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水庫密度流分層模式之發展與應用研究（三）

Development and Application of the Layer Averaged
Density Current Model in Reservoirs (3)

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摘 要

了解庫區泥砂運動特性對水庫永續利用有其必要性，密度流為庫區細粒泥砂輸送之重要行為，其運動機制受泥砂濃度、溫度、潛入與水庫回流等因子控制，現象複雜而難以量化並預測；隨著電腦計算能力的進步，許多三維雷諾平均方程數值模式已應用河川、海岸等方面之模擬預測，但仍有許多困難有待克服，諸如底床剪力、上下游邊界等處理。為了能更精確模擬水庫流場與泥砂運動並兼顧可行性，簡化泥砂運動二相流本質，本研究擬發展一水庫密度流分層積分模式，模擬區域將應用交錯格網並將水體分為數層，控制方程式植入形狀函數並沿各層積分得層平均方程式，由速度與剪力連續條件得代數方程組，水深直接由控制方程求解，另，上下游如單寬入流、溢洪道與排洪隧道等邊界之處理也將一並探討。本研究希望藉由水庫密度流分層積分模式之發展，能對水庫複雜流況與泥砂濃度分部有更進一步之了解，而能對水庫防淤工作有所助益，以利水庫永續利用。

關鍵詞：分層，水庫、模式

Abstract

Understanding sediment movements in reservoirs is important not only for maintaining acceptable water quality but also for extending reservoir's useful life time. The density current is often the governing process in reservoir sedimentation by transporting fine materials over long distance through the impoundment to the vicinity of the dam site. But phenomena of density currents in reservoirs are complex and dependent on turbidity, temperature, and other physical processes such as plunging, circulation, mixing, etc. With the rapid increase in computer power in recent years, several 3-D numerical models solving the Reynolds averaged equations have been developed to simulate flows in natural rivers. In order to simulate the complex flow field and the distribution of the sediment concentration in the reservoirs more precisely and feasibly, a finite volume layer averaged density current model will be developed in this study. This model is not only capable to simulate the flow field as well as suspended sediment concentration both in horizontal and vertical but also treats with variable boundary conditions such as surface spill, and bottom outlet. It is expected to aid engineers understanding the suspended sediment movement and its distribution of concentration in reservoirs. It may be helpful for determining the outlet location of sediment release, horizontally and vertically, at the dam site, etc.

Keywords: Layered 、 Reservoir 、 Model

一、緣由與目的

集水區泥砂一旦進入河道便會往庫區運移，最先入庫的渾水所挾帶的沉滓主要為細泥，由於細泥的比重輕，縱使經過沉砂設施仍無法如同砂一樣沉澱，因此入庫的泥多以懸浮狀態向壩址運移，在水體濁度隨著降雨持續而增加的情況下，渾水會順勢潛入水體底層，形成上層比重較小之清水層與下層比重較大之渾水層的水體分層現象；根據北區水資源局於 93 年艾利颱風過後在阿姆坪附近的量測資料顯示，在水位標高 220 公尺處水體的濁度小於 200 度，但在水位標高 220 公尺處水體濁度則介於 50,000 至 200,000 度，水庫若於底層設置排砂設施能在颱洪期間排渾蓄清，不僅降低水體濁度，並能減少泥砂淤積而延長水庫使用年限。

影響水庫泥砂運移與淤積的條件可分為兩種，一是影響泥砂進入水庫的質與量的因子，影響質的因子包含泥砂粒徑、沉降速度、比重、形狀、黏性與擴散係數等，影響量的因子包含集水區地質與地形、降雨量與降雨強度及其季節分布、土壤特性與植被、表土沖蝕與土地開發等；二是影響水庫泥砂運移能力的因子，包含水庫幾何如寬度、深度、形狀等，以及水理因子如坡降、粗糙度、水力半徑、流量及其時間分布、流速及其空間分布等；這些因子並非完全獨立，有些因子對淤砂的影響也不完全明朗。後者是控制河道河床質載(bed material load)輸送能力的主要因素，但河床質載於水庫上游端即因流速銳減而沉降，僅剩沖洗載(wash load)進入庫區，由此可知水庫泥砂淤積多由前者控制；至於河床質載與沖洗載如何區分並不明確，理論上，沖洗載為河道中無法淤積的部分，可以 D_{10} 作為區分，然實務上常以 200 號篩(0.0625mm)區分，也是土壤分類法中，砂(sand)與沉泥(silt)的界限。

目前發展中之三維模式如 Neary(1995,1999)發展完全三維分流模式，假設水面變化不大採固定水面網格，考慮紊流流場特性，探討取水工流場分布特性。Gessler et al(1999)發展三維動床模式探討彎道流場特性與底床變動現象，考慮紊流效應與懸浮載分布，並探討底床變動與河床沖蝕、懸浮載沉降、河床載傳輸等機制。Ahsan and Blumberg(1999)發展三維水理模式探討湖泊內水質問題與溫度分布，假設水庫內靜水壓分布，由水平方向動量方程式求解水平方向流場，代入連續方程式得到垂直方向速度，並考慮紊流擴散之效應。Gross et al(1999)以三維

水理模式 TRIM3D (Casulli and Cattani, 1994)為基礎，加入污染質傳輸與狀態方程式發展一半隱式(semi-implicit)有限差分鹽度模式，模擬南舊金山灣鹽度之長期變化。Cesare et al(2001)以二相流(two-phase flow)之商業計算流體力學(CFD)模式 CFX-4 植入輸砂連續方程式，藉以模擬異重流(density current)在水庫內的運動行為與底床淤積狀況。另外，以積分平均概念發展之擬似三維分層水理模式有 Choi(1998,1999)採消散 Galerkin 有限元素法發展分層平均泥石流模式，採用移動與固定格網並行方式探討泥石流進入水庫之前進速度與淤積現象。

本計畫之主要目的擬藉由一水庫密度流擬似三維分層模式之建立，探討深水庫之流場特性與密度流體於水體中之運動特性，期對水庫中泥砂運動特性有更進一步之了解，盼能對國內水庫之永續利用有所貢獻。本計畫為三年之連續計畫，第一年重點為水庫水理分層模式之建立，藉由垂直內外部邊界條件之處理，解決現今三維模式水深(自由液面)與底床糙度關聯性之瓶頸，並由針對上游單寬流量、下游固定水位與率定曲線等慣用邊界條件之處理，改進三維模式之實用性；第二年重點為建立密度流體與懸浮沉澱濃度之連動並垂直二維模式之比較，檢討密度流模式由定性轉為定量分析之可行性；第三年則為實際案例模擬及其各種邊界如上游浮動水面入流條件，下游溢洪道、排洪隧道等出流條件之處理，藉由密度流與非密流懸浮質泥砂運移模擬分析，探討排砂口平面位置與高度對排砂成效之影響，並檢討分層模式之實用性。

二、理論基礎

1.控制方程式

水理方程

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{\partial(uu)}{\partial x} + \frac{\partial(vu)}{\partial y} + \frac{\partial(wu)}{\partial z} = \\ -\frac{1}{\rho_w} \frac{\partial P}{\partial x} + \frac{1}{\rho_w} \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) - \frac{g}{\rho_w} \frac{\partial}{\partial x} \left(\int_z^{z_s} \rho' dz \right) \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial v}{\partial t} + \frac{\partial(uv)}{\partial x} + \frac{\partial(vv)}{\partial y} + \frac{\partial(wv)}{\partial z} = \\ -\frac{1}{\rho_w} \frac{\partial P}{\partial y} + \frac{1}{\rho_w} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) - \frac{g}{\rho_w} \frac{\partial}{\partial y} \left(\int_z^{z_s} \rho' dz \right) \end{aligned} \quad (3)$$

$$P = \rho g(H + z_b - z) \quad (4)$$

傳輸方程

$$\frac{\partial \rho \Psi}{\partial t} + \frac{\partial}{\partial x_j} (\rho \Psi U_j) = \frac{\partial}{\partial x_j} (-\rho \overline{\phi' u'_j} + \Gamma \frac{\partial \Psi}{\partial x_j}) + S_\Psi \quad (5)$$

2.分層模式

變數 ϕ 之層積分平均可定義為

$$\frac{1}{h} \int_{z_k(x,y)}^{z_{k+1}(x,y)} \phi(x, y, z, t) dz = \phi_{k+1/2}$$

各層平均與層邊界變數示如圖 1，利用萊布尼茲積分法則(Leibnitz's rule)將控制方程式對層厚度積分平均，可得層平均連續、動量與傳輸方程

$$\begin{aligned} \frac{\partial}{\partial t} (hU)_L + \frac{\partial}{\partial x} (hUU)_L + \frac{\partial}{\partial y} (hVU)_L + (FU)_{t-b} \\ = \frac{\partial}{\partial x} (h\nu \frac{\partial U}{\partial x})_L + \frac{\partial}{\partial y} (h\nu \frac{\partial U}{\partial y})_L + (\frac{\tau_{zx}}{\rho})_{t-b} - [gh \frac{\partial(H + z_B - z)}{\partial x}]_L - [gh \frac{\partial(\int_z^{z_s} \rho' dz)}{\partial x}]_L \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{\partial}{\partial t} (hV)_L + \frac{\partial}{\partial x} (hUV)_L + \frac{\partial}{\partial y} (hVV)_L + (FV)_{t-b} \\ = \frac{\partial}{\partial x} (h\nu \frac{\partial V}{\partial x})_L + \frac{\partial}{\partial y} (h\nu \frac{\partial V}{\partial y})_L + (\frac{\tau_{zy}}{\rho})_{t-b} - [gh \frac{\partial(H + z_B - z)}{\partial y}]_L - [gh \frac{\partial(\int_z^{z_s} \rho' dz)}{\partial y}]_L \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{\partial}{\partial t} (\rho h \overline{\Psi})_L + \frac{\partial}{\partial x} (\rho h \overline{U \Psi})_L + \frac{\partial}{\partial y} (\rho h \overline{V \Psi})_L + (\rho F \Psi)_{t-b} \\ = \frac{\partial}{\partial x} (h D_{xx})_L + \frac{\partial}{\partial y} (h D_{yy})_L + (f)_{t-b} + \overline{S_\Psi} \end{aligned} \quad (8)$$

3.形狀函數

對水平方向之每一水柱，變數在垂直方向分佈可以上、下邊界與層平均三點作二

次函數(quadratic function)近似，則可將層邊界上之一次導數表示為

$$\left. \frac{\partial \phi}{\partial z} \right|_{z_{k+1}} = \frac{1}{\Delta z} (4\phi_{k+1} + 2\phi_k - 6\phi_{k+1/2}) \quad (9)$$

$$\left. \frac{\partial \phi}{\partial z} \right|_{z_k} = \frac{1}{\Delta z} (-2\phi_{k+1} - 4\phi_k + 6\phi_{k+1/2}) \quad (10)$$

將(9)式與(10)式代入(6)~(8)式層平均方程式，則方程式可改寫為

$$\begin{aligned} & \frac{\partial}{\partial t} (hu_{k+1/2}) + \frac{\partial}{\partial x} (hu_{k+1/2} u_{k+1/2}) + \frac{\partial}{\partial y} (hv_{k+1/2} u_{k+1/2}) + (Fu)_{k+1} - (Fu)_k \\ &= -[gh \frac{\partial (H + z_B - z)}{\partial x}]_{k+1/2} + \frac{\partial}{\partial x} (hv_h \frac{\partial u_{k+1/2}}{\partial x}) + \frac{\partial}{\partial y} (hv_h \frac{\partial u_{k+1/2}}{\partial y}) \\ &+ v_v (6u_{k+1} - 12u_{k+1/2} + 6u_k) - [gh \frac{\partial (\int_z^{z_s} \rho' dz)}{\partial x}]_{k+1/2} \end{aligned} \quad (11)$$

$$\begin{aligned} & \frac{\partial}{\partial t} (hv_{k+1/2}) + \frac{\partial}{\partial x} (hu_{k+1/2} v_{k+1/2}) + \frac{\partial}{\partial y} (hv_{k+1/2} v_{k+1/2}) + (Fv)_{k+1} - (Fv)_k \\ &= -[gh \frac{\partial (H + z_B - z)}{\partial y}]_{k+1/2} + \frac{\partial}{\partial x} (hv_h \frac{\partial v_{k+1/2}}{\partial x}) + \frac{\partial}{\partial y} (hv_h \frac{\partial v_{k+1/2}}{\partial y}) \\ &+ v_v (6v_{k+1} - 12v_{k+1/2} + 6v_k) - [gh \frac{\partial (\int_z^{z_s} \rho' dz)}{\partial y}]_{k+1/2} \end{aligned} \quad (12)$$

$$\begin{aligned} & \frac{\partial}{\partial t} (h\Psi_{k+1/2}) + \frac{\partial}{\partial x} (hu_{k+1/2} \Psi_{k+1/2}) + \frac{\partial}{\partial y} (hv_{k+1/2} \Psi_{k+1/2}) + (F\Psi)_{k+1} - (F\Psi)_k \\ &= \frac{\partial}{\partial x} (h\Gamma_h \frac{\partial \Psi_{k+1/2}}{\partial x}) + \frac{\partial}{\partial y} (h\Gamma_h \frac{\partial \Psi_{k+1/2}}{\partial y}) + \Gamma_v (6\Psi_{k+1} - 12\Psi_{k+1/2} + 6\Psi_k) \end{aligned} \quad (13)$$

並應用形狀函數，使層介面間保持濃度連續與擴散通量連續，層介面邊界條件包含剪力或通量連續

$$\Gamma_{k-1} \left. \frac{\partial \Phi}{\partial z} \right|_{k-1}^t = \Gamma_k \left. \frac{\partial \Phi}{\partial z} \right|_k^b \quad (14)$$

