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計畫主持人：郭心怡

計畫參與人員：碩士班研究生-兼任助理人員：吳自勝
碩士班研究生-兼任助理人員：王勇量
碩士班研究生-兼任助理人員：彭晟祐
碩士班研究生-兼任助理人員：郭祐旻

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Multicoated elliptic fibrous composites of piezoelectric and piezomagnetic phases

Abstract

A theoretical framework is developed to investigate the magnetoelectroelastic potential in a multicoated elliptic fibrous composite with piezoelectric and piezomagnetic phases. We generalize the classic work of Lord Rayleigh (1892) to obtain the electrostatic potential in ordered conductive composites and its extension to a disordered system (Kuo and Chen, 2008; Kuo, 2010) to the current coupled magnetoelectroelastic multicoated elliptic composites. We combine the methods of complex potentials with a re-expansion formulae and the generalized Rayleigh's formulation to obtain a complete solution of the multi-field many-inclusion problem. It is shown that the coefficients of field expansions can be written in the form of an infinite set of linear algebraic equations. Numerical results are presented for several configurations. We use this method to study BaTiO₃-CoFe₂O₄ composites and find that, with appropriate coating, the effective magnetoelectric voltage coefficient can be enhanced with one order of magnitude compared with their non-coating counterpart.

1 Introduction

Magnetoelectric materials, which induce the polarization by a magnetic field, or conversely induce the magnetization by an electric field, have been the focus of research due to their varieties of microstructural phenomena and macroscopic properties. These make them promising for a wide range of applications, such as four-state memories, magnetic field sensors, and magnetically controlled opto-electric devices (Eerenstein *et al.*, 2006; Nan *et al.*, 2008). The study of magneto-electric coupling can be traced back to 1957 when Landau and Lifshitz (1984) showed the possibility of the coupling between the electric and magnetic fields in a substance with a certain magnetic symmetry class. This was subsequently experimentally confirmed in a single crystal Cr₂O₃ by Astrov (1960) and by Rado and Folen (1961) over fifty years ago. However, this coupling is weak in single phase materials, and this has motivated the study of composites of piezoelectric and piezomagnetic media. The basic idea is to couple a piezoelectric and a piezomagnetic material using strain: an applied electric

field creates a strain in the piezoelectric material which in turn induces a deformation in the piezomagnetic material, resulting in a magnetic field.

A number of micromechanical models hence were proposed to predict the effective moduli of multiferroic composites. For instance, Green's function approach was used by Nan (1994) and Huang and Kuo (1997) to study a fibrous composite consisting of Barium Titanate and Cobalt Ferrite. For such transversely isotropic fibrous composites, Benveniste (1995) derived exact connections among effective magnetoelectroelastic moduli based on a formalism discovered by Milgrom and Shtrikman (1989). Particulate composites were investigated by Harshé *et al.* (1993) using a cubic model, while a homogenization micromechanical method was employed by Aboudi (2001). Eshelby's equivalent inclusion approach and the mean field Mori-Tanaka model have been generalized to multiferroic composites by Li and Dunn (1998a, b), Huang (1998), Li (2000), Wu and Huang (2000) and Srinivas *et al.* (2006). Frequency dependence of magnetoelectric coefficients of multiferroic laminates was studied by Bichurin *et al.* (2001, 2010a, 2010b). For a good overview of the subject, the reader is referred to the review article by Nan *et al.* (2008).

In contrast to the variety of works on effective moduli of the composites, the study on the field solutions appears to be a relatively rarely explored area. However, a method that provides the detailed fields is useful to provide insights for developing better microstructures and more complex process such as the breakdown and failure (Li and Duxbury, 1989). Relevant works on the field distributions of magnetoelectric composites are not many. Lee *et al.* (2005) proposed a finite element analysis based micromechanics approach to determine the overall behaviors and reveal concentration of stress, electric and magnetic field within the unit cells. Tang and Yu (2008, 2009) used the variational asymptotic method for predicting the effective properties and local field distributions of smart materials. A two-scale asymptotic homogenization theory was adopted by Camacho-Montes *et al.* (2009) on the magnetoelectric coupling and cross-property connections in a two phase composite.

In a classic work, Lord Rayleigh computed the electrostatic potential for a conducting composite consisting of a periodic array of cylindrical or spherical inclusions. This was extended to arbitrary arrangements by Kuo and Chen (2008) and to elliptic cylinders by Kuo (2010). These works concern single fields. Later, Kuo and Bhattacharya (2010) generalized this methodology to electrostatic, magnetostatic and mechanical coupled fields. In this paper, we extend this Rayleigh's formulation to a multiferroic composite consisting of elliptic cylinders, specifically multicoated ellipses.

