



Maximum scan statistics and channel assignment problems in homogeneous wireless networks

Chih-Wei Yi*

Department of Computer Science, National Chiao Tung University, 1001 University Road, Hsinchu City 30010, Taiwan

ARTICLE INFO

Article history:

Received 14 October 2008

Received in revised form 1 February 2009

Accepted 14 February 2009

Communicated by D.-Z. Du

Keywords:

Wireless ad hoc networks

Maximum scan statistics

Channel assignment

Chromatic numbers

Clique numbers

Inductivities

ABSTRACT

In wireless networks, to avoid collisions of simultaneous transmissions over the same channel, adjacent nodes are assigned distinct channels, and the least number of channels used in an assignment is called the chromatic number. The determination of the chromatic number is NP-hard. In this paper, we introduce an analytic tool called maximum scan statistics. For a finite point set V and a convex compact set C , the maximum scan statistic of V with respect to the scanning set C is the largest number of points in V covered by a copy C . Based on the study of asymptotic maximum scan statistics, we obtain the asymptotics of the maximum degree and the clique number of homogeneous wireless networks. The results imply that the chromatic number is almost surely at most four times the clique number. We further prove that the approximation ratios of some vertex-ordering-based First-Fit channel assignment algorithms are almost surely bounded by 2. In the analysis, we also learn that the chromatic number is almost surely at most twice the clique number.

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1. Introduction

In homogeneous wireless ad hoc networks, each node has the same transmission range r , and two nodes have a link between them if the distance between them is at most r . Two nodes can communicate with each other either through a single hop direct link if they are within each other's transmission range or through a multi-hop communication session otherwise. The induced network topology is called r -disk graphs, or unit disk graphs (UDGs) if r is scaled to 1. Because there is no need for fixed infrastructures, wireless ad hoc networks can be flexibly deployed at low cost for varying missions such as decision making in battlefields, emergency disaster relief and environmental monitoring. In many applications, for example wireless sensor networks, a large number of devices will be randomly deployed. Consequently, wireless networks are represented by unit disk graphs over random point sets, and the induced network topology is called random disk graphs. In the literature, asymptotic analysis is interesting to the research community; see, e.g., [1–4].

In wireless networks, each node can be in either sending or receiving mode but not both at the same time. Due to the short transmission range, each node can only transmit messages to or receive messages from nearby nodes. To avoid collisions of simultaneous transmissions over the same channel, adjacent nodes are assigned distinct channels. The assignment of channels can be modeled by vertex coloring of graphs. A vertex coloring, assigning one color to each vertex, is proper if no adjacent vertices receive the same color. For a graph G , $\chi(G)$ denotes the least number of colors used in a proper coloring of G and is called the chromatic number of G .

Let $G(V, E)$ denote a graph with vertex set V and edge set E . A graph $H(V', E')$ is a subgraph of $G(V, E)$ if $V' \subseteq V$ and $E' \subseteq E$. In addition, if $E' = \{(u, v) \in E : \forall u, v \in V'\}$, H is called a induced subgraph of G (by V'). For any vertex $u \in V$, $\deg(u)$ denotes the number of vertices adjacent to u and is called the vertex degree of u . $\delta(G) = \min_{u \in V} \deg(u)$ and

* Tel.: +886 3 5131346.

E-mail address: yi@cs.nctu.edu.tw.

$\Delta(G) = \max_{u \in V} \deg(u)$ are called the minimum degree and maximum degree of G , respectively. $\delta^*(G)$ denotes the maximum of $\delta(H)$ over all subgraphs H of G (or over all induced subgraphs) and is called the inductivity of G . The clique number of G , denoted by $\omega(G)$, is the size of the largest clique subgraph of G . It is known that $\delta(G) \leq \omega(G) \leq \chi(G) \leq \delta^*(G) + 1 \leq \Delta(G) + 1$.

The determination of the chromatic number is NP-hard even for UDGs. The NP-completeness of the k -coloring of UDGs was proved for $k = 3$ by Clark [5] and for any arbitrary k by Gräf et al. [6]. Marathe et al. [7] showed that no polynomial time algorithm for the coloring problem of UDGs can have performance ratio better than $4/3$, unless $P = NP$. However, the chromatic number of disk graphs can usually be approximated by their clique number, which can be obtained in $O(|V|^3)$ time [5]. Peeters [8] showed that the ratio $\chi(G)/\omega(G)$ of UDGs is at most 3. In random geometric graphs, McDiarmid [9] showed that the ratio $\chi(G)/\omega(G)$ is with high probability equal to 1 in the sparse case and $2\sqrt{3}/\pi$ in the dense case. Penrose [10] even showed that if the random point process satisfies some sparse and regular conditions, the probability distribution of the chromatic number asymptotically focuses on two consecutive integers.

First-Fit channel assignment is a class of vertex-ordering-based greedy channel assignment algorithms. For a given vertex ordering, nodes are sequentially assigned the first available channel that is not used by prior adjacent nodes. The approximation ratios of First-Fit algorithms for heterogeneous networks are 3, 3, 3, and 5, respectively, in smallest-last ordering, distance-increasing ordering, lexicographic ordering, and radius-decreasing ordering [11]. The four orderings are abbreviated as SL, DI, LG, and RD, respectively, in what follows.

Let V be a finite point set on a plane, and C be a convex compact set. The maximum scan statistic (abbreviated as MSS) of V with respect to the scanning set C , denoted by $S_M(V, C)$, is the largest number of nodes that can be covered by a copy of C . In other words, $S_M(V, C) = \max_{C' \cong C} \#(V \cap C')$. Here $\#(A)$ denotes the cardinality of the finite point set A , and $C' \cong C$ means C' and C are congruent under shifting, rotating and reflecting. In [9], MSSs are used as a lower bound for the clique number of r -disk graphs. In [10], Penrose used MSSs to prove the focusing phenomenon of chromatic numbers. More applications of scan statistics can be found in [12].

Let $\mathbb{D}$ be a unit-area convex region, and $X_1, X_2, \dots$ be independent and uniformly distributed random points on $\mathbb{D}$. Let $Po(n)$ be a Poisson random variable with parameter n , independent of $\{X_1, X_2, \dots\}$. Given a positive number n , the point process $\{X_1, X_2, \dots, X_n\}$ is referred to as the *uniform n -point process*, and is denoted by $\mathcal{X}_n$; and the point process $\{X_1, X_2, \dots, X_{Po(n)}\}$ is referred to as the *Poisson point process* with mean n , and is denoted by $\mathcal{P}_n$. In the following, $\mathcal{V}_n$ is shorthand for $\mathcal{P}_n$ and $\mathcal{X}_n$.

