

A theoretical study of end effects for rectangular resonators on narrow channels

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In order to understand the end effects of rectangular resonators on narrow channels a linearized potential theory has been applied to study the deformation of waves by rectangular resonators in narrow channels, when the width of the channels is less than one half the incident wavelength. The "exact" solution is obtained by matched eigenfunction expansions; the resulting matrix equation is solved numerically. The numerical solution of the boundary integral equation method (BIEM) has also been obtained for comparison. The present two-dimensional mathematical model complements the previous laboratory investigation of James (J. Fluid Mech. 1970, 44, 615–621). The numerical results of the present analytical and numerical models are compared to James' laboratory measurements. Results show that both analytical solutions and BIEM solutions for the fundamental resonance mode are in reasonably good agreement with the laboratory experimental data. Therefore the considerable effect of main channel width on optimum resonator width and resonator length is further confirmed by theoretical analysis. The usual resonance theory, which predicts complete reflection on the incident wave train when the length of the branch canal is one quarter of a wavelength, was found incorrect by James' laboratory study. James' finding is proved theoretically by rigorous analytical and numerical models in the present study. In addition, higher resonance modes of extreme shape as well as intermediate shape rectangular resonators, not discovered in either James' laboratory investigation or his one-dimensional approximate analysis, were also discovered in the present study.

Keywords: end effect, resonator, deformation of wave, method of matched eigenfunction expansion, boundary integral equation method

Introduction

A resonator is taken to be a short rectangular branch canal on a narrow main water wave channel. Valembois¹ investigated models of short rectangular resonators built orthogonally onto parallel entrance breakwaters of a harbor but did not observe any end effects. Rayleigh² had previously developed an approximation for the end effects of an acoustic tube in a semi-infinite domain, an analogous situation. James³ examined the end effects for rectangular resonators on narrow channels by experimental study and observed very interesting phenomena. James classified the resonator into three types by their geometry: a long and narrow resonator, in which the motion is in a direction normal to that of the waves in the main channel; a short and wide resonator, in which the motion is parallel to that in the main channel; and a resonator intermediate in shape between the extremes. A one-dimensional approximate

theory, plane wave theory, was developed by James⁴ for extreme values of resonator geometry. But the plane wave theory can only discover the fundamental resonance mode of a long and narrow rectangular resonator. The generalized behaviors for short and wide resonators and intermediate shape resonators have not been treated mathematically before.

In the present paper, instead of James' one-dimensional approximation, a more reasonable two-dimensional analysis has been developed successfully, using both matched eigenfunction expansion and the boundary integral equation method (BIEM) to study the behaviors of resonators, of extreme shapes as well as intermediate shapes, built into channels or breakwaters of a harbor entrance. These methods have been widely used to solve water wave problems.^{5–10} Their accuracy is believed to be very good. In the present study the accuracies of the solution techniques are demonstrated by comparison of numerical results and James' laboratory data. Numerical results also show that more than one resonant mode exists, even for long and narrow resonators; this has not been observed in James' experimental study. In general, the dimensions of a resonator should be carefully chosen so that most of the incident wave can be reflected back into the upstream domain.

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Theoretical formulation of the problem

The fundamental assumption is the monochromatic small-amplitude motion of an inviscid, irrotational fluid with free surface. The geometry of a water wave channel with a built-in rectangular resonator is shown in Figure 1. A coordinate system is established with z positive upward and equal to zero at the free surface and the y -axis extending along the intersection of upstream channel and resonator. The channel depth is considered to be uniform in all regions.