Coating plays an important role in high-temperature systems and in various engineering applications. For instance, to reduce heat or stress concentration along the interface, interphase layers between the inclusions and the matrix are often introduced to act as thermal barrier. Graded materials can also be more damage-resistant than their conventional homogeneous counterpart (Suresh, 2001). Such interphase layer may have constant properties or spatially varying properties. Research into inhomogeneous multiferroics has primarily been confined to the study of bilayer and

multilayer structures. Among them, piezoelectric or piezomagnetic coefficients are assumed linear variation in the thickness direction by Chen and Lee (2003), Petrov *et al.* (2008) and Petrov and Srinivasen (2008), while exponentially graded assumption is adopted by Pan and Han (2005) and Wang *et al.* (2009). Fracture behavior of functionally graded piezoelectric and piezomagnetic composites was studied by Feng and Su (2006), Li and Lee (2008), and Zhou and Chen (2008). To our knowledge, the subject of piezoelectric/piezomagnetic fibrous composites with multicoated elliptic cylinders has not been examined in the literature before.

The plan of this article is organized as follows. First we consider a composite medium made of piezoelectric and piezomagnetic phases arranged in a microstructure consisting of parallel elliptic cylinders in a matrix in Section 2. The phases are transversely isotropic and under anti-plane shear with in-plane electromagnetic fields. In this situation, the fields are decoupled in the interior of every phase, and the coupling between the fields occurs only through the interface conditions (Kuo and Bhattacharya, 2010). We exploit this in Section 2.2 to obtain a representation of the solution. The basic idea is to follow Kuo (2010) and expand each field in each medium in a series. We consider periodic arrays in Section 2.3. In Section 3 we consider the case of multicoated elliptic cylinders. We show that a (6×6) array alone can mathematically simulate the effects of multiple coatings. We obtain effective properties in Section 4, and significantly show that the macroscopic properties depend solely on a single expansion coefficient (amongst the infinite). This methodology is illustrated in Section 5 using composites of BaTiO_3 and CoFe_2O_4 . We choose this material pair for its practical potential and also because it enables comparison with previous work. We observe that the composite medium has a non-trivial magnetoelectric coupling even through the individual components do not. Further, we show that the ME coefficient can be enhanced with an order of magnitude if the BTO fiber is coated with Terfenol-D.

2 Multiple elliptic cylinders

2.1 Basic formulations

Let us consider an infinite medium $\mathbb{R}^3$ containing N arbitrarily distributed, parallel and separated elliptic cylinders. The domain of the p th elliptic cylinder is denoted V_p , $p = 1, 2, \dots, N$, and the remaining matrix is denoted Ω_m . We assume that the cylinders and the matrix are made of distinct phases. Further, we assume that each phase is either piezoelectric or piezomagnetic with transversely isotropic symmetry (i.e. has $6mm$ symmetry) about the fiber axes. We introduce a Cartesian coordinate system positioned at a selected point O of the plane with the x - and y - axes in the plane of the cross-section and z - along the axes of the cylinders (Fig. 1). The centroids of the p th elliptic cylinders are designated as O_p , with O_px_p and O_py_p axes are directed along the major and minor axes of the ellipse. Each of ellipse

has the major and minor semi-axis, $l_x^{(p)}$ and $l_y^{(p)}$, and the inter-foci distance is $2d_p$, where $d_p^2 = l_x^{(p)2} - l_y^{(p)2}$. The ellipses are well separated so that any two inclusions will not get in touch with each other.

Let the composite be subjected to the anti-plane shear strain $\bar{\varepsilon}_{zx}$, $\bar{\varepsilon}_{zy}$, the in-plane electric fields $\bar{E}_x$, $\bar{E}_y$, and the magnetic fields $\bar{H}_x$, $\bar{H}_y$ at infinity. Thus the heterogeneous material is in a state of anti-plane shear problem (Chen, 1993; Benveniste, 1995; Kuo and Bhattacharya, 2010) and can be described by

$$\begin{aligned} u_x &= u_y = 0, \quad u_z = w(x, y), \\ \varphi &= \varphi(x, y), \\ \psi &= \psi(x, y), \end{aligned} \quad (2.1)$$

where u_x , u_y , u_z are the mechanical displacements along the x -, y -, and z -axes, and φ and ψ are the electric and magnetic potentials, respectively.