In this paper, we assume wireless ad hoc networks are represented by $\mathcal{V}_n$. Every node has the same transmission radius r_n that is given by $\pi r_n^2 = (\beta + o(1)) \ln n$ for some constant β . We derive the asymptotics of maximum scan statistics for scanning sets with area $(\beta + o(1)) \frac{\ln n}{n}$. Based on the results, we derive the asymptotics of the maximum degree and clique number of random geometric graphs. The ratio of the maximum degree and clique number implies that the chromatic number is almost surely at most four times the clique number. Furthermore, we prove that the approximation ratios of First-Fit algorithms in SL, DI, LG, and RD are almost surely upper bounded by 2, 2, 2, and 4, respectively. Our analysis also implies that the chromatic number is almost surely at most twice the clique number.

The rest of this paper is organized as follows. In Section 2, some probabilistic and geometric lemmas are given. In Section 3, we derive asymptotic MSSs. In Section 4, we apply MSSs to obtain asymptotics of some graph properties, including the maximum degree and clique number. In Section 5, the performance ratios of First-Fit algorithms are given. Our conclusions are given in Section 6.

2. Preliminaries

In what follows, a disk of radius r centered at x is denoted by $B(x, r)$. For $a, b \in \mathbb{R}^2$ and $A \subseteq \mathbb{R}^2$, $a + b$ denotes the addition of two vectors, and $A + b = \{a + b : \forall a \in A\}$. $\|x\|$ is the Euclidean norm of a point $x \in \mathbb{R}^2$. $|A|$ is shorthand for the 2-dimensional area of a measurable set $A \subset \mathbb{R}^2$. If A is a convex compact set, for a positive real number c , we use cA to denote the scaling of A by c w.r.t. the mass center of A . The diameter of a set A , denoted as $\text{diam}(A)$, is the supreme of the distance between any two points in the set, i.e. $\text{diam}(A) = \sup_{x, y \in A} \|x - y\|$. The symbols O , Ω , Θ , o , $\sim$ are defined in Table 1 and always refer to the limit $n \rightarrow \infty$. An event is said to be *asymptotic almost sure* (abbreviated by a.a.s.) if it occurs with a probability converging to one as $n \rightarrow \infty$. RV is an acronym of random variable. Table 1 lists the notations used in this paper.

2.1. Probability preliminaries

Let ϕ be the function $\phi(\mu) = 1 - \mu + \mu \ln \mu$ over $(0, \infty)$. ϕ is strictly convex and has the unique minimum zero at $\mu = 1$. See Fig. 1. Let $\phi_+^{-1} : [0, \infty) \rightarrow [1, \infty)$ be the inverse of the restriction of ϕ to $[1, \infty)$. We are interested in the function $\mu = \phi_+^{-1}(1/\beta)$, and the graph of $\mu = \phi_+^{-1}(1/\beta)$ is depicted in Fig. 2. Let

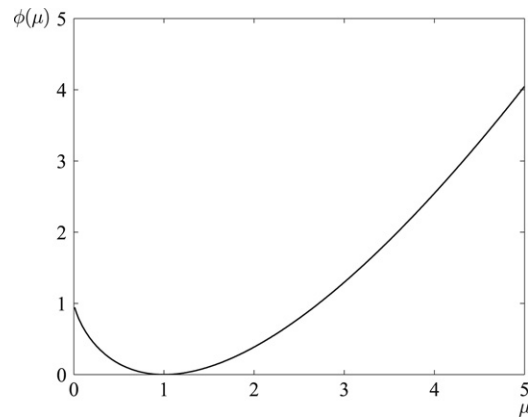
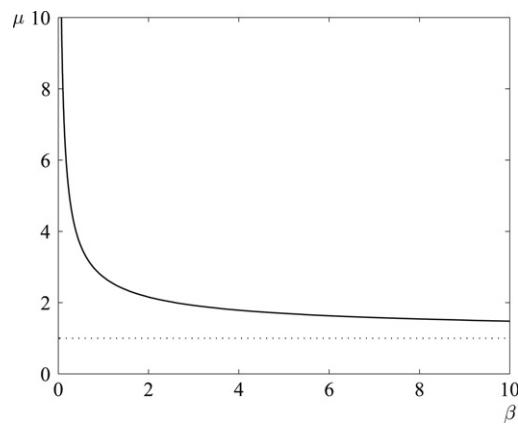
$$\mathcal{U}(\beta) = \beta \phi_+^{-1}(1/\beta) \quad \text{for } \beta \in (0, \infty).$$

$\mathcal{U}(\beta)$ is strictly increasing and is depicted in Fig. 3.

Table 1

Notations.

$G_r(V)$	The r -disk graph induced by a point set V
$\mathbb{D}$	A unit-area convex region
$\mathcal{P}_n$	A Poisson point process with mean n
$\mathcal{X}_n$	A uniform n -point process
$\mathcal{V}_n$	$\mathcal{P}_n$ or $\mathcal{X}_n$
r_n	The transmission radius
$Po(n)$	A Poisson RV with mean n
$\#(A)$	The cardinality of countable set A
$B(x, r)$	A disk with center x and radius r
$\ x\ $	The Euclidean norm of a point x
$ A $	The area of a measurable set A
$diam(A)$	The diameter of A
cA	The c -scaling of A
a.a.s.	An acronym of asymptotic almost sure
RV	An acronym of random variable
$f = O(g)$	$\exists N > 0, c > 0$ s.t. $f(n) \leq cg(n)$ for all $n \geq N$
$f = \Omega(g)$	$\exists N > 0, c > 0$ s.t. $cg(n) \leq f(n)$ for all $n \geq N$
$f = \Theta(g)$	$f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$
$f = o(g)$	$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$
$f \sim g$	$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 1$

**Fig. 1.** $\phi(\mu) = 1 + \mu \ln \mu - \mu$.**Fig. 2.** The curve is $\mu = \phi_+^{-1}(1/\beta)$, and $\mu = 1$ is the asymptotics as $\beta \rightarrow \infty$.