可得層介面剪力或濃度梯度連續條件式

$$2\Phi_{k-1} - 6\Phi_{k-\frac{1}{2}} + 4(1+r_k)\Phi_k - 6r_k\Phi_{k+\frac{1}{2}} + 2r_k\Phi_{k+1} = 0 \quad (15)$$

其中， Γ 為渦度滯性或擴散係數， r_k 則為相鄰兩層之渦度滯性或擴散係數比。

流場與傳輸方程之求解採非耦合演算法，經由上述步驟解得流場後，在將已知流場代入傳輸方程求解污染質濃度。最後，利用半隱式(implicit)法於z方向分

別求解u、v方程組，可得水平方向速度於垂直方向之分布，代入非自由液面層平均連續方程式可得垂直方向速度w

$$W_k = U_{2k-1} \frac{\partial z_{2k-1}}{\partial x} + V_{2k-1} \frac{\partial z_{2k-1}}{\partial y} - \sum_{m=1}^{k-1} \left[\frac{\partial}{\partial x} (h_m U_m) + \frac{\partial}{\partial y} (h_m V_m) \right] \quad k=2, \dots, N_k \quad (16)$$

自由液面層水位變化可表示為

$$\frac{H^{n+1} - H^n}{\Delta t} + \sum_{m=1}^{N_k} \left[\frac{\partial}{\partial x} (h_m U_m) + \frac{\partial}{\partial y} (h_m V_m) \right] = 0 \quad (17)$$

4. 渦動滯性(eddy viscosity)與邊界條件

由於水深、流速、流量與底床糙度係數乃相互作用影響，渦動滯性係流場函數而非流體性質，Elder(1959)提出垂直延散係數涵式

$$v = \alpha H u_* \quad (18)$$

本研究基於此涵式，並引入曼寧公式得下列渦動滯性概算式

$$\alpha = \frac{u_*}{3U} = \frac{\sqrt{gn}}{3H^{1/6}} \quad (19)$$

另，自由液面與底床之邊界濃度假設為零，且不考慮底床與水體間之泥砂交換(淤積與再懸浮)，上游入流濃度邊界可採用觀測流量-濃度率定曲線，下游邊界可假設濃度梯度為零。

5. 數值方法

(11)~(13)式中對流(convective)項採用上風(upwind)法

$$\phi_e = \begin{cases} \phi_P & \text{if } (\vec{v} \cdot \vec{n})_e > 0 \\ \phi_E & \text{if } (\vec{v} \cdot \vec{n})_e < 0 \end{cases} \quad (20)$$

擴散(diffusive)項採用中央(central)差分法

$$(h v_h \frac{\partial \phi}{\partial x} \Delta y)_e \cong (h v_h \Delta y)_e \frac{1}{\Delta x} (\phi_E - \phi_P) \quad (21)$$

將(20)與(21)式代入(11)~(13)式，以交錯格點有限體積法，變數配置如圖2，對層平均控制方程式(11)~(13)式對控制體積進行積分

$$\begin{aligned}
& [(\rho h \overline{\Phi \Delta x \Delta y})^n - (\rho h \overline{\Phi \Delta x \Delta y})^0] / \Delta t + \\
& (\rho h \overline{G_1 \Phi \Delta y})_{e-w} + (\rho h \overline{G_2 \Phi \Delta x})_{n-s} + (\rho F_R \Delta x \Delta y) \\
= & (h \overline{\Gamma_\Phi \Phi_x \Delta y})_{e-w} + (h \overline{\Gamma_\Phi \Phi_y \Delta x})_{n-s} + \\
& (\Gamma_\Phi \overline{\Phi_z \Delta x \Delta y})_{t-b} + \int_V S_\Phi dx dy
\end{aligned} \tag{22}$$

可得有限體積差分通式

$$\begin{aligned}
A_P \phi_P = & A_E \phi_E + A_W \phi_W + A_N \phi_N + A_S \phi_S \\
& + A_t \phi_t + A_b \phi_b + S^\phi
\end{aligned} \tag{23}$$

其中， ϕ 對應 x 、 y 動量方程式與傳輸方程式分別為 u 、 v 與 ψ ，而 A_P 、 A_N 、 A_E 、 A_S 、 A_W 、 A_t 、 A_b 、等係數可由(22)式計算而得。

四、結果與討論

1. 風剪引發之垂直環流場

在靜止的水庫或湖泊中，水面受到風吹力帶動，於表層湖水形成一順著風向之水流，水流流到岸邊時受到岸壁阻攔造成水面堆升，當水面堆升至一定程度時，底層湖水因水平壓力不平衡而產生與表層反向之水流，因而形成垂直環流場。對於單方向剪切、完全發展之流況，為確保質量守恆，流速對水深積分必須等於零，此時剪力分布為線性分布，Heaps(1984) 提出層流定常狀態下，垂直環流流速剖面如下

$$u(z) = \left(\frac{\tau_s}{4\rho v_v} \right) \frac{z(3z - 2H)}{H} \tag{24}$$

其中， u 流速， H 水深， v_v 垂直滯性； τ_s 為水面風剪， ρ 密度。

本測試案例使用長 400m、寬 160 m、水深 2 m 之矩形封閉水槽，由水體初始為靜止狀態，由 $t = 0s$ 到 $t = 10s$ 時刻水面受到風剪作用，風剪力由 0 N/m^2 增加到 0.5 N/m^2 ，當 $t > 10s$ 之候，水面風剪力維持在 0.5 N/m^2 ，模式使用參數設定如下： $\Delta x = 20m$ 、 $\Delta y = 8m$ 、 $\rho = 1000 \text{ kg/m}^3$ 、 $v_v = 0.015 \text{ m}^2/s$ 。

(1) 層數之影響

在層流狀態下，若黏滯係數為定值可使流場成二次分布，測試六組案例層數分別為5、10、15、20、25與30層，流速剖面比較如圖3。如同預期，最大流速發生於水面上，底部則產生回流，靜止點分別發生於底床及水面下三分之一水深處，另由圖中可發現，當層數由5層增至10層時，流速剖面變化最大，而當層數超過20時，流速剖面差異已可忽略。

(2)與解析解比較

由於層數超過20層後模擬結果差異不大，因此將20層之流速剖面模擬成果與解析解作比較如圖4。由圖中可看出流速剖面與解析解相近，僅稍微大些，而零流速點與解析解一致，發生於水面下三分之一水深處，而水面堆升之差異則較流速剖面明顯，就其原因應為解析解忽略動量所致。

(3)滯性分布型態對流速剖面之影響

天然流場中之渦動滯性為流場之函數，並非均勻分布，接著藉由滯性分布型態的改變對流速剖面之影響來觀察二次內插函數對求解天然流場流速剖面之能力。Elder(1959)提出渠道中渦動滯性沿水深方向呈二次分佈，且經由Jobson and Sayre (1970) 由實驗觀察證實，本測試案例滯性分佈使用均勻與二次分佈兩種型態，滯性分佈剖面與其相對之流速剖面模擬結果如圖5，結果發現，二次滯性分佈模擬所得流速剖面於接近底床與液面處較均勻滯性分佈有較大之速度梯度，而靠近中間水深處則流速梯度較均勻滯性平緩，此結果與符合物理現象。

2.直線渠道

由於風吹引起之環流場並無淨流量之問題，滯性大小並不會影響水深大小，在天然河川或渠道之一維或水深平均二維模式中，河床糙度對水深之影響由模式之底床剪應力即可反映，但在三維模式中，水深不單由底床剪應力大小決定，水體中滯性分佈型態亦會影響水深。

假設一矩形渠道長5km、寬100m、糙度係數0.035、底床坡降1/2000，通過單寬流量每公尺寬3.987cms，正常水深3.0m， α 值經(20)式概算為0.0304。上游邊界速度採用使用邊界入流量平均分配於該迭代之通水斷面，下游邊界則採用率定曲線，亦即速度之流向梯度為零，底床採不滑移邊界，側壁則採滑移邊界。

(1) 渦動滯性概算式之驗證

起始條件採用正常水深3.0m與均勻流速分佈1.329m/s，計算至收斂之流速剖面沿河道流量發展比較如圖6。藉助於層邊界速度與剪力連續條件之幫助，速度剖面於x=500m處即已接近二次曲線，再往下游處前進則有少許修正，此案例雖然為顯現河道入口端該有之發展段，究其原因在於本模式雖考慮渦動係數為流量、水深、糙度之函式，但仍未考慮垂直速度梯度對渦動係數之貢獻，因此，速度剖面雖未展現發展段特性，但仍符合模式之假設。

(2) 滯性分佈型態對河道流速剖面之影響

本模式之流速剖面模擬結果呈現二次剖面而非一般熟悉之對數剖面，其原因為渦動滯性沿水深方向為均勻分布，但因渦動滯性為流場之函數，必須由模式即時依流場自動計算，無法由人工設定，但未展現本模式之對不同流速剖面型態之模擬能力，本測試案例採用三種滯性分佈型態，示如圖7並表示如下：

$$\text{Type I: } \alpha = \frac{V_v}{Hu_*} = 0.0304$$

$$\text{Type II: } \alpha = 0.0304[0.5 + 4(\frac{z}{H})(1 - \frac{z}{H})]$$

$$\text{Type III: } \alpha = \kappa(\frac{z}{H})(1 - \frac{z}{H}) \quad , \kappa=0.41$$

其中，type I 由方程式(20)計算而得，type III 則為 Elder(1959)提出之二次分佈型態，type II 則為任意分布型態。三種滯性分佈型態之流速剖面模擬結果如圖 8 所示，圖中可看出流場依不同滯性分佈型態特性呈現其相對應之流速剖面形狀，其中 type I 流速剖面呈拋物線型態，type III 流速剖面則呈對數函數型態，type II 流速剖面則介於兩者之間，比較可發現本研究提出之二次內插函數可以分段之二次函數型態模擬各種型態之流速剖面，但必須注意，滯性分佈型態除影響流速剖面外，亦影響水深大小。

(3) 底床邊界對河道流速剖面之影響

另，從上述測試案例可看出本模式對各種流速分布型態之彈性，但在底床附近之流速梯度仍不夠大，與一般河川實測流速剖面仍有明顯之差異，深究其原因發現，本模式使用二次內插函數雖然具有剪力連續與收斂快速之特性，但天然河川底床之邊壁效應(wall effect)仍未納入考慮，因此底床附近之流速變化仍不夠顯

著；基於邊壁效應之概念，由於邊壁效應影響範圍厚度不大，若將底床之不滑移邊界改成水深平均速度之一定百分比，則可用來近似邊壁效應，圖 9 為不同百分比對無因次流速剖面之影響並與 Naot and Rodi (1982)之水槽試驗量測剖面比較，從圖中可看出當底床速度設定為水深平均流速之 0.5 倍時，模擬所得流速剖面已相當貼近實驗量測所得之流速剖面。

五、成果自評

本研究已完成水理模式於之建置與測試，並整理一篇英文論文“A layer-integrated approach for shallow water free surface flow computation”，投稿於SCI期刊*Communications in Numerical Methods in Engineering*，本文已正式接受刊登，電子全文預覽已刊登於該期刊網頁，紙本付梓中；而密度流子模式仍有部分問題尚待克服，目前尚未有具體成果。

參考文獻

1. 俞維昇(1991)，水庫沉滓運動特性之研究，國立台灣大學土木工程學研究所博士論文。
2. 經濟部水利處水利規劃試驗所(1999)，阿公店水庫防淤操作模型試驗及檢討計畫報告。
3. 國立交通大學防災工程研究中心(2003)，水庫沉滓運移模式研究與應用(2/2)——以石門水庫為例，經濟部水利署委託研究報告。
4. Ashan, A.K.M.Q., Blumberg, A.F., (1999), "Three-dimensional hydrothermal model of Onon- daga lake, New York", *J. Hydraulic Eng.*, ASCE, 125(6), 912-923.
5. Casulli, V., Cattani, E. (1994), "Stability, accuracy and efficiency of a semi-implicit method for three dimensional shallow water flow", *Comp. & Math. with App.*, 27(4), 99-112.
6. Cesare, G.D., Schleiss, A. (2001), and Hermann, F., "Impact of Turbidity Currents on Reservoir Sedimentation", *J. Hydraulic Eng.*, 127(1), 6-16.

7. Choi, S.U. (1998), "Layer-averaged modeling of two dimensional turbidity currents with a dissipative Galerkin finite element method, Part I: Formulation and application example", *J. Hydraulic Res.*, 36(3), 339-362.
8. Choi, S.U. (1999), "Layer-averaged modeling of two dimensional turbidity currents with a dissipative Galerkin finite element method, Part II: Sensitivity analysis and experimental verification", *J. Hydraulic Res.*, 37(2), 257-271.
9. Elder, J.W. (1959), "The dispersion of marked fluid in turbulent shear flow", *J. Fluid Mech.*, 5, 544-560.
10. Gessler, D., Hall, B., Spasojevic, M., Holly, F., Pourtaheri, H. and Raphael, N. (1999), "Application of 3D mobile bed, Hydrodynamic model", *J. Hydraulic Eng.*, ASCE, 125(7), 737-749.
11. Gross, E.S., Koseff, J.R., Monismith, S.G. (1999), "Three-dimensional salinity simulations of south San Francisco Bay", *J. Hydraulic Eng.*, ASCE, 125(11), 1199-1209.
12. Heaps, N.S. (1984), "Vertical structure of current in homogeneous stratified waters", *Hydrodynamics of lakes*, K. Hutter ed., 154-202.
13. Jobson, H.E. and Sayre, W.E. (1970), "Vertical transfer in open channel flow", *J. Hydraulic Div.*, 96, 703-724.
14. Naot, D. and Rodi, W. (1982), "Calculation of secondary currents in channel flow", *J. Hydraulic Div.*, 108, 948-968.
15. Neary, V.S. (1995), *Numerical modeling of diversion flows*, Ph.D. Thesis, Univ. of Iowa.
16. Neary, V.S., Sotiropoulos, F. and Odgaard, A.J. (1999), "Three-dimensional numerical model of lateral intake inflows", *J. Hydraulic Eng.*, ASCE, 125(2), 126-140.