Because of the assumption of inviscid and irrotational flow, a velocity potential Φ must exist in the entire domain and must satisfy the Laplace equation,

$$\nabla^2 \Phi = 0 \quad (1)$$

together with a free surface boundary condition,

$$\frac{\partial \Phi}{\partial z} - \frac{\sigma^2}{g} \Phi = 0 \quad (2)$$

where σ is the angular frequency of the monochromatic motion and g is the acceleration of gravity. The no-flow boundary conditions on the solid boundaries give

$$\frac{\partial \Phi}{\partial \mathbf{n}} = 0 \quad (3)$$

where $\mathbf{n}$ is a unit vector outward normal to any of the fixed boundary segments in Figure 1. Since we are dealing with a channel of uniform water depth h , the boundary condition at the channel bottom is

$$\frac{\partial \Phi}{\partial z} = 0 \quad (4)$$

on $z = -h$. Reflection and transmission waves in region 1 and region 3, respectively, must also satisfy the radiation condition¹¹ at $x = \pm\infty$; i.e., the disturbance generated by the presence of a resonator must be outgoing waves. The above equations complete the formulation of the problem of wave deformation in a constant depth channel with a rectangular resonator.

Since the incident wave is periodic with angular frequency σ , we may assume that the velocity potential is a product of functions of x , y , z , and t such as

$$\Phi(x, y, z, t) = \phi(x, y, z) e^{-i\sigma t} = F(x, y) G(z) e^{-i\sigma t} \quad (5)$$

Substituting the above expression into the Laplace

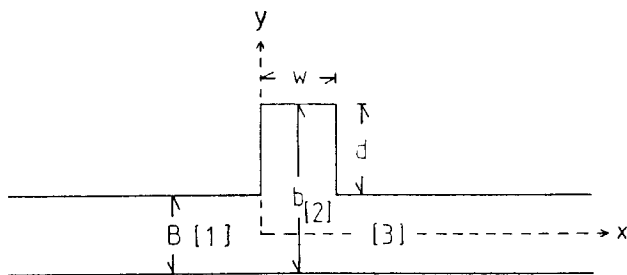


Figure 1. Geometrical sketch

equation, separating the functions of x , y , and z and converting them to a constant, say, μ^2 , one gets

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \mu^2 F = 0 \quad (6)$$

$$\frac{\partial^2 G}{\partial z^2} - \mu^2 G = 0 \quad (7)$$

Equation (6) is the Helmholtz equation, which is the governing equation for wave diffraction in a uniform depth wave field.¹² The solution of (7) together with the bottom boundary condition,

$$\frac{\partial G}{\partial z} = 0 \quad (8)$$

on $z = -h$ is simply

$$G(z) = \begin{cases} \cosh k(h+z) & k = \sqrt{\mu^2} \quad \mu^2 > 0 \\ \cos k(h+z) & k = \sqrt{-\mu^2} \quad \mu^2 < 0 \end{cases} \quad (9)$$

where k is related to the angular frequency σ and the water depth h through the free surface boundary condition. This relation can be obtained by substituting (5) and (9) into (2). One will find

$$\sigma^2 = \begin{cases} gk \tanh kh & \mu^2 = k^2 > 0 \\ -gk_n \tan k_n h & \mu^2 = -k_n^2 < 0 \end{cases} \quad n = 1, 2, \dots \quad (10)$$

where k is seen to be the usual wave number and k'_n , as was shown by Biesel,¹³ satisfies $(n - 0.5)\pi < k'_n h < n\pi$. Since the water depth in all regions of the present problem is uniform and the interfaces are vertical planes, it can be proved that the $G(z) = \cos[k_n(h+z)]$ term must disappear. The problem now is to obtain the solution of (6) with the boundary condition

$$\frac{\partial F}{\partial \mathbf{n}} = 0 \quad (11)$$

on the solid boundary. By using the separation of variables method the general solution of $F(x, y)$ can be obtained as

$$F(x, y) = \sum_{m=0}^{\infty} [A_m e^{i\alpha_m x} + B_m e^{-i\alpha_m x}] \cos[\beta_m (y + B/2)] \quad (12)$$

with

$$\beta_m = \frac{m\pi}{B} \quad \alpha_m = \sqrt{k^2 - \beta_m^2} \quad (13)$$

where m is any integer, A_m and B_m are unknown coefficients, and B is the channel width.