For any positive integer n , the factorial of n , $n! = 1 \cdot 2 \cdot \dots \cdot n$, can be estimated by Sterling's formula:

$$n! \sim (2\pi n)^{\frac{1}{2}} n^n e^{-n}. \quad (1)$$

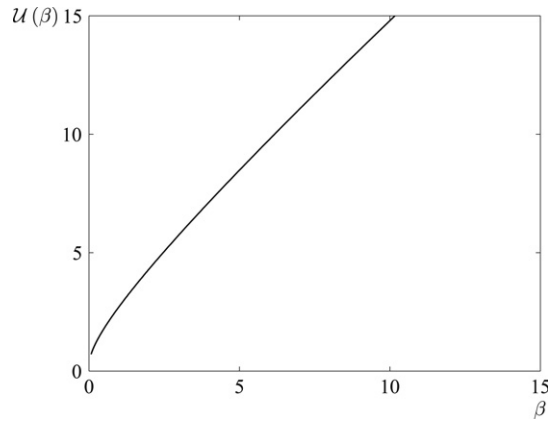


Fig. 3. The curve is $U(\beta)$.

Since

$$\frac{\Pr(Po(\lambda) = k+1)}{\Pr(Po(\lambda) = k)} = \frac{\frac{\lambda^{k+1}}{(k+1)!} e^{-\lambda}}{\frac{\lambda^k}{k!} e^{-\lambda}} = \frac{\lambda}{k+1},$$

for any $\mu > 1$, as $\lambda \rightarrow \infty$, the upper tail distribution of a Poisson RV can be estimated by

$$\begin{aligned} \Pr(Po(\lambda) \geq \mu\lambda) &= \sum_{k=\mu\lambda}^{\infty} \Pr(Po(\lambda) = k) \\ &= \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} \Pr(Po(\lambda) = \mu\lambda) \sim \sum_{k=0}^{\infty} \frac{\lambda^k}{(\mu\lambda)^k} \Pr(Po(\lambda) = \mu\lambda) \\ &= \frac{\mu}{\mu-1} \Pr(Po(\lambda) = \mu\lambda). \end{aligned} \quad (2)$$

From Eqs. (1) and (2), we have the following lemma.

Lemma 1. For any $\mu > 1$, as $\lambda \rightarrow \infty$,

$$\Pr(Po(\lambda) \geq \mu\lambda) \sim \frac{\sqrt{\mu}}{\mu-1} \frac{1}{\sqrt{2\pi\lambda}} e^{-\lambda\phi(\mu)}.$$

Proof. From Eq. (2) and applying Sterling's formula, equation (1), we have

$$\begin{aligned} \Pr(Po(\lambda) \geq \mu\lambda) &\sim \frac{\mu}{\mu-1} \frac{\lambda^{\mu\lambda}}{(\mu\lambda)!} e^{-\lambda} \sim \frac{\mu}{\mu-1} \frac{\lambda^{\mu\lambda}}{\sqrt{2\pi\mu\lambda} (\mu\lambda)^{\mu\lambda} e^{-\mu\lambda}} e^{-\lambda} \\ &= \frac{\mu}{\mu-1} \frac{1}{\sqrt{2\pi\mu\lambda} \mu^{\mu\lambda}} e^{-\lambda+\mu\lambda} = \frac{\mu}{\mu-1} \frac{1}{\sqrt{2\pi\mu\lambda}} e^{-\lambda+\mu\lambda-\mu\lambda \ln \mu} \\ &= \frac{\sqrt{\mu}}{\mu-1} \frac{1}{\sqrt{2\pi\lambda}} e^{-\lambda(1-\mu+\mu \ln \mu)} = \frac{\sqrt{\mu}}{\mu-1} \frac{1}{\sqrt{2\pi\lambda}} e^{-\lambda\phi(\mu)}. \end{aligned}$$

Thus, the lemma is proved. $\square$

The next lemma gives bounds for the maximum of a collection of Poisson RVs.

Lemma 2. Assume that $\lim_{n \rightarrow \infty} \frac{\lambda_n}{\ln n} = \beta$ for some $\beta > 0$. Let $Y_1, Y_2, \dots, Y_{I_n}$ be I_n Poisson RVs with means at most λ_n .

- (1) If $I_n = o(n\sqrt{\ln n})$, then for any $\mu > \phi_+^{-1}(1/\beta)$, $\max_{i=1}^{I_n} Y_i < \mu\lambda_n$ a.s.
- (2) If $I_n = \Omega\left(\frac{n}{\ln n}\right)$ and $Y_1, Y_2, \dots, Y_{I_n}$ are independent Poisson RVs with mean λ_n , then for any $1 < \mu < \phi_+^{-1}(1/\beta)$, $\max_{i=1}^{I_n} Y_i \geq \mu\lambda_n$ a.s.

Proof. Let Y be a Poisson RV with mean λ_n . We first show that, for any $\mu > 0$,

$$\Pr\left[\max_{i=1}^{I_n} Y_i \geq \mu\lambda_n\right] \leq I_n \Pr[Y \geq \mu\lambda_n].$$

Let X_i be the indicator of the event $Y_i \geq \mu\lambda_n$. Then X_i is a Bernoulli RV with probability $\Pr[Y_i \geq \mu\lambda_n]$. So

$$E[X_i] = \Pr[Y_i \geq \mu\lambda_n] \leq \Pr[Y \geq \mu\lambda_n].$$

Let $X = X_1 + \dots + X_{I_n}$. Since $\max_{i=1}^{I_n} Y_i \geq \mu\lambda_n$ if and only if $X \geq 1$, by Markov's inequality,

$$\begin{aligned} \Pr\left[\max_{i=1}^{I_n} Y_i \geq \mu\lambda_n\right] &= \Pr[X \geq 1] \leq E[X] = \sum_{i=1}^{I_n} E[X_i] \\ &\leq I_n \Pr[Y \geq \mu\lambda_n]. \end{aligned}$$

By Lemma 1,

$$\begin{aligned} \Pr\left[\max_{i=1}^{I_n} Y_i \geq \mu\lambda_n\right] &\lesssim I_n \frac{\sqrt{\mu}}{\mu-1} \frac{1}{\sqrt{2\pi\lambda_n}} e^{-\lambda_n\phi(\mu)} \\ &\sim \frac{1}{\sqrt{2\pi\beta}} \frac{\sqrt{\mu}}{\mu-1} \frac{I_n}{\sqrt{\ln n}} n^{-(\lambda_n/\ln n)\phi(\mu)} = \frac{1}{\sqrt{2\pi\beta}} \frac{\sqrt{\mu}}{\mu-1} \frac{I_n}{n\sqrt{\ln n}} n^{1-(\lambda_n/\ln n)\phi(\mu)}. \end{aligned}$$

Fix $\mu > \phi_+^{-1}(1/\beta)$. Then $1 - \beta\phi(\mu) < 0$ and

$$1 - (\lambda_n/\ln n)\phi(\mu) \rightarrow 1 - \beta\phi(\mu) < 0.$$

If $I_n = o(n\sqrt{\ln n})$, then

$$\Pr\left[\max_{i=1}^{I_n} Y_i \geq \mu\lambda_n\right] = o(1).$$

Thus, $\max_{i=1}^{I_n} Y_i < \mu\lambda_n$ a.s.