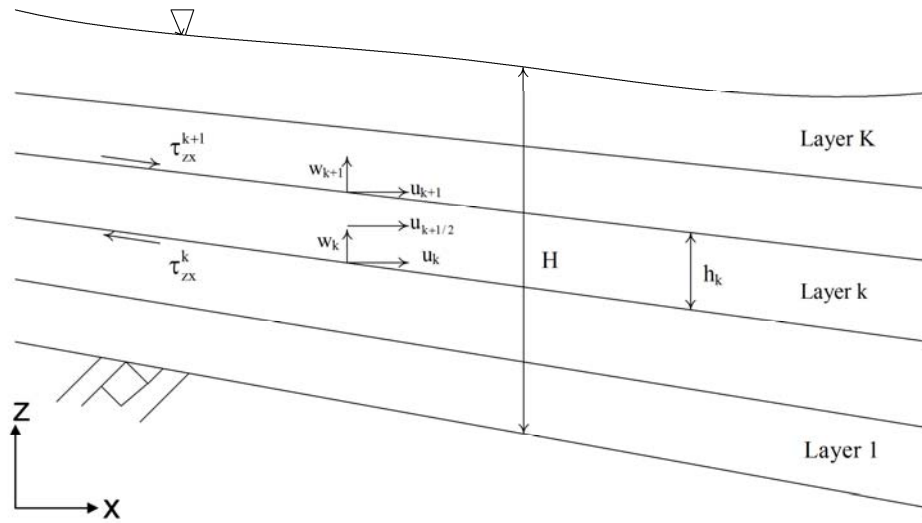


圖 1 不同滯性分佈對流速剖面之影響(均勻 vs 拋物線型)

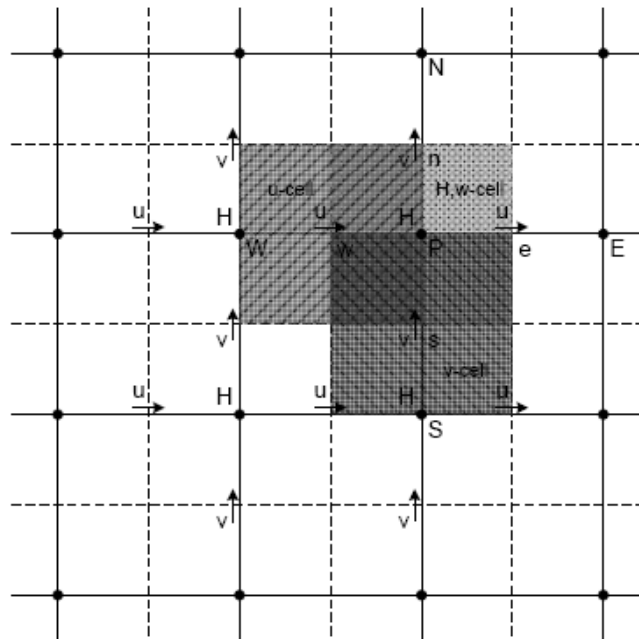


圖 2 交錯格點有限體積法變數配置圖

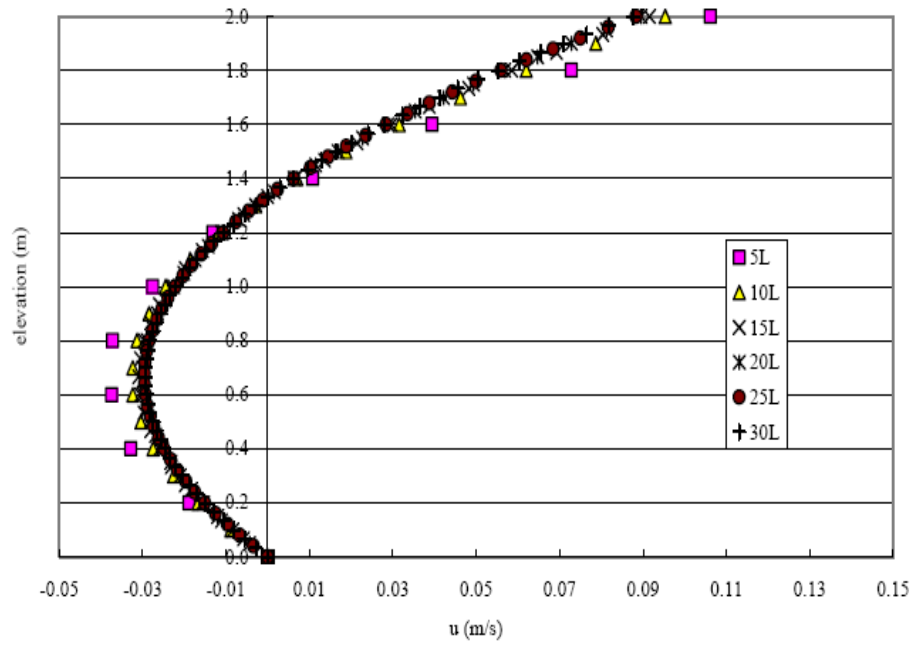


圖 3 層數對流速剖面之影響

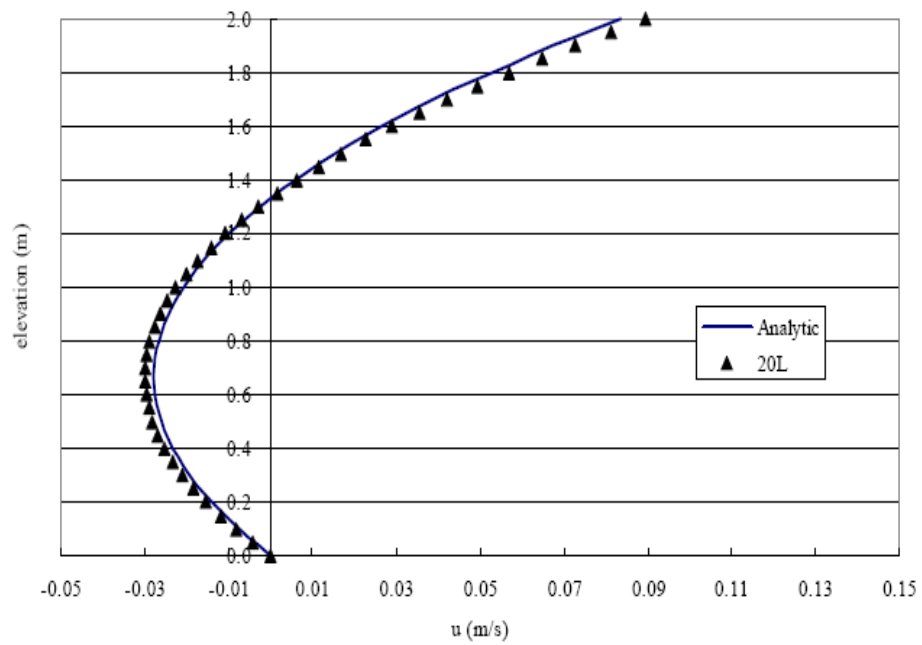


圖4 使用20層之流速剖面與解析解比較

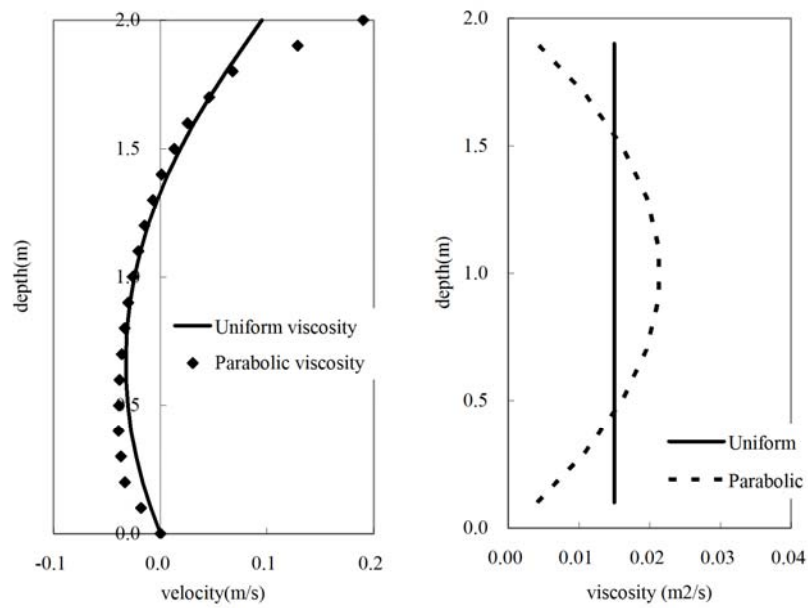


圖 5 不同滯性分佈對流速剖面之影響(均勻 vs 拋物線型)

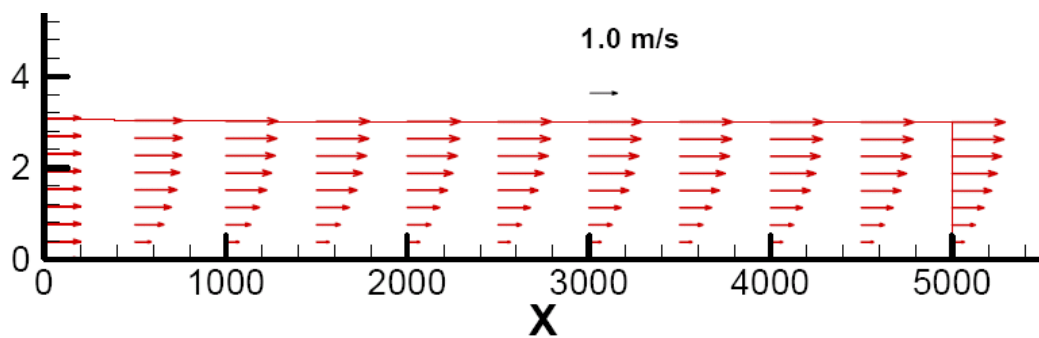


圖 6 均勻滯性分佈所得流速剖面沿河道流向之發展

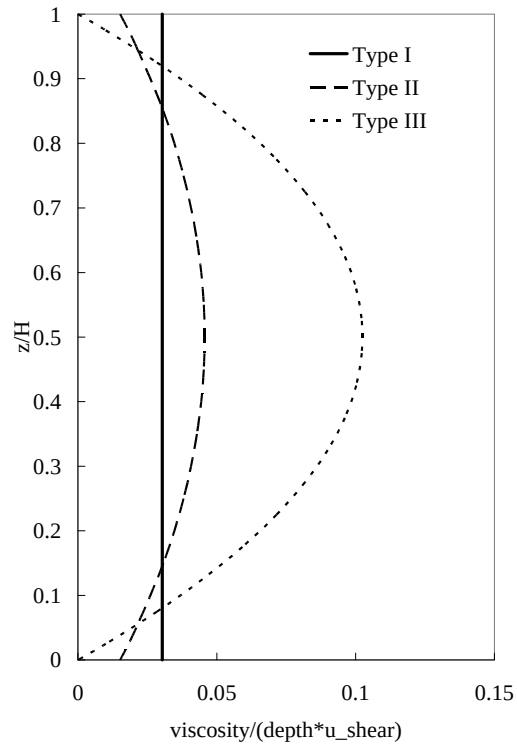


圖 7 三種不同滯性分佈型態

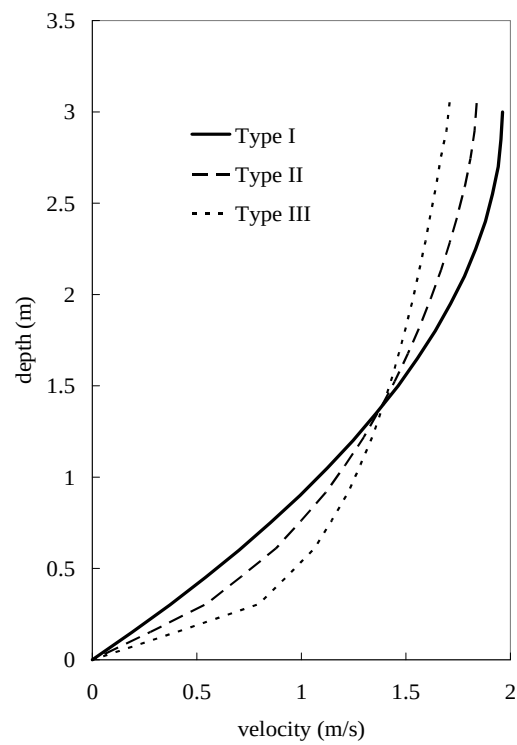


圖 8 不同滯性分佈對流速剖面之影響(均勻 vs 拋物線)