Substituting equations (9) and (12) into equation (5), and with the radiation condition, one gets ϕ at each region:

$$\phi_1(x, y, z) = \left\{ e^{ikx} + \sum_{m=0}^{M1} R_m e^{-i\alpha_m x} \psi_{\beta, m}(y) + \sum_{m=M1+1}^{\infty} R_m e^{-i\alpha_m x} \psi_{\beta, m}(y) \right\} \cosh k(h+z) \quad (14)$$

with

$$\beta_m = \frac{m\pi}{B} \quad \alpha_m = \sqrt{k^2 - \beta_m^2} \quad (15)$$

in region [1], where $M1$ is determined as an integer satisfying

$$\frac{M1\pi}{B} < k < \frac{(M1 + 1)\pi}{B}$$

$$\phi_2(x, y, z) = \left\{ \sum_{m=1}^{M2} [C_m e^{i\lambda_m x} + D_m e^{-i\lambda_m(x-w)}] \psi_{\gamma,m}(y) + \sum_{M2+1}^{\infty} [C_m e^{i\lambda_m x} + D_m e^{-i\lambda_m(x-w)}] \psi_{\gamma,m}(y) \right\} \cosh k(h+z) \quad (17)$$

with

$$\gamma_m = \frac{m\pi}{b} \quad \lambda_m = \sqrt{k^2 - \gamma_m^2} \quad (18)$$

in region 2, where b is the width of region 2, w is the length of the rectangular resonator, and $M2$ is an integer that must satisfy

$$M2 < \frac{2b}{L} < M2 + 1 \quad (19)$$

The functions $\psi(y)$ are given by

$$\psi_{\beta,m}(y) = \cos \beta_m(y + B/2)$$

$$\psi_{\gamma,m}(y) = \cos \gamma_m(y + B/2)$$

The velocity potential in region [3] is

$$\phi_3(x, y, z) = \left\{ \sum_{m=0}^{M1} T_m e^{i\alpha_m(x-w)} \psi_{\beta,m}(y) + \sum_{m=M1+1}^{\infty} T_m e^{i\alpha_m(x-w)} \psi_{\beta,m}(y) \right\} \cosh k(h+z) \quad (20)$$

In the assumption of narrow channels the channel width is less than one half the incident wavelength. Equations (14) and (20) can be reduced to

$$\phi_1(x, y, z) = \left\{ e^{ikx} + R_0 e^{-ikx} + \sum_{m=1}^{M1} R_m e^{-i\alpha_m x} \psi_{\beta,m}(y) \right\} \cosh k(h+z) \quad (21)$$

$$\phi_3(x, y, z) = \left\{ T_0 e^{ik(x-w)} + \sum_{m=1}^{M1} T_m e^{i\alpha_m(x-w)} \psi_{\beta,m}(y) \right\} \cosh k(h+z) \quad (22)$$

where α_m are imaginary numbers. The first term in equation (21) is the incident wave, which is presumed to be incident from $x = -\infty$ in region 1.

Solutions to the full problem must satisfy certain matching conditions over the vertical planes separating the fluid regions, namely,

$$\phi_1 = \phi_2 \quad (x=0; -h \leq z \leq 0; -B/2 \leq y \leq B/2) \quad (23)$$

or

$$M1 < \frac{2B}{L} < M1 + 1 \quad (16)$$

and L is the wavelength of the incident wave;

$$\phi_{1x} = \phi_{2x} \quad (x=0; -h \leq z \leq 0; -B/2 \leq y \leq B/2) \quad (24)$$

$$\phi_2 = \phi_3 \quad (x=w; -h \leq z \leq 0; -B/2 \leq y \leq B/2) \quad (25)$$

$$\phi_{2x} = \phi_{3x} \quad (x=w; -h \leq z \leq 0; -B/2 \leq y \leq B/2) \quad (26)$$

These conditions provide continuity of pressures and horizontal velocities normal to the vertical interface planes.

Since all terms in the summation series of equations (21) and (22) are decayed exponentially with distance x away from the interface planes, the wave motions persisting at some distance from the intersections are seen to be given by

$$\phi_1 = (e^{ikx} + R_0 e^{-ikx}) \cosh k(h+z) \quad (27)$$

in region [1] and

$$\phi_3 = T_0 e^{ik(x-w)} \cosh k(h+z) \quad (28)$$

in region [3]. Therefore R_0 and T_0 are the reflected and transmitted coefficients, respectively.