Next assume that the Y_i 's are i.i.d. Poisson RVs with mean λ_n . Then,

$$\begin{aligned} \Pr\left[\max_{i=1}^{I_n} Y_i \geq \mu\lambda_n\right] &= 1 - \Pr\left[\max_{i=1}^{I_n} Y_i < \mu\lambda_n\right] = 1 - \Pr[Y < \mu\lambda_n]^{I_n} \\ &= 1 - (1 - \Pr[Y \geq \mu\lambda_n])^{I_n} \geq 1 - e^{-I_n \Pr[Y \geq \mu\lambda_n]}. \end{aligned}$$

By Lemma 1,

$$I_n \Pr[Y \geq \mu\lambda_n] \sim \frac{1}{\sqrt{2\pi\beta}} \frac{\sqrt{\mu}}{\mu-1} \frac{I_n}{n\sqrt{\ln n}} n^{1-(\lambda_n/\ln n)\phi(\mu)}.$$

Fix $1 < \mu < \phi_+^{-1}(1/\beta)$. Then $1 - \beta\phi(\mu) > 0$ and

$$1 - (\lambda_n/\ln n)\phi(\mu) \rightarrow 1 - \beta\phi(\mu) > 0.$$

If $I_n = \Omega\left(\frac{n}{\ln n}\right)$, then

$$I_n \Pr[Y \geq \mu\lambda_n] \rightarrow \infty.$$

Hence,

$$\Pr\left[\max_{i=1}^{I_n} Y_i \geq \mu\lambda_n\right] \rightarrow 1.$$

Thus, $\max_{i=1}^{I_n} Y_i \geq \mu\lambda_n$ a.s. $\square$

From Lemma 2(1), if $I_n = o(n\sqrt{\ln n})$, for any given $\beta' \in (\beta, \infty)$, choose a $\mu \in \left(1, \frac{\mathcal{U}(\beta')}{\mathcal{U}(\beta)} \phi_+^{-1}(1/\beta)\right]$. Then, it is a.s. that

$$\begin{aligned} \max_{i=1}^{I_n} Y_i < \mu\lambda_n &= \frac{\mu}{\phi_+^{-1}(1/\beta)} \phi_+^{-1}(1/\beta) \beta \ln n = \frac{\mu}{\phi_+^{-1}(1/\beta)} \mathcal{U}(\beta) \ln n \\ &\leq \mathcal{U}(\beta') \ln n. \end{aligned}$$

Similarly, from Lemma 2(2), if $I_n = \Omega\left(\frac{n}{\ln n}\right)$ and $Y_1, Y_2, \dots, Y_{I_n}$ are independent and with mean λ_n , then for any $\beta' \in (0, \beta)$, it is a.s. that $\max_{i=1}^{I_n} Y_i \geq \mathcal{U}(\beta') \ln n$.

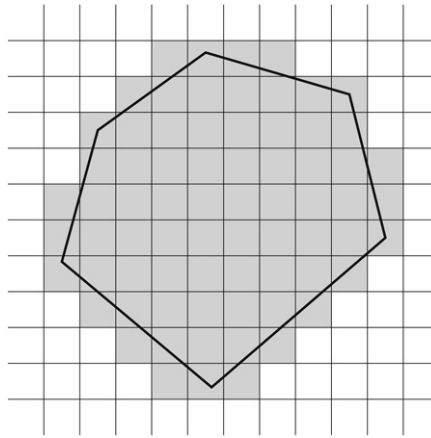


Fig. 4. The cells intersecting with the polygon form a polyquadrant.

2.2. Geometry preliminaries

Let $C \subset \mathbb{R}^2$ be a convex compact set. The r -neighborhood of C , denoted as C_r , is the union of all r -disks with centers in C , i.e. $C_r = \bigcup_{x \in C} B(x, r)$, and C_{-r} denotes the set of points in C apart from ∂C by at least r . $\text{diam}(C)$ denotes the diameter of C that is the supreme of the distance between any two points in C , i.e. $\text{diam}(C) = \sup_{x, y \in C} \|x - y\|$; and $\text{peri}(C)$ denotes the perimeter of C . According to the isodiametric inequality [13,14], a disk with diameter d not only has the largest area $\frac{1}{4}\pi d^2$ among all measurable sets with diameter d but also the longest perimeter πd among all convex compact sets with diameter d . The following lemma is from [15].

Lemma 3. Suppose that $C \subset \mathbb{R}^2$ is a convex compact set with diameter at most d . We have

$$|C_t| < |C| + \pi dt + \pi t^2, \text{ and } |C_{-t}| \geq |C| - \pi dt.$$

An ε -tessellation divides the plane by vertical and horizontal lines into equal-size square cells with width ε . Without loss of generality, we assume the origin is at a corner of cells. In a given tessellation, a polyquadrant is a collection of cells intersecting with a convex compact set. For example, in Fig. 4, the shaded cells form a polyquadrant induced by a polygon. The horizontal span of a polyquadrant is the horizontal distance measured in the number of cells from left to right. The vertical span of a polyquadrant is defined similarly but in the vertical direction. In an ε -tessellation, if the diameter of a polygon is d , the span of a induced polyquadrant is at most $\lceil d/\varepsilon \rceil + 1$. The following lemma is from [15].

Lemma 4. Let S be a region composed of m cells. For a positive constant integer τ , the number of polyquadrants with span at most τ and intersecting with S is $\Theta(m)$.

3. Maximum scan statistics

In this section, we derive the asymptotics of MSSs. Let $\{C_n\}$ denote a sequence of collections of convex compact sets, called regular if there exist constants β and c_0 such that $n|C_n| = (\beta + o(1)) \ln n$ and $\text{diam}(C_n) \leq c_0 \sqrt{\frac{\beta \ln n}{n}}$ for any $C_n \in C_n$. According to the isodiametric inequality, $\text{diam}(C_n) = \Theta\left(\sqrt{\frac{\ln n}{n}}\right)$ is implied by $\text{diam}(C_n) \leq c_0 \sqrt{\frac{\beta \ln n}{n}}$. Let

$$\mathcal{S}_M(\mathcal{V}_n, \mathbf{C}_n) = \max_{C_n \in \mathbf{C}_n} S_M(\mathcal{V}_n, C_n).$$

We have the following theorem.