The constitutive laws of the constituents for the non-vanishing fields become

$$\begin{pmatrix} \sigma_{zj} \\ D_j \\ B_j \end{pmatrix} = \begin{pmatrix} C_{44} & e_{15} & q_{15} \\ e_{15} & -\kappa_{11} & -\lambda_{11} \\ q_{15} & -\lambda_{11} & -\mu_{11} \end{pmatrix} \begin{pmatrix} \varepsilon_{zj} \\ -E_j \\ -H_j \end{pmatrix} \quad (2.2)$$

where j denotes the component x, y . We can write this compactly as

$$\Sigma_j^U = L_{UV} Z_j^V, \quad U, V = w, \varphi, \psi, \quad j = x, y, \quad (2.3)$$

where

$$\Sigma_j = \begin{pmatrix} \sigma_{zj} \\ D_j \\ B_j \end{pmatrix}, \quad \mathbf{L} = \begin{pmatrix} C_{44} & e_{15} & q_{15} \\ e_{15} & -\kappa_{11} & -\lambda_{11} \\ q_{15} & -\lambda_{11} & -\mu_{11} \end{pmatrix}, \quad \mathbf{Z}_j = \begin{pmatrix} \varepsilon_{zj} \\ -E_j \\ -H_j \end{pmatrix}. \quad (2.4)$$

Here σ_{zj} , D_j , B_j , ε_{zj} , E_j and H_j are the stress, electric displacement, magnetic flux, strain, electric field, and the magnetic field, respectively. C_{44} , κ_{11} , μ_{11} and λ_{11} are the elastic modulus, dielectric permittivity, magnetic permeability and magnetoelectric coefficients. The shear strains ε_{zx} and ε_{zy} , in-plane electric fields E_x , E_y , and in-plane magnetic fields H_x and H_y can be derived from the gradient of elastic displacement, electric potential, and magnetic potential as follows:

$$\begin{aligned} \varepsilon_{zx} &= \frac{\partial w}{\partial x}, \quad \varepsilon_{zy} = \frac{\partial w}{\partial y}, \\ E_x &= -\frac{\partial \varphi}{\partial x}, \quad E_y = -\frac{\partial \varphi}{\partial y}, \\ H_x &= -\frac{\partial \psi}{\partial x}, \quad H_y = -\frac{\partial \psi}{\partial y}. \end{aligned} \quad (2.5)$$

Further, the equilibrium equations, in the absence of body force, electric charge density and electric current density, are given by

$$\begin{aligned}\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} &= 0, \\ \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} &= 0, \\ \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} &= 0.\end{aligned}\tag{2.6}$$

Substitution of Eq.(2.3) into Eq.(2.6) yields

$$\begin{aligned}C_{44}\nabla^2 w + e_{15}\nabla^2 \varphi + q_{15}\nabla^2 \psi &= 0, \\ e_{15}\nabla^2 w - \kappa_{11}\nabla^2 \varphi - \lambda_{11}\nabla^2 \psi &= 0, \\ q_{15}\nabla^2 w - \lambda_{11}\nabla^2 \varphi - \mu_{11}\nabla^2 \psi &= 0,\end{aligned}\tag{2.7}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ represents the two-dimensional Laplace operator for the variable x and y . Since $\mathbf{L}$ is a nonsingular matrix, generically we can completely decouple (2.7) into three independent Laplace equations,

$$\nabla^2 w = 0, \quad \nabla^2 \varphi = 0, \quad \nabla^2 \psi = 0\tag{2.8}$$

in the interior of each phase.

In addition to these differential equations, we have to use interface conditions. We assume that the interfaces are perfectly bonded, and therefore the fields satisfy

$$[[\mathbf{Z}_j t_j]] = \mathbf{0}, \quad [[\mathbf{\Sigma}_j n_j]] = [[(\mathbf{LZ}_j) n_j]] = \mathbf{0}\tag{2.9}$$

where $[[\cdot]]$ denotes the jump in some quantity across the interface, t_j is the unit tangent to the interface and n_j is the unit outward normal to the interface, and the repeated index j denotes summing over the components x, y . Since $\mathbf{L}$ is different in each phase, the fields w, φ and ψ are generally coupled by the interface equations.