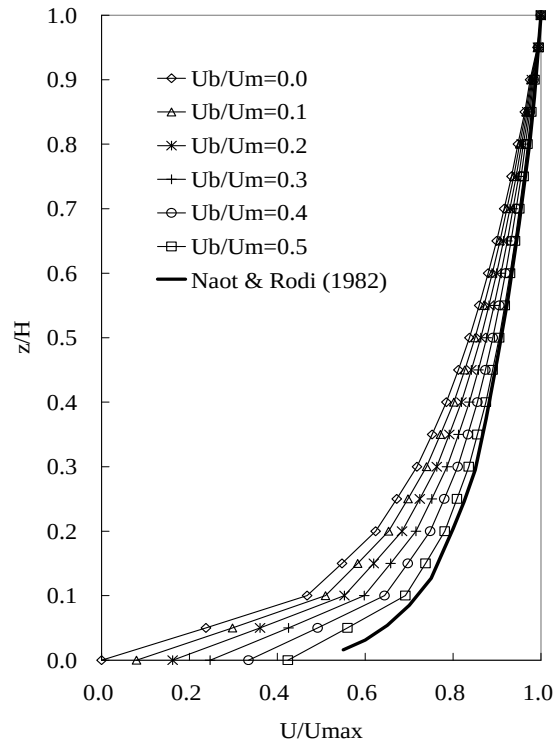


圖 9 不同底床邊界速度分佈對流速剖面之影響

A layer-integrated approach for shallow water free-surface flow computation

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SUMMARY

A novel semi-three-dimensional (semi-3D) layer-integrated approach was proposed in this study for the shallow water free-surface flow computation. A quadratic shape function proposed to approximate the velocity distribution in the layer ensures the continuity of velocities and shear stresses at interfaces. To discretize the governing equations, the finite volume formulation with staggered grid is used. As the two-dimensional (2D) sub-model for locating the free surface is not needed in this approach, the computational time consumption has been dramatically reduced. The model was verified through the wind-induced circulation in a closed basin by comparing the velocity profiles to the analytical solution. The formation of the velocity profiles due to change of viscosity distribution and the evolution process of water surface slope were also investigated. Further, the eddy viscosity is estimated by a function related to discharge, water depth, and Manning's n under the assumption of parabolic distribution of longitudinal velocity along the vertical direction for open channel flows. Two designed hypothetical cases were conducted to examine the proposed eddy viscosity relation and to demonstrate the capabilities of the proposed model. Copyright © 2007 John Wiley & Sons, Ltd.

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KEY WORDS: layer integrated; quadratic; shape function

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1. INTRODUCTION

Over the past decades, two-dimensional (2D) depth-averaged numerical models have been widely used to simulate unsteady free-surface flows in open channel, reservoir, and coastal water. The depth-averaged approach is computationally efficient with trade-off of discarding depth-related information but incapable of understanding the mechanism of the natural phenomenon in which vertical variations of velocities are the crucial information, such as flows in channel bends, density currents in reservoirs, or seawater intrusions in river estuaries.

With the rapid increase in computer power in recent years, several three-dimensional (3D) numerical models solving the Reynolds-averaged equations have been developed to simulate flows in natural rivers. Demuren and Rodi [1] developed a 3D model using the 'rigid-lid' approach, which takes the free surface into account as a symmetry plane. Hervouet and Van Haren [2] proposed a 3D Navier–Stokes solver in a domain with a moving boundary method, incorporating a 2D shallow water equations solver assuming hydrostatic pressure. Because of the pressure field calculated by 3D Navier–Stokes solver that may locally differ from the hydrostatic pressure, a procedure is employed to correct the free surface given by 2D shallow water equations according to the 3D pressure field. Meselhe and Sotiropoulos [3] presented a new code solving Reynold-averaged Navier–Stokes equations, in which the free-surface elevation is determined by allowing the computational mesh to deform during the iterative solution procedure so that the proper kinematic and dynamic conditions are satisfied at convergence. The model developed by Meselhe and Sotiropoulos [3] will be severely computationally time demanding for unsteady simulations because the mesh has to be regenerated at each iteration. As volume of fluid (VOF) [4] method has been widely used to simulate free-surface flow in computational fluid dynamics (CFD) and chemical engineering, the CFX-4 [5], reserving some modules can be extended by users for their own modeling purposes, is one of the most popular commercial CFD packages. Faure *et al.* [6] embedded the treatment of inlet/outlet boundary conditions in using the CFX-4 to simulate flow with free surface. This treatment is indispensable to the application of common 3D CFD codes *in situ* due to the lack of field velocity investigations to set up the boundary conditions.

As the fully 3D numerical models mentioned above are severely computationally time demanding and hard to be applied to the field problem due to its large demand of boundary conditions not being investigated *in situ*, the layer-integrated concept with hydrostatic pressure assumption was proposed to simplify the 3D shallow water free-surface flow computation and has been widely used in coastal water simulations. Li and Zhan [7] presented a multi-level finite element model with vertical integration for each water layer and applied to the tidal current flow in Tokyo Bay. Kim and Lee [8] applied the finite difference model BACHOM-3 based on the multi-level concept to simulate the tidal and wind-induced current in Suyoung Bay, Korea. Shankar *et al.* [9] developed a finite difference model using the same concept to simulate flow patterns for the sudden contraction and expansion flows and show reasonable flow characteristics downstream of an opening. Li and Gu [10] applied the layer-integrated concept to the mass exchange and tidal flushing problem in the harbor. The multi-level or multi-layer approach allows free surface on top layer to vary with time or iteration. The mesh does not need to be regenerated in each iteration. It significantly reduces the computational time consumption. In large-scale coastal simulation, this approach discards some local phenomena but gains practicability. Lin and Falconer [11] extended the use of layer-integrated approach to the modeling of tidal circulation with flooding and drying in Hummer Estuary along the northeast coast of England. Owing to the lack of mechanism on balancing the water depth

due to change of flow discharges, a 2D depth-averaged model is still needed to locate the water surface in their modeling.

In this paper, a semi-3D layer-integrated numerical model based on interface continuity of velocity and shear stress was proposed for the shallow water free-surface flow computation. A quadratic shape function, being applied as an auxiliary function for deriving the velocities distribution within a layer, is to assure the continuity of velocity and shear stress at interfaces. This quadratic shape function was examined through advection–diffusion equation by Tsai *et al.* [12]. The proposed model was verified through the wind-induced circulation in a closed basin by comparing the simulated velocity profiles to the analytical solution. The developing process of water surface and velocity profile was also investigated. Further, the flexibility of the proposed approach was tested through the formation of the velocity profiles due to change of viscosity distribution. As the turbulence model is not embedded, an estimation of eddy viscosity, related to discharge, water depth, and Manning’s roughness n under the assumption of uniform distribution of viscosity along the vertical direction, was also derived for open channel flows. Two designed hypothetical cases were conducted to examine the proposed eddy viscosity relation and to demonstrate the capabilities of the proposed model.

2. MATHEMATICAL MODEL

2.1. Governing equations

The governing equations of unsteady flows in open channels generally based on the 3D Navier–Stokes equations under hydrostatic pressure assumption can be expressed in the conservative form as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} + \frac{\partial(uu)}{\partial x} + \frac{\partial(vu)}{\partial y} + \frac{\partial(wu)}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) \quad (2)$$

$$\frac{\partial v}{\partial t} + \frac{\partial(uv)}{\partial x} + \frac{\partial(vv)}{\partial y} + \frac{\partial(wv)}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) \quad (3)$$

$$p = \rho g(H + z_B - z) \quad (4)$$

where t is the time; x, y, z are the Cartesian co-ordinates in the horizontal plane (x, y) and vertical axis (z); u, v, w are components of velocity in the x, y, z directions, respectively; p is the pressure; ρ is the density of water; g is the gravitational acceleration; and $\sigma_{xx}, \tau_{yx}, \tau_{zx}, \tau_{xy}, \sigma_{yy}, \tau_{zy}$ are components of the stress tensor in the x – z and y – z planes; H is the flow depth, and z_B is the elevation of channel bed, respectively.

2.2. Layer-integrated equations

Under the assumption of incompressible fluid and hydrostatic pressure, the layer-averaged equations can be obtained from integrating Navier–Stokes equations along the flow depth by layers. The layer interfaces are inclined in the longitudinal and horizontal in the transverse. A sketch of the layers

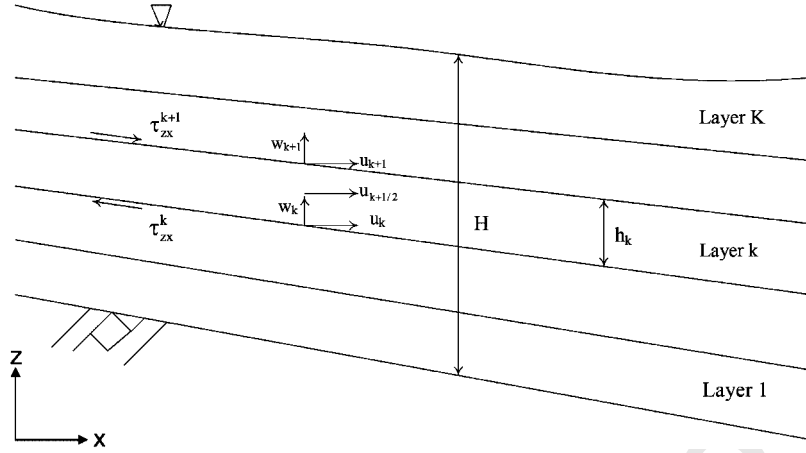


Figure 1. Schematic illustration of layer-integrated concept.

- 1 and the relative position of the variables in the x - z plane is illustrated in Figure 1. As illustrated,
 3 three layer types exist, including top layer, bottom layer, and internal layer. The thickness of top
 and bottom layers varies with the x, y co-ordinates to prescribe both the free surface and bottom
 5 topography, respectively. In contrast, the thickness of internal layers can be either uniform or
 non-uniform. Letting

$$\phi_{k+1/2} = \frac{1}{h} \int_{z_k(x,y)}^{z_{k+1}(x,y)} \phi(x, y, z, t) dz \quad (5)$$

- 7 where ϕ represents the physical variables such as u, v , and p , $\phi_{k+1/2}$ represents its layer-average of
 k th layer; z_{k+1} refers to the elevations of the layer interfaces between the $(k+1)$ th and k th layers,
 9 and z_k refers to that between k th and $(k-1)$ th layers, respectively; K is the total number of layers;
 and h is the thickness of the layer. The governing equations for mass and momentum are first
 11 integrated over the k th layer, where $k = 1, 2, 3, \dots, K$. Applying Leibniz's rule to the layer-integral
 and neglecting high-order terms, the k th layer-averaged momentum equations become

$$\begin{aligned} & \frac{\partial}{\partial t}(hu_{k+1/2}) + \frac{\partial}{\partial x}(hu_{k+1/2}u_{k+1/2}) + \frac{\partial}{\partial y}(hv_{k+1/2}u_{k+1/2}) + (Fu)_{k+1} - (Fu)_k \\ &= - \left[gh \frac{\partial(H+z_B-z)}{\partial x} \right]_{k+1/2} + \frac{\partial}{\partial x} \left(hv_h \frac{\partial u_{k+1/2}}{\partial x} \right) + \frac{\partial}{\partial y} \left(hv_h \frac{\partial u_{k+1/2}}{\partial y} \right) + \left(\frac{\tau_{zx}}{\rho} \right)_{k+1} - \left(\frac{\tau_{zx}}{\rho} \right)_k \end{aligned} \quad (6)$$

$$\begin{aligned} & \frac{\partial}{\partial t}(hv_{k+1/2}) + \frac{\partial}{\partial x}(hu_{k+1/2}v_{k+1/2}) + \frac{\partial}{\partial y}(hv_{k+1/2}v_{k+1/2}) + (Fv)_{k+1} - (Fv)_k \\ &= - \left[gh \frac{\partial(H+z_B-z)}{\partial y} \right]_{k+1/2} + \frac{\partial}{\partial x} \left(hv_h \frac{\partial v_{k+1/2}}{\partial x} \right) + \frac{\partial}{\partial y} \left(hv_h \frac{\partial v_{k+1/2}}{\partial y} \right) + \left(\frac{\tau_{zy}}{\rho} \right)_{k+1} - \left(\frac{\tau_{zy}}{\rho} \right)_k \end{aligned} \quad (7)$$

- 1 where the subscript $(k+1/2)$ denotes the layer-averaged quantities; subscripts $(k+1)$ and k denote
 2 the quantities at top and bottom layer interfaces, respectively; h is the layer thickness; v_h is the
 3 horizontal eddy viscosity; and the layer across flux F can be expressed as

$$F_k = w_k - \left(u \frac{\partial z}{\partial x} \right)_k - \left(v \frac{\partial z}{\partial y} \right)_k \quad (8)$$

- 5 where w_k is the vertical velocity at interface between $(k-1)$ th and k th layers. Integration of
 continuity equations and application of impermeable bottom boundary condition gives vertical
 7 velocity at the interface between k th and $(k+1)$ th layer:

$$w_{k+1} = u \frac{\partial z}{\partial x} \Big|_{k+1} + v \frac{\partial z}{\partial y} \Big|_{k+1} - \sum_{m=1}^k \left[\frac{\partial}{\partial x} (hu_{m+1/2}) + \frac{\partial}{\partial y} (hv_{m+1/2}) \right] \quad (9)$$

- 9 where m is the dummy index for summation. From Equations (6), (7), and (9), one can solve the
 velocity components u , v , and w for each layer. The depth variation will be solved only for the top
 11 layer from continuity equation. Re-applying Equation (9) with the kinematic boundary condition
 on free surface and integrating along the time step, the change of water depth, $\Delta H = H^{t+\Delta t} - H^t$,
 13 in a time step can be determined by

$$\frac{H^{t+\Delta t} - H^t}{\Delta t} + \sum_{m=1}^K \left[\frac{\partial}{\partial x} (hu_{m+1/2}) + \frac{\partial}{\partial y} (hv_{m+1/2}) \right] = 0 \quad (10)$$

- 15 where Δt is the time interval and K is the total number of layers.