Theorem 5. For the asymptotics of $\mathcal{S}_M(\mathcal{V}_n, \mathbf{C}_n)$, it is almost sure that

$$\Pr \left[\frac{\mathcal{S}_M(\mathcal{V}_n, \mathbf{C}_n)}{\ln n} \sim \mathcal{L}(\beta) \right] \rightarrow 1.$$

To prove Theorem 5, we first consider that $\mathcal{V}_n$ is a Poisson point process. In Section 3.1, a lower bound for $\mathcal{S}_M(\mathcal{P}_n, \mathbf{C}_n)$ (in Lemma 6) and an upper bound for $\mathcal{S}_M(\mathcal{P}_n, \mathbf{C}_n)$ (in Lemma 7) are given. Since the two bounds are tight, Theorem 5 on Poisson point processes follows. In Section 3.2, applying a de-Poissonization argument, we extend the results to uniform point processes.

3.1. Theorem 5 on Poisson point processes

A lower bound for $\mathcal{S}_M(\mathcal{P}_n, \mathbf{C}_n)$ is given by the following lemma.

Lemma 6. For any $\varepsilon > 0$, we have $\Pr\left[\frac{\mathcal{S}_M(\mathcal{P}_n, \mathbf{C}_n)}{\ln n} \geq (1 - \varepsilon) \mathcal{U}(\beta)\right] \rightarrow 1$.

Proof. Tessellate the plane into equal-size square cells with width $c_0 \sqrt{\frac{\beta \ln n}{n}}$. For any given $C_n \in \mathbf{C}_n$, since $\text{diam}(C_n) \leq c_0 \sqrt{\frac{\beta \ln n}{n}}$, we can place one copy of C_n in each cell that is fully contained in $\mathbb{D}$, and those copies of C_n are pairwise disjoint. Let I_n be the number of copies of C_n . $I_n = \Omega\left(\left(\frac{1}{c_0 \sqrt{\frac{\beta \ln n}{n}}}\right)^2\right) = \Omega\left(\frac{n}{\ln n}\right)$. Let Y_i denote the number of nodes in the i -th copy of C_n for any $1 \leq i \leq I_n$. $Y_1, Y_2, \dots, Y_{I_n}$ are i.i.d. Poisson RVs with rate $(\beta + o(1)) \ln n$. Since $\mathcal{S}_M(\mathcal{P}_n, \mathbf{C}_n) \geq \max_{1 \leq i \leq I_n} Y_i$, from Lemma 2(2), it is a.a.s. that, for any given $\varepsilon > 0$,

$$\mathcal{S}_M(\mathcal{P}_n, \mathbf{C}_n) \geq (1 - \varepsilon) \mathcal{U}(\beta) \ln n.$$

So, this lemma is proved. $\square$

An upper bound for $\mathcal{S}_M(\mathcal{P}_n, \mathbf{C}_n)$ is given by the following lemma.

Lemma 7. For any $\varepsilon > 0$, we have $\Pr\left[\frac{\mathcal{S}_M(\mathcal{P}_n, \mathbf{C}_n)}{\ln n} \leq (1 + \varepsilon) \mathcal{U}(\beta)\right] \rightarrow 1$.

Proof. Let $d_n = c_0 \sqrt{\frac{\beta \ln n}{n}}$. Recall that $\text{diam}(C_n) \leq d_n$ for all $C_n \in \mathbf{C}_n$. Since $\mathcal{U}(\beta)$ is an increasing function, for a given $\varepsilon > 0$, we choose $\beta_1 > \beta$ such that $\mathcal{U}(\beta_1) = (1 + \varepsilon) \mathcal{U}(\beta)$. Let $\delta = \min\left(\frac{\beta_1 - \beta}{4\pi c_0^2 \beta}, \frac{2 - \sqrt{2}}{2}\right)$ and $M = \left\lceil \frac{1}{\delta d_n} \right\rceil$. Divide $\mathbb{D}$ by a δd_n -tessellation. Let P be a polyquadrate corresponding to C_n . Since $P \subseteq (C_n)_{\sqrt{2}\delta d_n}$, we have

$$\begin{aligned} |P| &\leq |(C_n)_{\sqrt{2}\delta d_n}| \\ &\leq |C_n| + \sqrt{2}\delta d_n \cdot \text{peri}(C_n) + \pi \left(\sqrt{2}\delta d_n\right)^2 \quad (\text{by Lemma 3}) \\ &\leq |C_n| + \sqrt{2}\delta d_n (\pi d_n) + \pi \left(\sqrt{2}\delta d_n\right)^2 \quad (\text{by isodiametric inequality}) \\ &= |C_n| + \left(\sqrt{2} + 2\delta\right) \pi \delta d_n^2 \leq |C_n| + 2\pi \delta d_n^2 \sim |C_n| + 2\pi \delta c_0^2 |C_n| \\ &= (1 + 2\pi c_0^2 \delta) |C_n| \leq \frac{\beta_1 + \beta}{2\beta} |C_n| = \frac{\beta_1 + \beta}{2} \frac{\ln n}{n}. \end{aligned}$$

If Y is the number of nodes in P , Y is a Poisson RV with rate at most $\frac{\beta_1 + \beta}{2} \ln n$. Let I_n denote the number of polyquadrates that are induced by a copy of $C_n \in \mathbf{C}_n$, and $Y_1, Y_2, \dots, Y_{I_n}$ be the number of nodes in each polyquadrate. Since the span of P is at most $m = \lceil d_n M \rceil + 1 \leq \frac{1}{\delta} + d_n + 1$, that asymptotically depends only on the constant δ , by Lemma 4, we have

$$I_n = O(M^2) = O\left(\frac{1}{d_n^2}\right) = O\left(\frac{1}{|C_n|}\right) = O\left(\frac{n}{\ln n}\right).$$

Since $\frac{\beta_1 + \beta}{2} < \beta_1$, from Lemma 2(1), it is a.a.s. that

$$\mathcal{S}_M(\mathcal{P}_n, \mathbf{C}_n) \leq \max_{1 \leq i \leq I_n} Y_i < \mathcal{U}(\beta_1) \ln n = (1 + \varepsilon) \mathcal{U}(\beta) \ln n.$$

So, the lemma is proved. $\square$

3.2. De-Poissonization

Here we introduce a de-Poissonization technique that allow us to extend results on Poisson point processes to uniform point processes. By Chebyshev inequality, it is almost sure that $Po\left(n - n^{\frac{3}{4}}\right) \leq n \leq Po\left(n + n^{\frac{3}{4}}\right)$. Thus,

$$\mathcal{S}_M\left(\mathcal{P}_{n - n^{\frac{3}{4}}}, C_n\right) \leq \mathcal{S}_M(\mathcal{X}_n, C_n) \leq \mathcal{S}_M\left(\mathcal{P}_{n + n^{\frac{3}{4}}}, C_n\right). \quad (3)$$