Numerical solutions

In order to construct a solution the infinite series in (14), (17), and (20) must be truncated to a finite number of terms, given by M . We then must solve for $4M + 4$ unknown coefficients R_m , C_m , D_m , and T_m ($m = 0, 1, 2, \dots, M$). Matching and boundary conditions are manipulated in the following manner. Using the eigenfunction expansions for each region in the matching conditions (23)–(26) and employing the orthogonality of these functions, we obtain the following equations for computing coefficients in the eigenfunction expansion^{8,10}:

$$\int_{-B/2}^{B/2} \phi_1(0, y, z) dy = \int_{-B/2}^{B/2} \phi_2(0, y, z) dy \quad (29)$$

$$\int_{-B/2}^{B/2} \phi_1(0, y, z) \psi_{\beta,n}(y) dy = \int_{-B/2}^{B/2} \phi_2(0, y, z) \psi_{\beta,n}(y) dy \quad (30)$$

with $n = 1, 2, \dots, M$;

$$\int_{-B/2}^{B/2} \frac{\partial \phi_1}{\partial x}(0, y, z) dy = \int_{-B/2}^{B/2} \frac{\partial \phi_2}{\partial x}(0, y, z) dy$$

$$= \int_{-B/2}^{b-B/2} \frac{\partial \phi_2}{\partial x}(0, y, z) dy \quad (31)$$

$$\int_{-B/2}^{B/2} \frac{\partial \phi_1}{\partial x}(0, y, z) \psi_{\gamma,n}(y) dy$$

$$= \int_{-B/2}^{B/2} \frac{\partial \phi_2}{\partial x}(0, y, z) \psi_{\gamma,n}(y) dy$$

$$= \int_{-B/2}^{b-B/2} \frac{\partial \phi_2}{\partial x}(0, y, z) \psi_{\gamma,n}(y) dy \quad (32)$$

with $n = 1, 2, \dots, M$;

$$\int_{-B/2}^{B/2} \phi_2(w, y, z) dy = \int_{-B/2}^{B/2} \phi_3(w, y, z) dy \quad (33)$$

$$\int_{-B/2}^{B/2} \phi_2(w, y, z) \psi_{\beta,n}(y) dy$$

$$= \int_{-B/2}^{B/2} \phi_3(w, y, z) \psi_{\beta,n}(y) dy \quad (34)$$

with $n = 1, 2, \dots, M$;

$$\int_{-B/2}^{B/2} \frac{\partial \phi_3}{\partial x}(w, y, z) dy = \int_{-B/2}^{B/2} \frac{\partial \phi_2}{\partial x}(w, y, z) dy$$

$$= \int_{-B/2}^{b-B/2} \frac{\partial \phi_3}{\partial x}(w, y, z) dy \quad (35)$$

$$\int_{-B/2}^{B/2} \frac{\partial \phi_3}{\partial x}(w, y, z) \psi_{\gamma,n}(y) dy$$

$$= \int_{-B/2}^{B/2} \frac{\partial \phi_2}{\partial x}(w, y, z) \psi_{\gamma,n}(y) dy$$

$$+ \int_{-B/2}^{b-B/2} \frac{\partial \phi_2}{\partial x}(w, y, z) \psi_{\gamma,n}(y) dy \quad (36)$$

with $n = 1, 2, \dots, M$, where the shift of integration limits in the last term of equations (31), (32), (35), and (36) is possible, since there is no contribution to the horizontal velocity field from the contours $[B/2 \leq y \leq (b - B/2); x = 0 \text{ and } x = w]$, i.e., equation (11). The resulting set of $4M + 4$ simultaneous equations (29)–(36) is solved numerically as a linear matrix equation.

There is still a matter of the choice of M ,¹⁴ and the infinite series must be truncated. Numerical tests show that solutions seem to converge faster for longer waves. To reduce computation times and as a matter of convenience for the present study, 11 terms ($M = 10$) in every domain are used in all computations. The accuracy of these numerical results will be checked by comparing with solutions obtained from the BIEM in a later section.