However, there exist two more unknowns for momentum equation in the x and y directions,
 17 respectively, which are the shear stress, $(\tau/\rho)_k$, $(\tau/\rho)_{k+1}$ appearing at the top and bottom interfaces
 of each layer. To evaluate the shear stress in terms of the velocity gradient, one may need a function
 19 to be able to present the velocity variation in the layer. Obviously, three unknowns including the
 velocity at center, top, and bottom of the layer exist in one layer. A quadratic function might be
 21 a reasonable and suitable choice. The formulation of layer interface constraint relation integrated
 with a quadratic shape function is described in the following paragraph.

23 2.3. Formulation of layer interface constraint relation

The shear stress at interfaces which remained in the layer-integrated momentum equations (6)
 25 and (7) can be evaluated by transforming the vertical velocity gradient into a combination of the
 nearby components of velocity. The transformation can be conducted by applying a quadratic shape
 27 function. A thorough analysis for the use of quadratic shape function for the dispersion equation
 has been performed by Tsai *et al.* [12]. This quadratic shape function is applied as an auxiliary
 29 function either to improve the velocity resolution or to assure the continuity of velocity and shear
 stress at interfaces. The following shows the derivation of first derivatives at interfaces in each
 31 layer. By adopting the quadratic polynomial interpolation function in each layer, ϕ (can be u or v)
 in the interval $[z_k; z_{k+1}]$ can be represented as

$$\phi(z) = a + bz + cz^2 \quad (11)$$

1 Using the two nodal values of ϕ at the boundary points in a layer, namely,

$$\phi(z=0) = a = \phi_k \quad (12)$$

$$\phi(z=\Delta z) = a + b\Delta z + c\Delta z^2 = \phi_{k+1} \quad (13)$$

and the definition of $\phi_{k+1/2}$, i.e.

$$\phi_{k+1/2} = \frac{1}{\Delta z} \int_0^{\Delta z} (a + bz + cz^2) dz \quad (14)$$

The coefficients a , b , and c in Equation (11) can be specified as follows:

$$a = \phi_k \quad (15)$$

$$b = \frac{1}{\Delta z} (-2\phi_{k+1} - 4\phi_k + 6\phi_{k+1/2}) \quad (16)$$

$$c = \frac{1}{\Delta z} (3\phi_{k+1} + 3\phi_k - 6\phi_{k+1/2}) \quad (17)$$

5 where the node z_k is taken as the origin and $\Delta z = z_{k+1} - z_k$. Thus, the velocity gradient at the interface of the k th layer can be expressed as top interface of the k th layer

$$\left. \frac{\partial \phi}{\partial z} \right|_{z_t, k} = \frac{1}{\Delta z} (4\phi_{k+1} + 2\phi_k - 6\phi_{k+1/2}) \quad (18)$$

bottom interface of the k th layer

$$\left. \frac{\partial \phi}{\partial z} \right|_{z_b, k} = \frac{1}{\Delta z} (-2\phi_{k+1} - 4\phi_k + 6\phi_{k+1/2}) \quad (19)$$

11 Substituting Equations (18) and (19) into Equations (6) and (7) produces the layer-averaged momentum equations

$$\begin{aligned} & \frac{\partial}{\partial t} (hu_{k+1/2}) + \frac{\partial}{\partial x} (hu_{k+1/2}u_{k+1/2}) + \frac{\partial}{\partial y} (hv_{k+1/2}u_{k+1/2}) + (Fu)_{k+1} - (Fu)_k \\ &= - \left[gh \frac{\partial (H + z_B - z)}{\partial x} \right]_{k+1/2} + \frac{\partial}{\partial x} \left(h v_h \frac{\partial u_{k+1/2}}{\partial x} \right) + \frac{\partial}{\partial y} \left(h v_h \frac{\partial u_{k+1/2}}{\partial y} \right) \\ &+ v_{v, k+1/2} (6u_{k+1} - 12u_{k+1/2} + 6u_k) \end{aligned} \quad (20)$$

$$\begin{aligned} & \frac{\partial}{\partial t} (hv_{k+1/2}) + \frac{\partial}{\partial x} (hu_{k+1/2}v_{k+1/2}) + \frac{\partial}{\partial y} (hv_{k+1/2}v_{k+1/2}) + (Fv)_{k+1} - (Fv)_k \\ &= - \left[gh \frac{\partial (H + z_B - z)}{\partial y} \right]_{k+1/2} + \frac{\partial}{\partial x} \left(h v_h \frac{\partial v_{k+1/2}}{\partial x} \right) + \frac{\partial}{\partial y} \left(h v_h \frac{\partial v_{k+1/2}}{\partial y} \right) \\ &+ v_{v, k+1/2} (6v_{k+1} - 12v_{k+1/2} + 6v_k) \end{aligned} \quad (21)$$

1 The quadratic velocity distribution in one layer introduces two more unknowns which are
 3 the velocities at the top and bottom layer interfaces. Therefore, one needs the internal interface
 5 boundary condition to close the system of equations for the flow field. By following that shear
 7 stress at the interface between two adjacent, $(k-1)$ th and k th layers should be continuous, the
 velocity gradient at the bottom of the k th layer multiplying the viscosity of k th layer should be
 equal to the velocity gradient at the top of the $(k-1)$ th layer multiplying the viscosity of the
 $(k-1)$ th layer:

$$(v_{v,k-1}) \left(\frac{\partial \phi}{\partial z} \Big|_{z_t,k-1} \right) = (v_{v,k}) \left(\frac{\partial \phi}{\partial z} \Big|_{z_b,k} \right) \quad (22)$$

9 Then the layer interface restriction equations are obtained

$$2u_{k-1} - 6u_{k-1/2} + 4(1+r_k)u_k - 6r_k u_{k+1/2} + 2r_k u_{k+1} = 0 \quad (23)$$

$$2v_{k-1} - 6v_{k-1/2} + 4(1+r_k)v_k - 6r_k v_{k+1/2} + 2r_k v_{k+1} = 0 \quad (24)$$

where $r_k = v_{v,k+1/2}/v_{v,k-1/2}$ is the ratio of the vertical viscosities of two adjacent layers.

11 2.4. Boundary conditions

The treatment of boundary conditions for the model is described as follows:

- 13 • The boundary condition for the lateral walls can be slip or non-slip.
- 15 • At the free surface, the wind shear and the kinematic free-surface boundary conditions were
 used. At the bottom boundary, the no slip condition and impermeable condition were used.
- 17 • At the layer interfaces, the continuity of velocity and shear stress is automatically satisfied
 by using Equations (23) and (24) under the assumption of quadratic distribution in the layer.
- 19 • At the upstream end, assuming the vertical and lateral velocities are zero, the longitudinal
 velocity assumed uniform along the depth can be set as the discharges per unit width divided
 by wetted area obtained from water depth in every iteration.
- 21 • At the downstream end, the normal depth is used and velocities were assumed homogeneous
 in the longitudinal direction, i.e. $\partial u / \partial x = 0$.

23 3. NUMERICAL ALGORITHM

3.1. Finite volume formulation

25 The control volume-based finite volume formulation implies that the integral conservation of mass
 27 and momentum is exactly satisfied over any group of control volume and, of course, over the whole
 computational domain. By following the control volume concept in a staggered grid (plane view)
 29 shown in Figure 2, as the momentum equation has been integrated along the layer thickness, the
 finite volume formulation was applied only in the plane coordinates. Integrating the layer-averaged
 31 momentum equations (20) and (21) along the control volume and time interval, the conservative
 form can be derived and given as

$$[(hu_{k+1/2}\Delta x\Delta y)^{t+\Delta t} - (hu_{k+1/2}\Delta x\Delta y)^t]/\Delta t$$

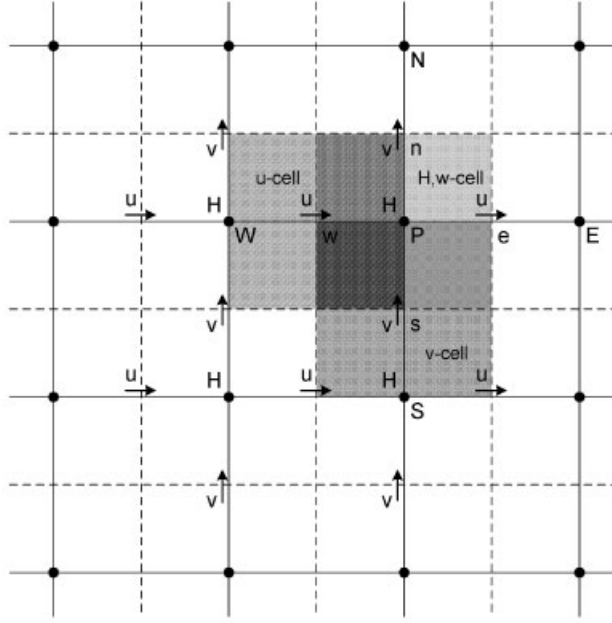


Figure 2. The location of variables and their control volumes in the x - y plane.

$$\begin{aligned}
 & + (hu_{k+1/2}u_{k+1/2}\Delta y)_{e-w} + (hv_{k+1/2}u_{k+1/2}\Delta x)_{n-s} + ((Fu)_{k+1} - (Fu)_k)\Delta x\Delta y \\
 & = \left(hv_h \left(\frac{\partial u_{k+1/2}}{\partial x} \right) \Delta y \right)_{e-w} + \left(hv_h \left(\frac{\partial u_{k+1/2}}{\partial y} \right) \Delta x \right)_{n-s} \\
 & + v_v(6u_{k+1} - 12u_{k+1/2} + 6u_k)\Delta x\Delta y - [gh(H + z_B - z)_{k+1/2}]_{e-w}\Delta y
 \end{aligned} \tag{25}$$

$$\begin{aligned}
 & [(hv_{k+1/2}\Delta x\Delta y)^{t+\Delta t} - (hv_{k+1/2}\Delta x\Delta y)^t]/\Delta t \\
 & + (hu_{k+1/2}v_{k+1/2}\Delta y)_{e-w} + (hv_{k+1/2}v_{k+1/2}\Delta x)_{n-s} + ((Fv)_{k+1} - (Fv)_k)\Delta x\Delta y \\
 & = \left(hv_h \left(\frac{\partial v_{k+1/2}}{\partial x} \right) \Delta y \right)_{e-w} + \left(hv_h \left(\frac{\partial v_{k+1/2}}{\partial y} \right) \Delta x \right)_{n-s} \\
 & + v_v(6v_{k+1} - 12v_{k+1/2} + 6v_k)\Delta x\Delta y - [gh(H + z_B - z)_{k+1/2}]_{n-s}\Delta x
 \end{aligned} \tag{26}$$

- 1 where the superscripts t and $t + \Delta t$ indicate time, and the subscripts (e, w) , (n, s) indicate the faces of the CV on the x - and y -axis, respectively.

3.2. Difference schemes

The approximation to the surface integral in Equations (25) and (26) requires the values of variables at locations other than computational nodes. The upwind difference scheme is applied to the convective flux due to its dependence on the flow direction:

$$\phi_e = \begin{cases} \phi_P & \text{if } (\mathbf{v} \cdot \mathbf{n})_e > 0 \\ \phi_E & \text{if } (\mathbf{v} \cdot \mathbf{n})_e < 0 \end{cases} \quad (27)$$

where variable ϕ is either u or v , subscripts e , P , E indicate locations of variables as shown in Figure 2. The upwind difference scheme has been widely used in the practice. The scheme is only 1st order accurate but can be easily extended to the 3D problem. The central difference scheme is applied to the diffusive flux due to its physical nature.

$$\left(h v_h \frac{\partial \phi}{\partial x} \Delta y \right)_e \cong (h v_h \Delta y)_e \frac{1}{\Delta x} (\phi_E - \phi_P) \quad (28)$$

where variable ϕ is either u or v , subscript P indicates the center of the CV, and subscript E indicates the center of the neighboring CVs as shown in Figure 2.

3.3. Algebraic equation system for the water column

By substituting Equations (27) and (28) into Equations (25) and (26) and then summing all the flux approximations and source terms, one can produce an algebraic equation that relates the variable value at the center of that CV to the values at the surrounding CVs.

$$\begin{aligned} & A_t^u u_{i,j,k+1}^{t+\Delta t} + A_P^u u_{i,j,k+1/2}^{t+\Delta t} + A_b^u u_{i,j,k}^{t+\Delta t} \\ & = A_W^u u_{i-1,j,k+1/2}^t + A_E^u u_{i+1,k+1/2}^t + A_N^u u_{i,j+1,k+1/2}^t + A_S^u u_{i,j-1,k+1/2}^t + S^u \end{aligned} \quad (29)$$

$$\begin{aligned} & A_t^v v_{i,j,k+1}^{t+\Delta t} + A_P^v v_{i,j,k+1/2}^{t+\Delta t} + A_b^v v_{i,j,k}^{t+\Delta t} \\ & = A_W^v v_{i-1,j,k+1/2}^t + A_E^v v_{i+1,k+1/2}^t + A_N^v v_{i,j+1,k+1/2}^t + A_S^v v_{i,j-1,k+1/2}^t + S^v \end{aligned} \quad (30)$$

where subscripts i , j , and k are the indices of the variable along the x , y , and z axes, and superscripts u and v indicate the coefficients for u and v momentum, respectively, the subscript P indicates the center of the CV, and subscripts N , E , W , S indicate the center of the neighboring CVs as shown in Figure 2, S is the summation of the remaining terms including the pressure gradient and previous time step contribution. The coefficients A_P^u , A_P^v , ... can be computed from Equations (25) and (26). Applying Equation (29) to the water column from bottom up to water surface layer by layer, K layer-averaged equations will be established for the K layers. Furthermore, taking boundary conditions on the bottom bed, water free surface, and $(K-1)$ layer interface constraint (23) into account, a series of x -momentum equations consisting of $(2K+1)$ equations can be established for a water column. Consequently the $(2K+1)$ unknowns can be solved by the $(2K+1)$ equations system. Similarly, a series of y -momentum equations consisting of $(2K+1)$ equations can be established for a water column from Equations (30) and (24) in the same way.