For $|C_n| = (\beta + o(1)) \frac{\ln n}{n}$, we have

$$\frac{\left(n - n^{\frac{3}{4}}\right) |C_n|}{\ln \left(n - n^{\frac{3}{4}}\right)} \sim \frac{\left(n + n^{\frac{3}{4}}\right) |C_n|}{\ln \left(n + n^{\frac{3}{4}}\right)} \sim \frac{n |C_n|}{\ln n} = \beta + o(1).$$

Hence,

$$\mathcal{G}_M \left(\mathcal{P}_{n-n^{\frac{3}{4}}}, C_n \right) \sim \mathcal{G}_M \left(\mathcal{P}_{n+n^{\frac{3}{4}}}, C_n \right) \sim \mathcal{G}_M \left(\mathcal{P}_n, C_n \right). \quad (4)$$

From Eqs. (3) and (4), we have

$$\mathcal{G}_M \left(\mathcal{P}_n, C_n \right) \sim \mathcal{G}_M \left(\mathcal{X}_n, C_n \right).$$

4. Asymptotics of graph properties

In what follows, we assume that a randomly deployed wireless ad hoc network is composed of homogeneous wireless devices represented by $\mathcal{P}_n$ or $\mathcal{X}_n$. Let r_n be the transmission radius or sensing radius given by $n\pi r_n^2 = (\beta + o(1)) \ln n$ for some constant β . The induced network topology is an r_n -disk graph over $\mathcal{V}_n$. If β is fixed, r_n decreases as n increases. We remark that although r_n is scaled with respect to the parameter n , all results in this paper can be scaled back to a standard unit disk model, like the one used in [4], without too much effort by applying the technique used in [16]. In addition, the simplest method to interpret the model is to read $n\pi r_n^2$ as the average number of neighbors and β as a tunable parameter of the node density. We have the following asymptotic theorem on the clique number and the maximum degree of the induced network topology.

Theorem 8. Let $n\pi r_n^2 = (\beta + o(1)) \ln n$ for some constant $\beta > 0$. Then,

- (1) $\Pr \left[\frac{\omega(G_{r_n}(\mathcal{V}_n))}{\ln n} \sim \mathcal{U} \left(\frac{\beta}{4} \right) \right] \rightarrow 1.$
- (2) $\Pr \left[\frac{\Delta(G_{r_n}(\mathcal{V}_n))}{\ln n} \sim \mathcal{U}(\beta) \right] \rightarrow 1.$

The proof of Theorem 8 is given in Sections 4.1 and 4.2. Before giving the proof, we present a corollary on the chromatic number.

Corollary 9. $\Pr \left[1 \leq \frac{\chi(G_{r_n}(\mathcal{V}_n))}{\omega(G_{r_n}(\mathcal{V}_n))} \lesssim 4 \right] \rightarrow 1.$

Proof. Since $\omega(G) \leq \chi(G) \leq \Delta(G) + 1$, we have $1 \leq \frac{\chi(G)}{\omega(G)} \leq \frac{\Delta(G)+1}{\omega(G)}$. From Theorem 8, we know that $\frac{\Delta(G_{r_n}(\mathcal{V}_n))}{\omega(G_{r_n}(\mathcal{V}_n))}$ converges to $\frac{\mathcal{U}(\beta)}{\mathcal{U}(\beta/4)}$ in probability. The corollary can be proved by showing that (1) $\frac{\mathcal{U}(\beta)}{\mathcal{U}(\beta/4)}$ is increasing w.r.t. β ; (2) $\lim_{\beta \rightarrow 0} \frac{\mathcal{U}(\beta)}{\mathcal{U}(\beta/4)} = 1$; and (3) $\lim_{\beta \rightarrow \infty} \frac{\mathcal{U}(\beta)}{\mathcal{U}(\beta/4)} = 4$. Since the remaining arguments are purely mathematical, we leave them to the readers. $\square$

4.1. Clique numbers

This subsection is dedicated to the proof of Theorem 8(1).

In an r -disk graph, nodes in a disk with radius $\frac{1}{2}r$ form a clique. Let $\mathbf{C}_n = \{B(\mathbf{o}, \frac{1}{2}r_n)\}$. Obviously, $\omega(G_{r_n}(\mathcal{V}_n)) \geq \mathcal{G}_M(\mathcal{V}_n, \mathbf{C}_n)$. Applying Theorem 5, for any $\varepsilon > 0$, we have

$$\Pr \left[(1 - \varepsilon) \mathcal{U} \left(\frac{1}{4}\beta \right) \leq \frac{\omega(G_{r_n}(\mathcal{V}_n))}{\ln n} \right] \rightarrow 1. \quad (5)$$

Since the distance between two adjacent nodes is at most r , the diameter of the convex hull of a clique is at most r . According to the isodiametric inequality, the area of the convex hull is at most $\frac{1}{4}\pi r^2$. Let $\mathbf{C}_n$ be the collection of all convex compact sets with area $\frac{1}{4}(\beta + o(1)) \frac{\ln n}{n}$ and diameter at most r_n . Any convex hull of cliques is contained in a $\mathbf{C}_n \in \mathbf{C}_n$. Applying Theorem 5, for any $\varepsilon > 0$, we have

$$\Pr \left[\frac{\omega(G_{r_n}(\mathcal{V}_n))}{\ln n} \leq (1 + \varepsilon) \mathcal{U} \left(\frac{1}{4}\beta \right) \right] \rightarrow 1. \quad (6)$$

Hence, Theorem 8(1) follows Eqs. (5) and (6).

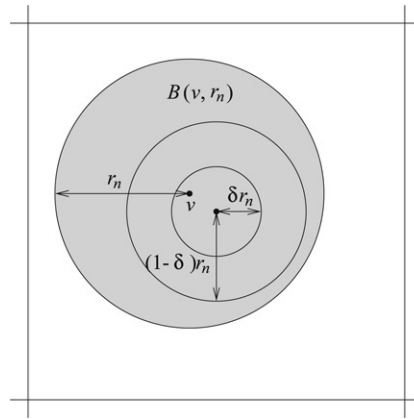


Fig. 5. Draw two disks with radius δr_n and $(1 - \delta)r_n$ at the center of a cell. If there exists a node v in the small disk, $B(v, r_n)$ contains the annulus with radii δr_n and $(1 - \delta)r_n$.