Numerical results and comparisons

James^{3,4} studied the end effects of rectangular resonators on narrow channels. His study included both theoretical and experimental works. He developed a one-dimensional approximate theory, the plane wave theory, examining the resonance reflection of waves due to long and narrow rectangular resonators on narrow channels. His solution provides the reflection and transmission coefficient values. However, his one-dimensional approximate theory can only investigate the fundamental resonance of long and narrow resonators.

To get a more accurate solution and to find other possible resonance modes, a two-dimensional analysis is required. The present two-dimensional analytical model is used to study the end effect of rectangular resonators, of extreme as well as intermediate shapes,

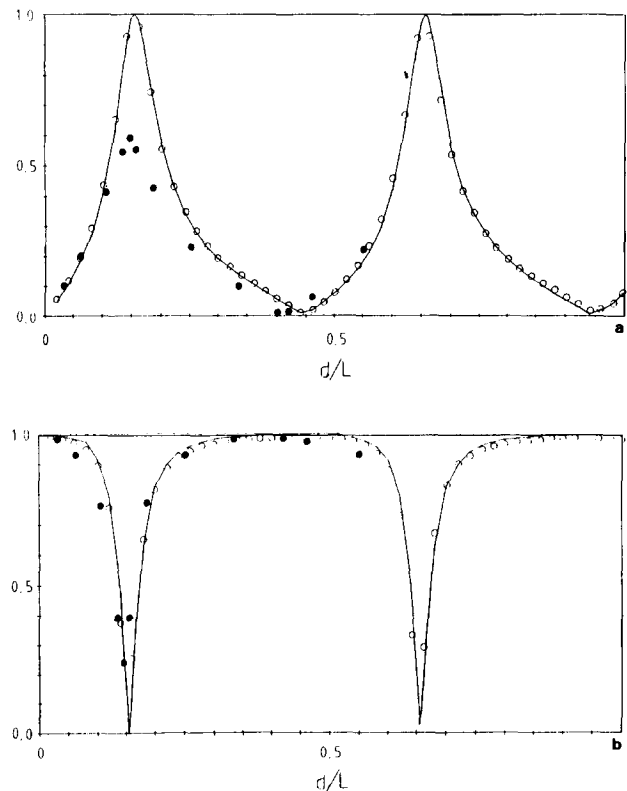


Figure 2. Typical results for narrow resonator; $w/L = 0.34$, $B/L = 0.385$. (a) Reflection coefficient and (b) transmission coefficient: solid line, analytical solution, ●, James' experimental data, and ○, BIEM solution

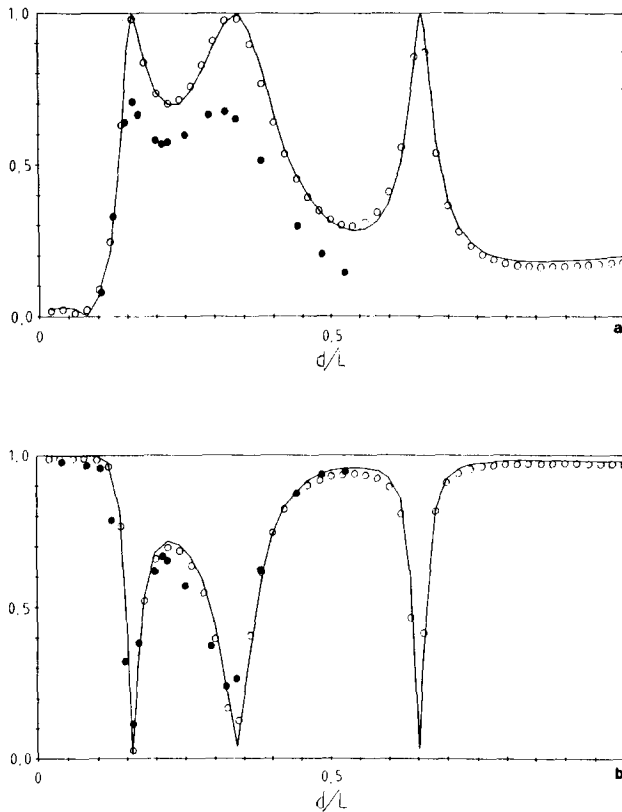


Figure 3. Typical results for wide resonators; $w/L = 0.60$, $B/L = 0.385$. (a) Reflection coefficient and (b) transmission coefficient: solid line, analytical solution, ●, James' experimental data, and ○, BIEM solution