The layer-integrated equations and interface shear continuous conditions constitute the algebraic equation system of a water column. Consequently, the flow field can be solved implicitly in the vertical to satisfy the boundary conditions at the bed, interface, and the free surface simultaneously.

3.4. Computational procedure

A fully implicit time integration method is applied in the vertical direction and leads to a pentadiagonal system of equations with respect to the vertical direction. The vertical coupling of the discretized momentum equations is solved implicitly in the vertical direction by the Thomas algorithm and column by column iteratively implicitly in the horizontal direction. The main procedure can be summarized as follows and presented as a flowchart in Figure 3:

- (a) Set the initial depth and velocities.
- (b) Compute the eddy viscosity.
- (c) Solve the $(2K + 1)$ equations of x -momentum equation system ((23) and (29)) to obtain velocity u for each water column by using the Thomas algorithm.
- (d) Solve the $(2K + 1)$ equations of y -momentum equation system ((24) and (30)) to obtain velocity v for each water column by using the Thomas algorithm.
- (e) Repeat steps (c) and (d) if velocity field does not converge.
- (f) The vertical velocity w and water depth H are determined by Equations (9) and (10), respectively.
- (g) Repeat steps (b)–(f) until both velocity and depth converge, then march to the next time step.

3.5. Numerical stability limitations

The momentum equations were solved by using a combined implicit (vertical) and iteratively implicit (horizontal) schemes, but the continuity equation was solved explicitly. For the 3D shallow water equations, the horizontal velocity components are coupled in the vertical direction by the vertical advection and viscosity term. An explicit time integration of the vertical exchange in shallow areas would lead to the time step limitations [13]:

$$\Delta t \leq \frac{\Delta z^2}{2\nu_v} \quad (31)$$

and

$$\Delta t \leq \frac{\Delta z}{w} \quad (32)$$

4. ESTIMATION OF EDDY VISCOSITY FOR CHANNEL FLOWS

The law of flow resistance in open channel flow can be derived by balancing the retarding shear force at the boundary against the propulsive force acting in the flow direction. The water depth is determined by the interaction of the mean flow and bottom roughness in 1D or plane 2D simulations. But from the point of view of 3D modeling, the bottom roughness only acts on the bottom layer. Hence, the velocity profile is determined not only by the mean flow discharge and

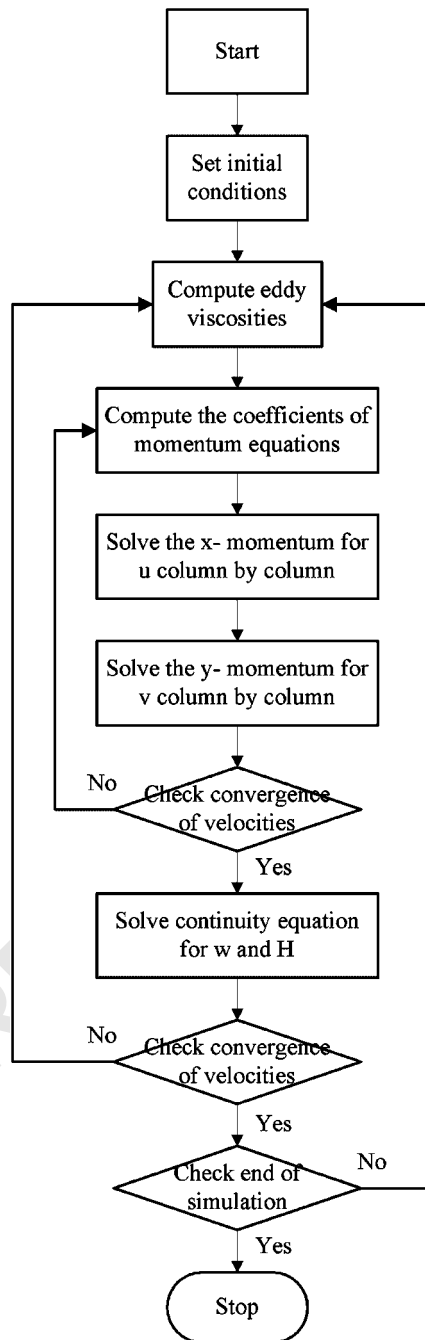


Figure 3. Flow chart of the computational procedure.

bottom friction but also by the eddy viscosity, an improper use of eddy viscosity may result in the incorrect water depth. However, the eddy viscosity is indeed a function of the flow field, not a fluid property. As the turbulent model has not been embedded in the proposed model at the present stage, an estimation of eddy viscosities is still required. The Elder's relation [14] between water depth, shear velocity, and eddy viscosity, $\nu = \alpha H u_*$, is frequently used in practice. Following the Elder's relation, the determination of α was derived as below.

In order to find a clear and simple relation for α in Elder's relation, uniform turbulent intensity (i.e. uniform eddy viscosity) along the depth is assumed and will lead to parabolic distribution of horizontal velocities in the vertical direction. Consider 3D Navier–Stokes equation in the mean flow direction

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \left(\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) + g_x \quad (33)$$

Assuming that flow is a steady 2D uniform flow and slope is small, the equation can be reduced to

$$\nu_v \frac{\partial^2 u}{\partial z^2} = -g S_0 \quad (34)$$

Integrating Equation (34) along the water depth becomes

$$\frac{\partial u}{\partial z} = -\frac{g S_0}{\nu_v} z + C_1$$

and applying bottom shear condition $\partial u / \partial z|_{z=0} = \tau_0 / \rho \nu_v$ yields $C_1 = \tau_0 / \rho \nu_v$, then the vertical gradient of velocity can be expressed as

$$\frac{\partial u}{\partial z} = -\frac{g S_0}{\nu_v} z + \frac{\tau_0}{\rho \nu_v} \quad (35)$$

By applying the surface condition

$$\left. \frac{\partial u}{\partial z} \right|_{z=H} = \frac{\tau_s}{\rho \nu_v}$$

Equation (35) becomes

$$\frac{\tau_s}{\rho \nu_v} = -\frac{g S_0}{\nu_v} H + \frac{\tau_0}{\rho \nu_v}$$

or

$$-g S_0 H + \frac{\tau_0}{\rho} = \frac{\tau_s}{\rho}$$

Therefore, bottom shear τ_0 can be determined by water depth, bottom slope, and surface (wind) shear

$$\tau_0 = \tau_s + \gamma H S_0 \quad (36)$$

Integrating Equation (35) along water depth again becomes

$$u = -\frac{g S_0}{2 \nu_v} z^2 + \frac{\tau_0}{\mu_v} z + C_2$$

- 1 Applying no slip condition at the bottom bed, C_2 will be diminished. Then the velocity profile
can be expressed as

$$3 \quad u = -\frac{gS_0}{2\nu_v}z^2 + \frac{\tau_0}{\mu_v}z \quad (37)$$

- 5 For mass conservation, integration of the velocity along water depth should be consistent to the
flow discharge, i.e.

$$\int_0^H u \, dz = q \quad (38)$$

- 7 Substituting Equation (37) into Equation (38), then

$$\frac{gS_0}{3\nu_v}H^3 + \frac{\tau_s}{2\mu_v}H^2 = q \quad (39)$$

- 9 The eddy viscosity cannot be determined by Equation (39) alone due to another remaining
unknown H . If there were no wind shear acting on water surface, the energy slope is equal to the
11 bottom slope because of uniformity in flow direction, then the kinematic eddy viscosity can be
expressed as

$$13 \quad \nu_v = \frac{gS_0}{3q}H^3 \quad (40)$$

or in the form of Elder's relation $\nu_v = \alpha H u_*$, in which,

$$15 \quad \alpha = \frac{u_*}{3U}$$

Consider Manning's equation in a wide rectangular channel, the depth-averaged velocity,

$$\begin{aligned} U &= \frac{1}{n} H^{2/3} S_0^{1/2} \\ &= \frac{H^{1/6}}{n\sqrt{g}} u_* \end{aligned} \quad (41)$$

- 17 Then coefficient in Elder's relation can also be expressed as

$$\alpha = \frac{u_*}{3U} = \frac{n\sqrt{g}}{3H^{1/6}} \quad (42)$$

- 19 Hence, the eddy viscosity can be determined by Manning' roughness n , water depth h , and
shear velocity u_* . Also, the bed slope in Equation (41) can be replaced by the friction slope
21 so that the non-uniform flow situation can also be satisfied. Consequently, Equation (42) could
be useful for the practical problems. Thus, α can be estimated easily by Manning' roughness n ,
23 water depth H *in situ* if no investigated data were available. The value of α is about 0.014–0.058
calculated by Equation (42) for natural river flows (about $n=0.02$ – 0.05 , $H=0.5$ – 10 m). This is
25 slightly smaller but of the same order compared with data investigated by Nezu and Rodi [15] due
to the assumption of uniform turbulent intensity.

5. MODEL VERIFICATION

5.1. Wind-induced flow

Wind-induced flow in a closed basin in which wind shear acting on the water surface results in a drift current in the direction it blows and a bottom return flow in the opposite direction is chosen to verify the model's accuracy and applicability. The analytical solution for steady laminar flow under no-slip condition, given by Heaps [16], for constant vertical eddy viscosity is

$$u(z) = \left(\frac{\tau_s}{4\rho\nu_v} \right) \frac{z(3z-2H)}{H} \quad (43)$$

where $u(z)$ is the velocity at any depth z ; H the water depth; ν_v the vertical eddy viscosity; τ_s the wind stress acting on the surface; and ρ the water density.

This numerical experiment is carried out to simulate a steady wind-induced flow in a uniform rectangular channel of length 400 m, width 160 m, and depth 2 m. The water initially at rest was subject to a uniform wind stress, increasing linearly from 0 N/m^2 at $t=0 \text{ s}$ to 0.5 N/m^2 at $t=10 \text{ s}$, and maintaining 0.5 N/m^2 at $t>10 \text{ s}$. The following parameters were adopted: $\Delta x=20 \text{ m}$, $\Delta y=8 \text{ m}$, $\rho=1000 \text{ kg/m}^3$, $\nu_v=0.015 \text{ m}^2/\text{s}$.

A constant viscosity leads to a parabolic velocity profile in the case of laminar flow. Effect of number of layers was also carried out with four cases of 5, 10, 20, and 30 layers. Comparison of model predictions at the middle length of the basin to the analytical solution is shown in Figure 4. The major improvement of predictions of velocity profile takes place between 5 and 10 layers, and no more significant difference takes place between predictions over 20 layers. Comparison of the u profile at middle length using 20 layers to the analytical solution is as expected. The upper layer currents are in the wind direction while the lower layer currents are in the opposite direction. It can be seen that the prediction has a slightly larger velocity. The predicted depth with zero

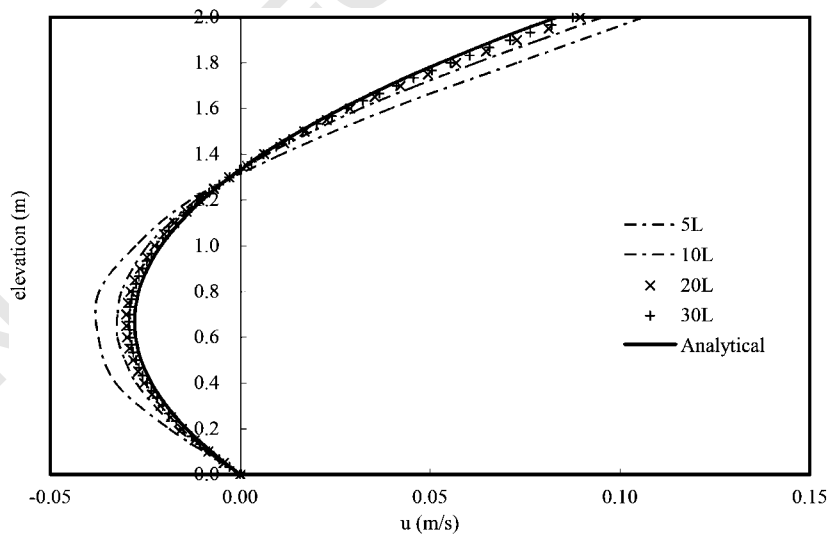


Figure 4. Effect of number of layers on velocity profile prediction.