4.2. Maximum degrees

This subsection is dedicated to the proof of [Theorem 8\(2\)](#).

For each node in r -disk graphs, all its neighbors are in a disk of radius r centered at this node. Let $\mathbf{C}_n = \{B(\mathbf{o}, r_n)\}$. Obviously, $\Delta(G_{r_n}(\mathcal{V}_n)) \leq \mathcal{J}_M(\mathcal{V}_n, \mathbf{C}_n) - 1$. Hence, applying [Theorem 5](#), for any $\varepsilon > 0$, we have

$$\Pr \left[\frac{\Delta(G_{r_n}(\mathcal{V}_n))}{\ln n} \leq (1 + \varepsilon) \mathcal{U}(\beta) \right] \rightarrow 1. \quad (7)$$

In the following lemma, a lower bound for the maximum degree is provided.

Lemma 10. Let $n\pi r_n^2 = (\beta + o(1)) \ln n$ for some $\beta > 0$. For any $\varepsilon > 0$, we have

$$\Pr \left[\frac{\Delta(G_{r_n}(\mathcal{V}_n))}{\ln n} \geq (1 - \varepsilon) \mathcal{U}(\beta) \right] \rightarrow 1.$$

Proof. Since $\mathcal{U}(\beta)$ is an increasing function, for any given $\varepsilon > 0$, we can choose $\beta_1 < \beta$ such that $\mathcal{U}(\beta_1) = (1 - \varepsilon) \mathcal{U}(\beta)$. Let $\delta = \frac{\beta - \beta_1}{3\beta}$. Divide the plane into equal-size square cells by a $4r_n$ -tessellation. For each cell fully contained in $\mathbb{D}$, draw two concentric disks at the center with radius δr_n and $(1 - \delta)r_n$. Since $\delta < \frac{1}{3}$, the disk with radius δr_n is contained in the disk with radius $(1 - \delta)r_n$. The area of small disks is $\pi(\delta r_n)^2$, and the area of annuli of radii δr_n and $(1 - \delta)r_n$ is

$$\pi((1 - \delta)r_n)^2 - (\delta r_n)^2 = (1 - 2\delta)\pi r_n^2.$$

Consider a cell and the two concentric disks in it. See [Fig. 5](#). If there exists a node v in the small disk, $B(v, r_n)$ contains the annulus and is contained in the cell.

First, we consider that $\mathcal{V}_n$ is a Poisson point process. Let I_n be the number of non-empty small disks, and $Y_1, \dots, Y_{I_n}$ be the number of nodes in the corresponding annuli of those non-empty small disks. For each small disk, the event that the disk is not empty is a Bernoulli trial with probability $1 - e^{-n\pi(\delta r_n)^2}$. So, I_n is a binomial RV with $\Theta(\frac{n}{\ln n})$ trials. By Chebyshev inequality, we can prove $I_n = \Theta(\frac{n}{\ln n})$. In addition, $Y_1, \dots, Y_{I_n}$ are Poisson RVs with means

$$(1 - 2\delta)n\pi r_n^2 = \frac{\beta + 2\beta_1}{3\beta}n\pi r_n^2 = \left(1 + \frac{\beta - \beta_1}{3\beta_1}\right)\beta_1 \ln n,$$

and since those annuli are pairwise disjoint, $Y_1, \dots, Y_{I_n}$ are i.i.d. Since $\Delta(G_{r_n}(\mathcal{P}_n)) \geq \max_{1 \leq i \leq I_n} Y_i$, applying [Lemma 2](#), we have

$$\frac{\Delta(G_{r_n}(\mathcal{P}_n))}{\ln n} \geq \frac{\max_{1 \leq i \leq I_n} Y_i}{\ln n} > \mathcal{U}(\beta_1) = (1 - \varepsilon) \mathcal{U}(\beta).$$

Applying the de-Poissonization technique introduced in [Section 3.2](#), we can obtain the same result on $\mathcal{X}_n$. Hence, the lemma is proved. $\square$

[Theorem 8\(2\)](#) follows [Eq. \(7\)](#) and [Lemma 10](#).

5. Channel assignment problems

In wireless ad hoc networks, to avoid transmission collisions, two adjacent nodes in wireless networks cannot use the same channel. The assignment of channels can be modeled by vertex coloring problems. A vertex coloring is an assignment of colors to each vertex of graph such that any adjacent vertices receive different colors. Let $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$ represent the color set. A vertex coloring g which is a function from V to $\mathbb{Z}^+$ is proper if $g(u) \neq g(v)$ for any edge $(u, v) \in E$.

5.1. Graph inductivities

The inductivity of G , denoted as $\delta^*(G)$, is the largest minimum vertex degree over all subgraphs of G , i.e.

$$\delta^*(G) = \max_{H: \text{subgraph of } G} \delta(H),$$

and it is known that

$$\chi(G) \leq \delta^*(G) + 1.$$

For any subgraph H , there must exist a vertex u whose neighbors in H are all in its right-hand-side half plane. The vertex degree of u in H , denoted as $\deg_H(u)$, is at least $\delta(H)$. Based on this observation, we can prove the following theorem.

Theorem 11. Let $n\pi r_n^2 = (\beta + o(1)) \ln n$ for some constant β . Then,

$$\Pr \left[\frac{\delta^*(G_{r_n}(\mathcal{V}_n))}{\ln n} \leq (1 + \varepsilon) \mathcal{U} \left(\frac{1}{2} \beta \right) \right] \rightarrow 1. \quad (8)$$

Furthermore, it is a.a.s. that

$$\frac{\delta^*(G_{r_n}(\mathcal{V}_n))}{\omega(G_{r_n}(\mathcal{V}_n))} \leq \frac{\mathcal{U}(\frac{1}{2}\beta)}{\mathcal{U}(\frac{1}{4}\beta)}, \text{ and } 1 \leq \frac{\delta^*(G_{r_n}(\mathcal{V}_n))}{\omega(G_{r_n}(\mathcal{V}_n))} \leq 2.$$

Proof. Let H be a subgraph of $G_{r_n}(\mathcal{V}_n)$ and $u(H)$ be one of the leftmost vertices of H . All u 's neighbors in H locate in the right half disk of $B(u, r_n)$. Let $C_n = \{(x, y) \in B(\mathbf{o}, r_n) : x \geq 0\}$ be the right half disk of $B(\mathbf{o}, r_n)$ and $\mathbf{C}_n = \{C_n\}$. Then, we have

$$\delta^*(G) = \max_{H \subseteq G} \delta(H) \leq \max_{H \subseteq G} \deg_H(u(H)) \leq \delta_M(\mathcal{V}_n, \mathbf{C}_n).$$