2.2 Representation of the solution

We start by considering the case that the cylinders are homogeneous. We showed above that the fields are decoupled in the interior of every phase, but are coupled at the interfaces. Therefore, we may follow Kuo (2010) and use a series expansion for each field in the interior of each phase and then obtain the coefficients by enforcing the interface and boundary conditions.

Since w, φ and ψ are harmonic, we can construct an analytic function $\Phi(z) = U(z) + iU^*(z)$, of the complex variable $z = x + iy$, where U^* is the conjugate harmonic

function, related to U by the Cauchy-Riemann equation

$$\frac{\partial U}{\partial x} = \frac{\partial U^*}{\partial y}, \quad \frac{\partial U}{\partial y} = -\frac{\partial U^*}{\partial x}, \quad U = w, \varphi, \psi. \quad (2.10)$$

Further, the shape of the cross section of the cylinders defines elliptic coordinates: $(\mu > 0, -\pi < \theta \leq \pi)$ (Moon and Spencer, 1998)

$$z = x + iy = d \cosh \omega = d \cosh (\mu + i\theta) \quad (2.11)$$

are the most appropriate system for the solution of Laplace's equation.

We now consider a situation where the composite is subjected to a macroscopically uniaxial loading

$$w_{\text{ext}} = \bar{\varepsilon}_{zx}x, \quad \varphi_{\text{ext}} = -\bar{E}_x x, \quad \psi_{\text{ext}} = -\bar{H}_x x, \quad (2.12)$$

for constants $\bar{\varepsilon}_{zx}$, $\bar{E}_x$ and $\bar{H}_x$. We may rewrite this in short as

$$\Phi_{\text{ext}} = \Gamma^\Phi z, \quad (2.13)$$

where Φ represents the appropriate field – the anti-plane deformation w , electric potential φ , or magnetic potential ψ – and $\Gamma^\Phi = \Gamma_R^\Phi + i\Gamma_I^\Phi$ the corresponding applied field – $\bar{\varepsilon}_{zx}$, $-\bar{E}_x$ or $-\bar{H}_x$.

We rewrite the governing equation, Eq. (2.8), in elliptic coordinates (μ, θ) ,

$$\nabla^2 \Phi = \frac{1}{d^2 (\cosh^2 \mu - \cos^2 \theta)} \left(\frac{\partial^2 \Phi}{\partial \mu^2} + \frac{\partial^2 \Phi}{\partial \theta^2} \right) = 0. \quad (2.14)$$

The potential field for the p th elliptic cylinder and its surrounding matrix can be expanded with respect to its centroid O_p as (Yardley *et al.*, 1999)

$$\Phi_i^{(p)}(z_p) = \sum_{n=-\infty}^{\infty} C_n^{\Phi(p)} e^{-n\omega_p}, \quad C_n^{\Phi(p)} = C_{-n}^{\Phi(p)} \quad (2.15)$$

for the inclusion, and

$$\Phi_m^{(p)}(z_p) = \sum_{n=-\infty}^{\infty} A_n^{\Phi(p)} e^{-n\omega_p} + \sum_{n=1}^{\infty} B_n^{\Phi(p)} e^{-n\omega_p}, \quad A_n^{\Phi(p)} = A_{-n}^{\Phi(p)} \quad (2.16)$$

for the matrix. Here $\omega_p = \mu_p + i\theta_p$ is the local elliptic coordinate centered at the origin of the p th ellipse, the subscripts i and m denote the inclusion and matrix, respectively. The coefficients $A_n^{\Phi(p)} = A_{nR}^{\Phi(p)} + iA_{nI}^{\Phi(p)}$, $B_n^{\Phi(p)} = B_{nR}^{\Phi(p)} + iB_{nI}^{\Phi(p)}$ and $C_n^{\Phi(p)} = C_{nR}^{\Phi(p)} + iC_{nI}^{\Phi(p)}$ are some complex unknowns to be determined. The superscripts p appearing in (2.15) and (2.16) indicate that the fields are expanded with respect to the p th ellipse centroid.

We recall the interface conditions (2.9) which we rewrite as

$$\operatorname{Re} \Phi_i^{(p)} \Big|_{\partial V_p} = \operatorname{Re} \Phi_m^{(p)} \Big|_{\partial V_p}, \quad (\boldsymbol{\Sigma}^\Phi)_m^{(p)} \cdot \mathbf{n}_p \Big|_{\partial V_p} = (\boldsymbol{\Sigma}^\Phi)_i^{(p)} \cdot \mathbf{n}_p \Big|_{\partial V_p} \quad (2.17)$$

where

$$\boldsymbol{\Sigma}^w = (\sigma_{zx}, \sigma_{zy}), \quad \boldsymbol{\Sigma}^\varphi = (D_x, D_y), \quad \boldsymbol{\Sigma}^\psi = (B_x, B_y), \quad (2.18)$$

$\partial V_p : \mu_p = a_p$ denotes the interface between the matrix and the p th elliptic cylinder, and $\mathbf{n}_p$ is the unit outward normal of the interface ∂V_p .