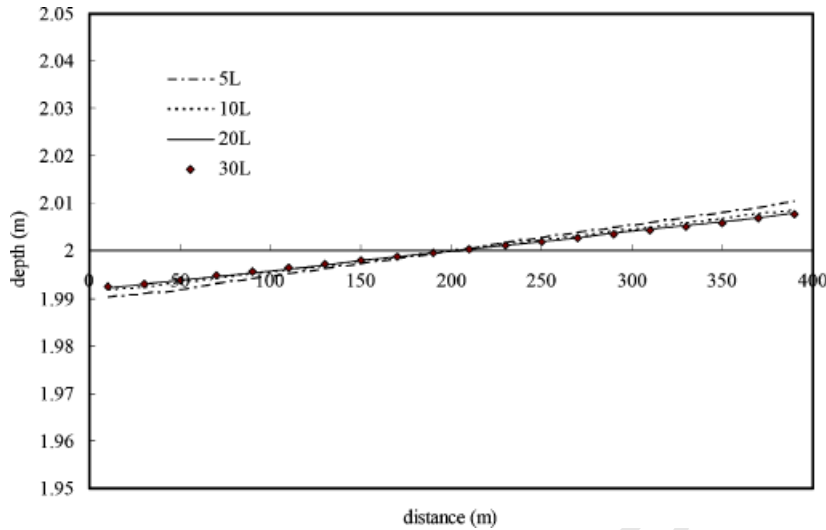


Figure 5. Effect of number of layers on water depth prediction.

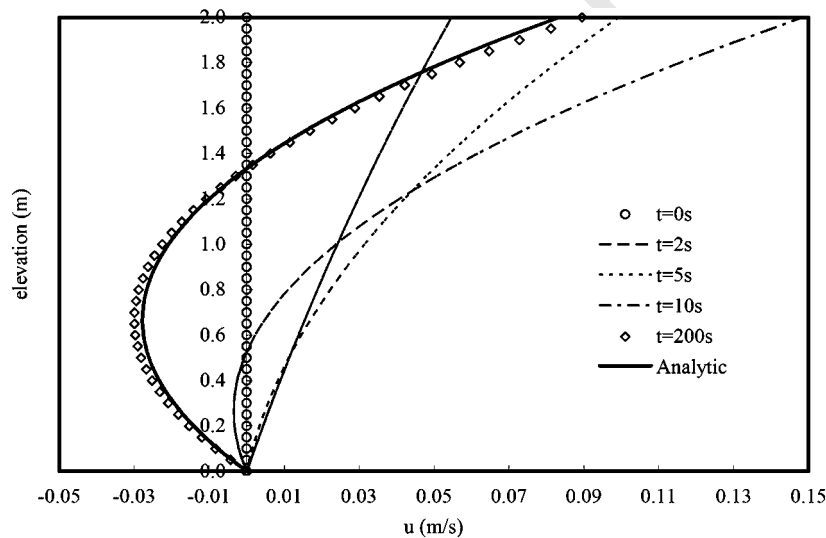


Figure 6. Variation of velocity in the wind-induced flow at middle length ($x=200\text{m}$).

1 velocity is at $\frac{1}{3}$ of water depth under the free surface. It is consistent with the analytical solution.
 2 The wind setup of the water surface has the same tendency as shown in Figure 5. With increase
 3 in the surface wind shear or decrease in the eddy viscosity, the maximum Reynolds number tested for limited stable condition can be raised up to around 300.

5 The evolution of the wind-induced circulation could be classified into two steps: (a) surface
 6 flow and water setup induced by wind shear and (b) bottom reverse flow induced by pres-
 7 sure difference. The transient process was also carried out to examine the physical mecha-

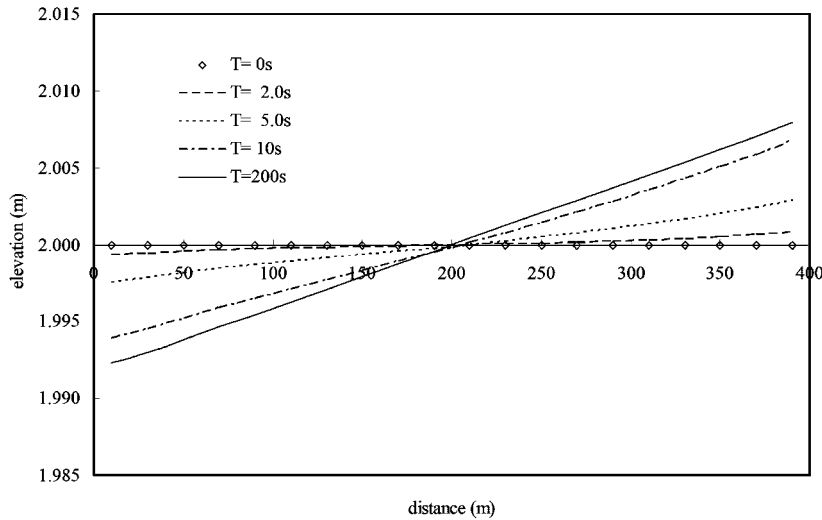


Figure 7. Variation of water depth in the wind-induced flow.

nism. Figures 6 and 7 show the simulated evolution of velocity profile at middle length and water surface setup varying with time with 20 layers, respectively. From Figure 6 it is obvious that the shear-induced surface flow in the beginning and the pressure-induced reverse flow at middle length were successfully predicted by the proposed model. These simulation results include surface setup and pressure-induced circulation, two significant components of the physical mechanism.

5.2. Effect of viscosity distribution on wind-induced flow

Owing to the distribution of vertical eddy viscosities existing in natural flows not being uniform, three more types of viscosity distribution were considered to further investigate the capability of the proposed quadratic shape function for solving free-surface flows. The parabolic law of eddy viscosity distribution is the most frequently assumed type in natural flows [15]. It has the maximum viscosity at middle depth and the minimum near the surface and the bottom. Figure 8 shows the simulated velocity profile with parabolic viscosity distribution compared to the one with uniform viscosity. The larger velocity gradient is found near both surface and bottom due to its smaller viscosity. Contrarily, the smaller velocity gradient is found at middle depth due to its larger viscosity. The simulated profile behaves in a well-consistent tendency with its physical mechanism. To further examine the flexibility of the auxiliary shape function, the trapezoidal distribution of viscosity was also conducted. The comparison of the velocity profiles due to change of the viscosity distribution is shown in Figure 9. For the case of trapezoidal type I with increased viscosity from top to bottom, the simulated velocity profile is well adapted as its gradient decreases from top to bottom by the quadratic shape function. By preserving the condition of zero net flow, the point of zero is risen a little compared with the uniform law. Figure 9 also shows that the quadratic shape function can also adapt the velocity profile to the trapezoidal type II of viscosity distribution as well.

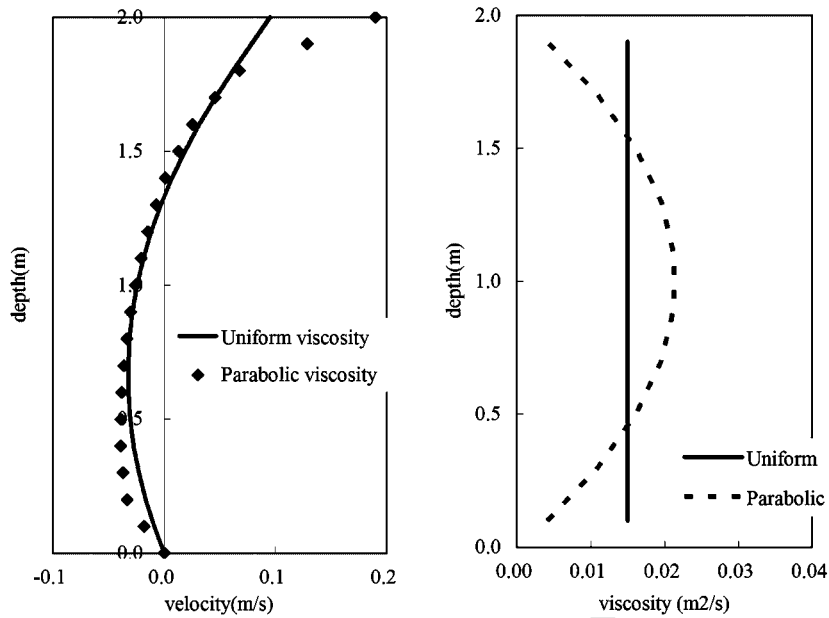


Figure 8. Velocity profiles corresponding to their own viscosities (uniform vs parabolic).

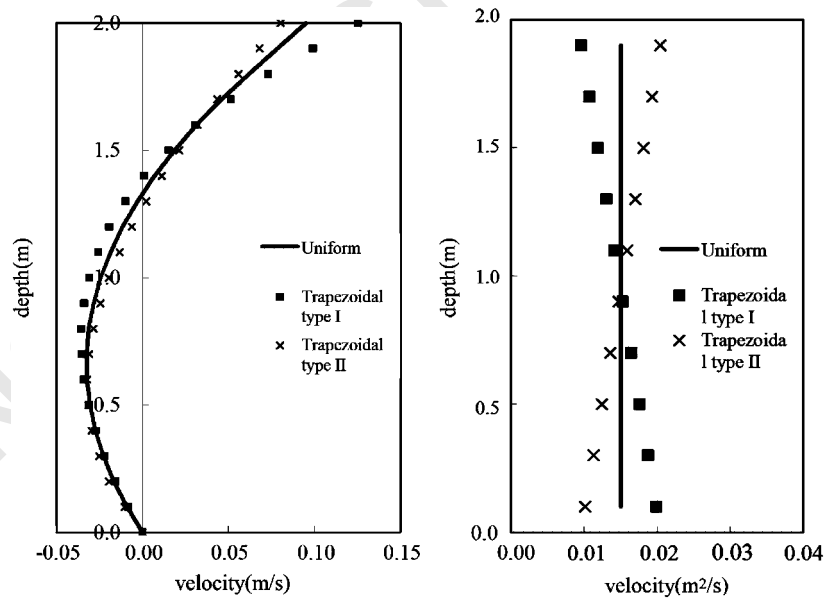


Figure 9. Velocity profiles corresponding to their own viscosities (uniform vs trapezoidal).

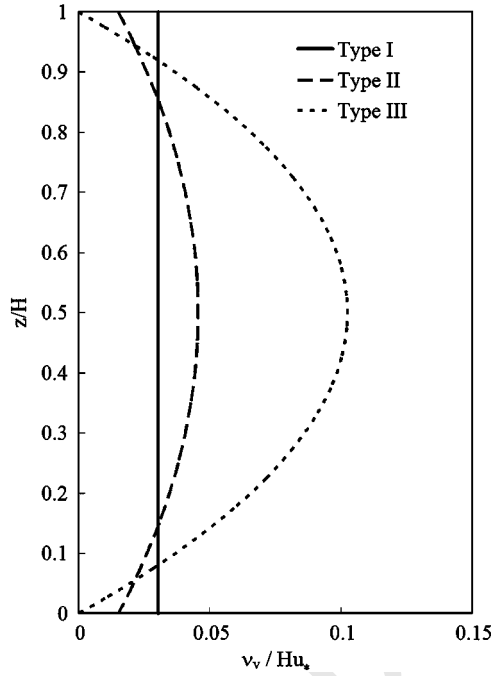


Figure 10. Three types of viscosity variation distribution employed in the study.

5.3. Examination of proposed eddy viscosity relation

Because there is no net flow in the wind-induced flow in a closed basin, the non-uniformity of viscosity distribution does not affect the mean water depth but the velocity profile. As far as 1D or plane 2D is concerned, the normal depth is well known as a function of flow discharge and roughness in natural rivers. However, it is not sufficient in the 3D free-surface flow computations because the water depth and velocity profile are also affected by the eddy viscosity. The following case is designed hypothetically to investigate the effect of viscosity variation on channel flow. A steady flow is carried in a uniform rectangular channel with length 5000 m, width 100 m, slope 0.0005, Manning's roughness n of 0.035, and discharge of 3.987 cm per unit width. The normal depth determined by Manning's equation is 3.0 m. The slip condition was used at the side walls but no-slip condition at the channel bed. The water body was divided into four layers of equal thickness. The following parameters were specified: $\Delta x = 500$ m, $\Delta y = 10$ m, $\rho = 1000$ kg/m³. Upstream, the lateral and vertical velocities v and w were set as zero, and the longitudinal velocity u was equal to the inflow discharge divided by the instantaneous flow area. Downstream, zero longitudinal velocities gradient $\partial u / \partial x = 0$, $\partial v / \partial x = 0$ were applied. Three types of viscosity variation distribution employed as shown in Figure 10 and the coefficient α in Elder's relation can be expressed as follows:

$$\text{Type I: } \alpha = \frac{v_v}{Hu_*} = 0.0304$$

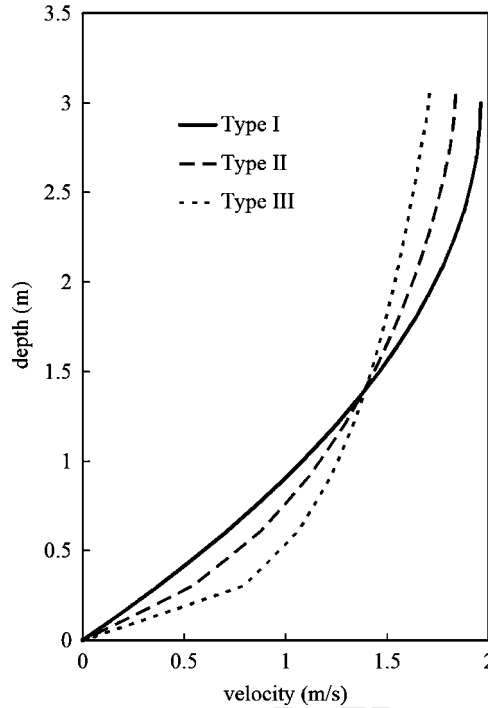


Figure 11. Velocity profiles corresponding to the three types of viscosity variation distribution.

$$\text{Type II: } \alpha = 0.0304 \left[0.5 + 4 \left(\frac{z}{H} \right) \left(1 - \frac{z}{H} \right) \right]$$

$$\text{Type III: } \alpha = \kappa \left(\frac{z}{H} \right) \left(1 - \frac{z}{H} \right), \quad \kappa = 0.41$$

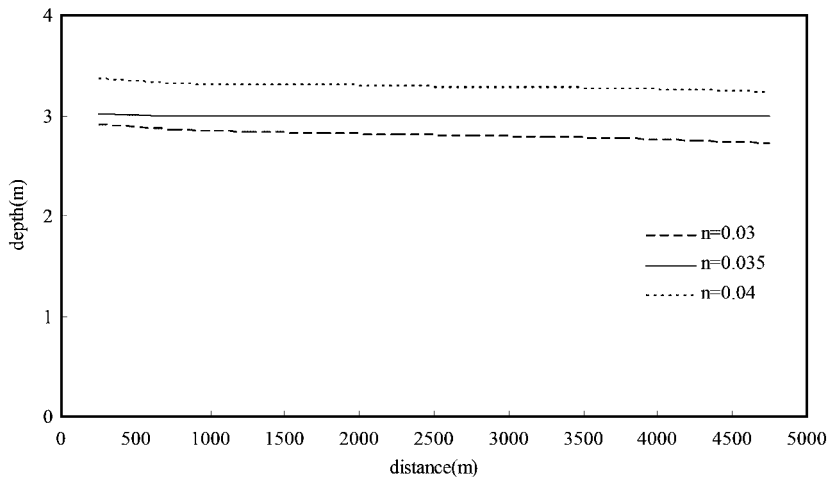
where type I is calculated by Equation (42); type III is a parabolic distribution function proposed by Elder [14]; and type II is an arbitrary function.