Applying Lemma 2, we have

$$\lim_{n \rightarrow \infty} \frac{\delta^*(G_{r_n}(\mathcal{V}_n))}{\ln n} \leq (1 + \varepsilon) \mathcal{U} \left(\frac{1}{2} \beta \right).$$

Combining with Lemma 8, we can prove the inequalities of the ratio of $\delta^*(G_{r_n}(\mathcal{V}_n))$ and $\omega(G_{r_n}(\mathcal{V}_n))$. $\square$

Note that McDiarmid [9] showed that the ratio $\frac{\delta^*(G_{r_n}(\mathcal{V}_n))}{\omega(G_{r_n}(\mathcal{V}_n))}$ is 1 and 2 respectively in the sparse case in which $\frac{n\pi r_n^2}{\ln n} \rightarrow 0$ and in the dense case in which $\frac{n\pi r_n^2}{\ln n} \rightarrow \infty$. Since $\frac{\mathcal{U}(\frac{1}{2}\beta)}{\mathcal{U}(\frac{1}{4}\beta)}$ is increasing w.r.t. β , Theorem 11 shows that the ratio increases monotonically from 1 to 2 as networks evolve from sparse to dense.

5.2. First-Fit algorithms

In this subsection, we introduce a class of vertex-ordering-based greedy channel assignment algorithms, called First-Fit algorithms.

A vertex ordering h is a one-to-one function from V to $\{1, 2, \dots, |V|\}$. For a vertex ordering h and a vertex u , $\text{ind}_h(u)$ denotes the number of prior adjacent nodes of u in ordering h , i.e. $\text{ind}_h(u) = \#\{(v \in V : (u, v) \in E, h(v) < h(u))\}$. The inductivity of G with respect to h , denoted by $\text{ind}_h(G)$, is given by $\max_{u \in V} \text{ind}_h(u)$. It was known that

$$\delta^*(G) = \min_{h: \text{vertex ordering}} \text{ind}_h(G).$$

The First-Fit channel assignment heuristics are a class of vertex-ordering-based greedy channel assignment algorithms. For a given vertex ordering h , channels are assigned to each vertex sequentially with the smallest available channel number that is not used by prior adjacent nodes. The channel number of node u is at most the number of prior adjacent vertices plus 1, i.e. $\text{ind}_h(u) + 1$, and the total number of channels used is upper bounded by the inductivity of the ordering plus 1, i.e. $\delta_h(G) + 1$. In [11], four vertex orderings were discussed:

- (1) Smallest-last ordering (SL): The vertices are greedily selected in reverse order. Each time, a vertex with the smallest degree in the remaining induced subgraph is selected and removed. This vertex ordering has the least inductivity, i.e. $ind_{SL}(G) = \delta^*(G)$. [17]
- (2) Distance-increasing ordering (DI): The vertices are sorted in the increasing order of their Euclidean distances from an arbitrary fixed point.
- (3) Lexicographic ordering (LG): The vertices are sorted in the lexicographic order of their coordinates.
- (4) Radius-decreasing ordering (RD): The vertices are sorted in the decreasing order of their transmission radii. If all nodes have the same transmission radii, the ordering is random.

For any vertex set V and transmission radius r , the approximation ratios of $ind_{SL}(G_r(V))$, $ind_{DI}(G_r(V))$, $ind_{LG}(G_r(V))$, and $ind_{RD}(G_r(V))$ to $\chi(G_r(V))$, respectively, are 3, 3, 3, and 5 [11]. For any vertex ordering g , we have

$$\omega(G_r(V)) \leq \chi(G_r(V)) \leq ind_g(G_r(V)) + 1 \leq \Delta(G_r(V)) + 1.$$

Hence, from Theorem 8, it is a.a.s. that $\frac{ind_g(G_{r_n}(V_n))}{\omega(G_{r_n}(V_n))} \lesssim 4$. In addition, since $ind_{SL}(G_{r_n}(V_n)) = \delta^*(G_{r_n}(V_n))$, from Theorem 11, it is a.a.s. that $\frac{ind_{SL}(G_{r_n}(V_n))}{\omega(G_{r_n}(V_n))} \lesssim 2$. Moreover, applying the argument used in Theorem 11 to the vertex ordering DI and LG, we can prove that it is a.a.s. that $\frac{ind_{DI}(G_{r_n}(V_n))}{\omega(G_{r_n}(V_n))} \lesssim 2$ and $\frac{ind_{LG}(G_{r_n}(V_n))}{\omega(G_{r_n}(V_n))} \lesssim 2$. Therefore, we have the following theorem.

Theorem 12. *It is a.a.s. that the approximation ratio of any First-Fit algorithm is at most 4. Furthermore, it is a.a.s. that the approximation ratios of First-Fit in SL, DI, LG are at most 2. Furthermore, the chromatic number is a.a.s. at most twice of the clique number.*

6. Conclusion

Channel assignment is an essential problem for wireless networks and is NP-hard. In this paper, we assume that homogeneous wireless ad hoc networks are modeled by r -disk graphs over random point processes with mean density n . First, we introduce an analytic tool called maximum scan statistics and show that the maximum scan statistics of Poisson point processes or uniform point processes with respect to regular scanning sets of area $(\beta + o(1)) \frac{\ln n}{n}$ can be asymptotically approximated by $\mathcal{U}(\beta) \ln n$. Next, we consider that the transmission radius r_n is given by $n\pi r_n^2 = (\beta + o(1)) \ln n$. Based on maximum scan statistics, we prove that the maximum degree and clique number of r_n -disk graphs are a.a.s. $\mathcal{U}(\beta) \ln n$ and $\mathcal{U}(\frac{\beta}{4}) \ln n$, respectively. This result implies that the chromatic number is a.a.s. upper bounded by four times the clique number. Last, we give analysis of several vertex-ordering-based First-Fit channel assignment algorithms, and show that the approximation ratio of any First-Fit algorithm is a.a.s. bounded by 4 and the approximation ratios of First-Fit algorithms in SL, DI, LG are a.a.s. bounded by 2. The analysis also implies that the chromatic number is a.a.s. upper bounded by twice the clique number.

Acknowledgments

This work of C.-W. Yi is supported in part by the NSC under Grant Nos. NSC95-2221-E-009-059-MY3 and NSC97-2221-E-009-052-MY3, by the ITRI under Grant No. 8352BA4124, and by the MoE ATU plan.

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