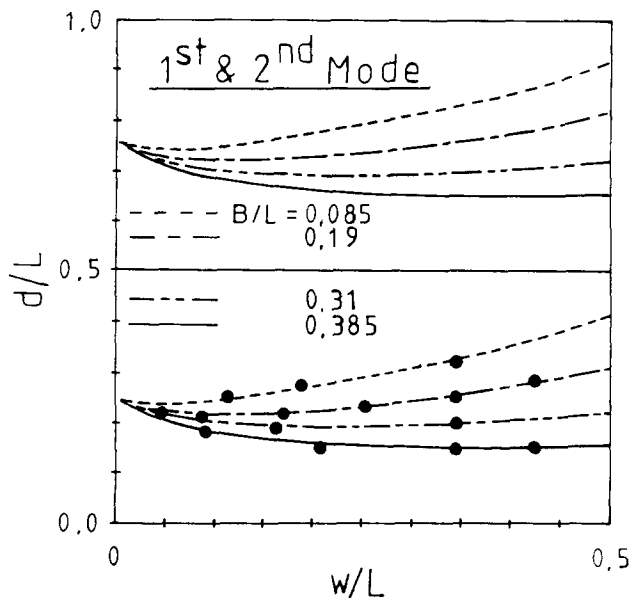


Figure 4. Geometry for resonance: lines, analytical solutions and ●, James' experimental data

on narrow channels. The numerical results of a narrow resonator with $w/L = 0.34$ and $B/L = 0.385$ are plotted in Figure 2; the numerical results of a wide resonator with $w/L = 0.60$ and $B/L = 0.385$ are plotted in Figure 3. James' laboratory experimental data are also plotted in Figures 2 and 3 for comparison. The notation d is the width of a rectangular resonator defined in Figure 1. The agreement is reasonable. Since viscosity has been neglected in the present theoretical analysis, slight differences between analytical solutions and the laboratory observation data in Figures 2 and 3 are expected. A series of computer runs are made to find the geometry for resonance for different values of w/L and are plotted in Figure 4 to compare with James' laboratory results. Again, the agreement is excellent for the fundamental resonance mode. More resonance modes have been discovered in the present two-dimensional analysis; two of these higher resonance modes are shown in Figure 4 for reference.

BIEM solutions

The BIEM has been used to solve a variety of problems in theoretical hydrodynamics.¹⁵ For water waves the BIEM has been used by Lee,⁵ Yeung,⁶ Salmon et al.,¹⁶ Lennon et al.,¹⁷ Liu and Abbaspour,⁷ and Dalrymple and Kirby.⁹ It is believed that the accuracy of the BIEM is excellent. In order to further confirm the correction of the above two-dimensional analysis a BIEM with a constant element model will be used to study the end effects of rectangular resonators on narrow channels. More sophisticated models exist¹⁸; however, as long as a sufficient number of elements are used, the accuracy is the same. Since the cited references provide detailed explanations of the BIEM technique, only a brief discussion is provided here.