Figure 11 shows the velocity profiles corresponding to the three types of viscosity variation distributions, where the shape of velocity profile apparently varies with the type of viscosity variation distribution employed. The shapes of velocity profiles for type I and type III are parabolic and logarithmic, respectively, while the shape for type II is in between.

The water depth in 1D and 2D shallow water flow computation is determined by the flow discharge, energy slope, and channel roughness. From Figure 11 one can see that the water depth is affected not only by these three factors but also by the viscosity variation distribution in the 3D free-surface flow computations. However, the flow regime is indeed turbulent because of their high roughness and irregular bed forms and the eddy viscosity is a function of flow field, not a fluid property. The difficulty of the 3D channel flow simulation is that the eddy viscosity changes anytime and anywhere so that manually calibrating eddy viscosity in the model is almost impossible. Before the turbulent model is considered in the proposed model, an estimation of eddy viscosity, related to Manning's roughness n , water depth, and flow discharge is needed. The case

Table I. Comparison of computed water depth at mid-length from simulation and Manning's equation.

	$n=0.03$	$n=0.035$	$n=0.04$
Manning's equation	2.74	3.00	3.25
Simulated	2.79	3.00	3.28

Figure 12. Simulation of water surface profiles with different n values.

designed hypothetically is the same as above, used to examine the eddy viscosity relation derived previously in this study. α is estimated to be about 0.0304 by Equation (42), and the corresponding eddy viscosity is about $0.011 \text{ m}^2/\text{s}$.

This numerical experiment with same conditions as the previous case is carried out to verify the relation between flow depth and Manning's roughness n . Three different values 0.03, 0.035, and 0.04 of Manning's roughness n were considered in the numerical experiments. Table I shows the comparison of computed water depths with those calculated by Manning's equation. Both the water depth and velocity variation obviously demonstrate the response of the change of Manning's roughness n . The normal depths are 2.74, 3.0, and 3.25 m for $n=0.03$, 0.035, and 0.04, respectively. Computed water depths are a slightly larger than those from Manning's equation. Comparison of the water depths under different n values is shown in Figure 12. The velocity profiles of inner cross section at middle length for various Manning's n are also compared in Figure 13. The larger the value of roughness considered, the larger the depth and smaller the velocity obtained in the simulation.

5.4. Channel flow with wind shear

The final case demonstrated in the following is to solve the wind shear effect on the flow patterns. All conditions were the same as the previous experiment with $n=0.035$ except that the wind shears acting on the surface are 0.0, 2.0, and 5.0 N/m^2 directing to the upstream. Figure 14 shows

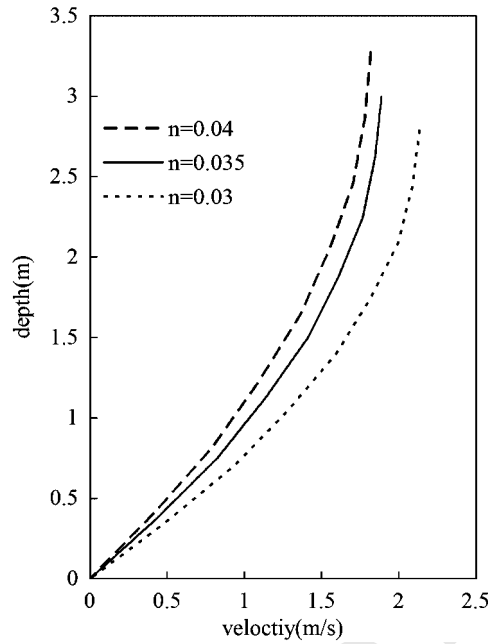


Figure 13. Comparison of velocity profiles with different n values at $x = 2500\text{m}$.

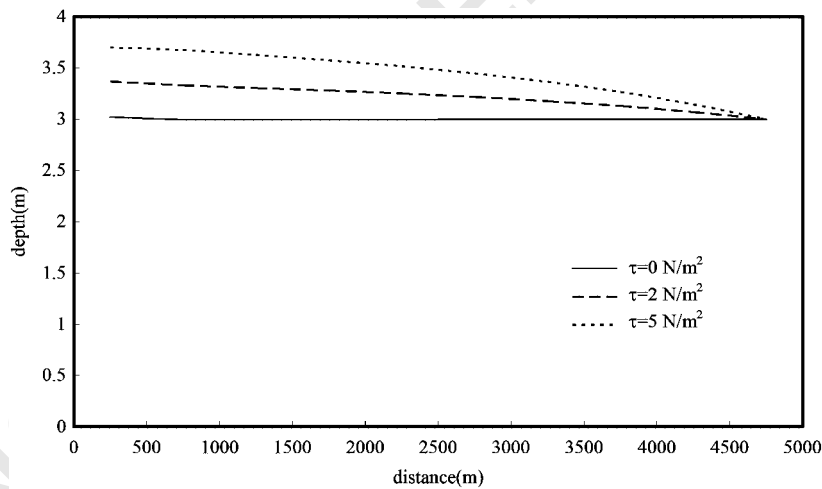


Figure 14. Simulation of water surface profile under different wind shears.

- 1 the comparison of wind-induced setups under different shear stresses. The water surface profile
- behaves like the M_2 curve under the action of wind shears because the rating curve was used as
- 3 downstream boundary conditions. Increase of the water depth drives the flow against the additional
- resistance on the surface. The maximum velocity takes place at one-fourth of depth under free
- 5 surface due to wind shear resistance acting on the water surface. The converged water depth is

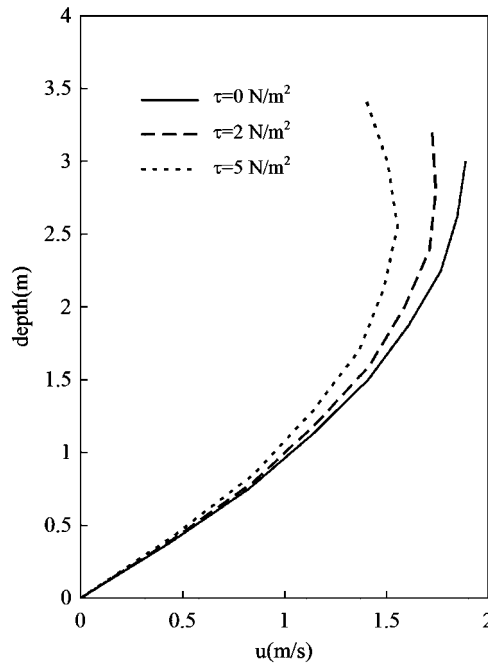


Figure 15. Comparison of velocity profiles under different wind shears at $x=2500$ m.

- 1 3.0 m downstream and increases gradually to 3.17 m upstream under wind shear of 2.0 N/m^2 due
- 2 to wind-induced setup. The velocity profiles were also compared in Figure 15 under different
- 3 values of wind shears.

6. CONCLUSION

- 5 There were a few 3D free-surface flow models developed in the past decades but least applied
- 6 to field problem due to its heavy computational time consumptions. A novel interface continuity-
- 7 based approach proposed in this study makes the layer approach applicable in the shallow water
- 8 free-surface flow computation. Use of a quadratic shape function guarantees that there is no discon-
- 9 tinuity situation taking place between two adjacent layers further. The modeling procedure adopted
- 10 provides a simple and robust method for simulating 3D free-surface shallow water flows. The
- 11 verification performed in the case of wind-induced flow in a closed rectangular basin by comparing
- 12 with the analytical solution shows a reasonable agreement in the simulated steady-state profiles.
- 13 The evolution of water surface slope and the velocity profiles at the middle length of the basin
- 14 were examined to investigate whether the model responding to the wind shear is consistent with
- 15 the physical nature. Alternate shapes of velocity profile formed by their correspondent distribution
- 16 of vertical viscosity also show the flexibility of the proposed approach either in wind-induced
- 17 flows or in channel flows. The variations of velocity profile due to change of viscosity distribution
- 18 and near bed velocity are also investigated. Further, a simplified relation between discharge, water
- 19 depth, and eddy viscosity is derived as the closure of the governing equations. The proposed model

- 1 applicability for the 3D shallow free-surface water flow simulations was examined through two hypothetical cases and shows convincing results. Further development of considering the turbulent
- 3 model is needed for real-world application.

NOTATIONS

h	layer thickness
n	Manning's roughness
p	hydrostatic pressure
u	x -component of velocity
u_*	shear velocity
v	y -component of velocity
z_B	elevation of bottom bed
A	coefficients of the algebraic equation for water columns
C	arbitrary constant
F	layer across flux
H	water depth
K	number of layers
S	slope
S^u, S^v	source terms
U	depth-averaged velocity
α	arbitrary constant for Elder's relation
γ	specific weight of water
μ	dynamic eddy viscosity
ν	kinematic eddy viscosity
ρ	water density
τ	shear stress
τ_s	wind shear

Subscripts

- | | |
|--------------|--|
| e, s, w, n | denote faces of the considered cell |
| h | horizontal direction |
| k | denote the layer interface quantity between k th and $(k - 1)$ th layers |
| $k + 1/2$ | denote the k th layer-averaged quantity |
| m | dummy index used for integration along water depth |
| v | vertical direction |
| E, S, W, N | denote centroid of neighbors of the considered cell |
| 5 P | denote centroid of the considered cell |

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REFERENCES

1. Demuren AO, Rodi W. Side discharges into open channels: mathematical model. *Journal of Hydraulic Engineering* 1983; **109**(12):1707–1722.
2. Hervouet JM, Van Haren L. Recent advances in numerical methods for fluid flows. In *Floodplain Processes*, Chapter 6, Anderson, Walling, Bates (eds). Wiley: New York, 1996.
3. Meselhe EA, Sotiropoulos F. Three-dimensional numerical model for open channel with free surface variations. *Journal of Hydraulic Research* 2000; **38**(2):115–121.
4. Hirt CW, Nicholls BD. Volume of fluid (VOF) method for dynamics of free boundaries. *Journal of Computational Physics* 1981; **39**:201–221.
5. CFX. *CFX-4.2: Solver*. AEA Technology: U.K., 1997.
6. Faure JB, Buil N, Gay B. 3-D modeling for unsteady free-surface flow in open channel. *Journal of Hydraulic Research* 2004; **42**(3):263–272.
7. Li YS, Zhan JM. An efficient three-dimensional semi-implicit finite element scheme for simulation of free surface flows. *International Journal for Numerical Methods in Fluids* 1993; **16**:187–198.
8. Kim CK, Lee JS. A three-dimensional PC-based hydrodynamic model using an ADI scheme. *Coastal Engineering* 1994; **23**:271–287.
9. Shankar NJ, Chan ES, Zhang QY. Three-dimensional numerical simulation for an open channel flow with a constriction. *Journal of Hydraulic Research* 2001; **39**(2):187–201.
10. Li CW, Gu J. 3D layered-integrated modeling of mass exchange in semi-enclosed water bodies. *Journal of Hydraulic Research* 2001; **39**(4):403–412.
11. Lin B, Falconer RA. Three-dimensional layer-integrated modeling of estuarine flows with flooding and drying. *Estuarine, Coastal and Shelf Science* 1997; **44**:737–751.
12. Tsai TL, Yang JC, Huang LH. An accurate integral-based scheme for advection–diffusion equation. *Communications in Numerical Methods in Engineering* 2001; **17**:701–713.
13. Delft Hydraulics. *User Manual of Delft3D-FLOW—Simulation of Multi-dimensional Hydrodynamic Flows and Transport Phenomena, Including Sediments*. Delft Hydraulics, 2003.
14. Elder JW. The dispersion of marked fluid in turbulent shear flow. *Journal of Fluid Mechanics* 1959; **5**:544–560.
15. Nezu I, Rodi W. Open-channel flow measurements with a laser Doppler anemometer. *Journal of Hydraulic Engineering* 1986; **112**:332–355.
16. Heaps NS. Vertical structure of current in homogeneous stratified waters. *Hydrodynamics of Lakes*, Hutter K (ed.). 1984; 154–202.
17. Jobson HE, Sayre WE. Vertical transfer in open channel flow. *Journal of Hydraulic Division* 1970; **96**:703–724.