.893	108.793

Note: [a]Denotes the largest relative coverage among the four bootstrap confidence intervals

Table VI.

Therefore, the relative coverage becomes larger when the sample size n becomes larger. The simulation results reported in Tables III and IV, also indicate that the relative coverage increases when the failure rate of up time decreases. Among the four bootstrap confidence intervals, the PB method has the largest relative coverage in most situations. Consequently, based on relative coverage, the PB method has the best performance among the four bootstrap methods for interval estimation of availability A for a parallel system with distribution-free up and down times.

Conclusions

This paper proposed four feasible and efficient interval estimations of availability A for a parallel system with distribution-free up and down times. Based on CAN estimator $\hat{A}$, the four bootstrap methods SB, PB, BCPB and BCa are applied to construct confidence intervals for system availability A. The relative coverage is adopted to understand, compare, and assess performance of the resulted bootstrap confidence intervals. The simulation results imply that the PB method has the best performance on relative coverage. Consequently, the PB method is the best one made by practitioners who want to obtain an efficient confidence interval of availability A for a practical parallel system. Note that the bootstrap estimation methods presented in this paper would be easily applied to practical parallel systems. Further research may consider comparison investigations of system characteristics for two or more parallel systems by means of bootstrap technique.

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