As shown in Figure 2, in region 2, Green's identity formula is applied, and the Hankel function of the first kind and zero order $H_0^{(1)}(kr)$ is chosen to be the fundamental solution of the two-dimensional Helmholtz equation,¹⁹ i.e., equation (6). Therefore the wave function F_2 at any position in the domain of interest can be expressed in integral form as a function of the value of F_2 and its normal derivative $\partial F_2/\partial n$ on the boundaries:

$$F_2(x_0, y_0) = -\frac{i}{4} \int_S \left[F_2(x, y) \frac{\partial}{\partial n} (H_0^{(1)}(kr)) - H_0^{(1)}(kr) \frac{\partial}{\partial n} (F_2(x, y)) \right] ds(x, y) \quad (37)$$

where r is the distance from the interior point (x_0, y_0) to the boundary, S . The second part of the integrand of (37) represents a distribution of sources (or sinks) along the boundaries with strength $\partial F_2/\partial n$. On the other hand, the first part is a distribution of doublets located on the boundaries with a strength $F_2(x, y)$. In order to be able to determine the wave function F_2 , at any interior point of region 2, either the value F_2 or the value $\partial F_2/\partial n$ on the boundaries of the region must be known.

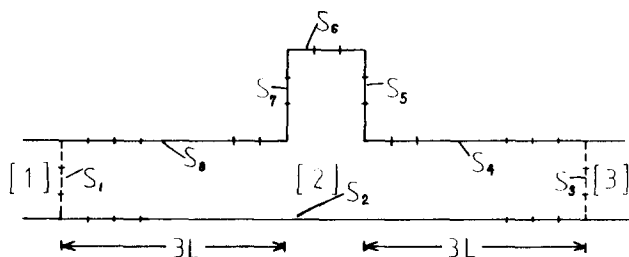


Figure 5. Discretization of boundaries for BIEM

For the present problem the boundary is divided into eight parts, $S_1, S_2, \dots, S_8$, as shown in Figure 5. Since the boundaries of $S_2, S_4, \dots, S_8$ are a rigid wall, their boundary conditions can be expressed as

$$\frac{\partial F_2}{\partial n} = 0 \quad (38)$$

which is the same as equation (11), for the upwave and downwave lateral boundaries, S_1 and S_3 , where the channel is assumed to have a uniform cross section. The wave function F_1 can be represented analytically by using an eigenfunction expansion with unknown coefficients, as equation (12) shows. By placing the lateral boundaries sufficiently far from the rectangular resonator, only the progressive waves in the eigenfunction expression are necessary, which is due to the assumption of a narrow channel, and the evanescent modes will exponentially decay away from the intersections. Therefore the wave functions on S_1 and S_3 will be

$$F_1(x, y) = [e^{ikx} + R_0 e^{-ikx}] \frac{1}{\cosh kh} \quad (39)$$

$$F_3(x, y) = T_0 e^{ikx} \frac{1}{\cosh kh} \quad (40)$$

where R_0 and T_0 are the reflection and transmission coefficients to be determined. Because of the continuity of pressure and normal velocity over the intersection boundaries the boundary conditions on S_1 and S_3 can be established.⁶

In the present study the lateral boundaries are placed three wavelengths away from the rectangular resonator, and boundaries are discretized to 232 elements. The numerical results of BIEM are plotted in Figures 2 and 3. The agreement among solutions of matched eigenfunction expansions, BIEM numerical solutions, and James' laboratory data are clear. Therefore the accuracy of the present two-dimensional model is further confirmed.

Conclusion

James' one-dimensional approximation method of examining the fundamental resonance of an extreme shape resonator on a long channel has been extended to a

two-dimensional study. The two-dimensional analyses, both analytical and numerical models, provide an accurate means to determine the fundamental resonance mode as well as other possible resonance modes for extreme shapes as well as intermediate shape resonators. The accuracy of the two-dimensional analyses is confirmed by comparing numerical results with James' experimental data.

The present study is constrained to narrow channel conditions; i.e., the width of the channel is less than one half the incident wavelength. Otherwise, if channel width is larger than one-half wavelength, lateral oscillation waves become important and even dominate in the channel sometimes. This phenomenon is due to the cutoff frequencies.²⁰ Therefore the efficiency of the resonator will be doubted. Penny and Price²¹ also have shown that the maximum zone of protection in the downstream domain provided by a resonator at resonance is limited to a width of one half the incident wavelength, from the view of linear wave theory. To examine this complex phenomenon, we may need to consider the higher order effects, instead of the present linear analysis, which therefore are not included in the present study.

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