Using the orthogonality properties of trigonometric functions, the interface conditions (2.17) provide

$$\mathbf{a}_{n\Lambda}^{(p)} = \mathbf{T}_{an\Lambda}^{(p)} \mathbf{b}_{n\Lambda}^{(p)}, \quad \mathbf{c}_{n\Lambda}^{(p)} = \mathbf{T}_{cn\Lambda}^{(p)} \mathbf{b}_{n\Lambda}^{(p)}, \quad (2.19)$$

and $A_0^{\Phi(p)} = C_0^{\Phi(p)}$, where

$$\mathbf{a}_{n\Lambda}^{(p)} = \begin{pmatrix} A_{n\Lambda}^{w(p)} \\ A_{n\Lambda}^{\varphi(p)} \\ A_{n\Lambda}^{\psi(p)} \end{pmatrix}, \quad \mathbf{b}_{n\Lambda}^{(p)} = \begin{pmatrix} B_{n\Lambda}^{w(p)} \\ B_{n\Lambda}^{\varphi(p)} \\ B_{n\Lambda}^{\psi(p)} \end{pmatrix}, \quad \mathbf{c}_{n\Lambda}^{(p)} = \begin{pmatrix} C_{n\Lambda}^{w(p)} \\ C_{n\Lambda}^{\varphi(p)} \\ C_{n\Lambda}^{\psi(p)} \end{pmatrix}, \quad (2.20)$$

$$\begin{aligned} \mathbf{T}_{anR}^{(p)} &= (\mathbf{L}^{(m)} - \mathbf{L}^{(p)})^{-1} (\cosh na_p \mathbf{L}^{(m)} + \sinh na_p \mathbf{L}^{(p)}) \frac{e^{-na_p}}{\sinh 2na_p}, \\ \mathbf{T}_{anI}^{(p)} &= (\mathbf{L}^{(m)} - \mathbf{L}^{(p)})^{-1} (\sinh na_p \mathbf{L}^{(m)} + \cosh na_p \mathbf{L}^{(p)}) \frac{-e^{-na_p}}{\sinh 2na_p}, \\ \mathbf{T}_{cnR}^{(p)} &= \mathbf{T}_{anR}^{(p)} + \frac{e^{-na_p}}{2 \cosh na_p} \mathbf{I}, \quad \mathbf{T}_{cnI}^{(p)} = \mathbf{T}_{anI}^{(p)} - \frac{e^{-na_p}}{2 \sinh na_p} \mathbf{I}, \end{aligned} \quad (2.21)$$

Λ represents R , the real part, or I , the imaginary part of the coefficients, and $\mathbf{I}$ is the 3×3 identity tensor

We now need to relate the solutions to the applied boundary conditions. We do so by applying the Green's second identity (Arfken and Weber, 2001) to the matrix domain Ω_m . This gives

$$\begin{aligned} & \int_{\Omega_m} [G(\mathbf{x}; \mathbf{x}') \nabla'^2 \Phi_m(\mathbf{x}') - \Phi_m(\mathbf{x}') \nabla'^2 G(\mathbf{x}; \mathbf{x}')] dA' \\ &= \int_{\partial \Omega_m} [G(\mathbf{x}; \mathbf{x}') \nabla' \Phi_m(\mathbf{x}') - \Phi_m(\mathbf{x}') \nabla' G(\mathbf{x}; \mathbf{x}')] \cdot \mathbf{n}' ds', \end{aligned} \quad (2.22)$$

where the prime $'$ denotes the operation in reference to the $\mathbf{x}'$ coordinate, $\mathbf{n}'$ is the outward unit normal to the matrix's boundary $\partial \Omega_m$, dA' represents the area element for the $\mathbf{x}'$ coordinate, ds' is the differential arc length. Here $G(\mathbf{x}; \mathbf{x}')$ is the free-space Green's function for Laplace operator satisfying $\nabla^2 G(\mathbf{x}; \mathbf{x}') = -\delta(\mathbf{x} - \mathbf{x}')$, where

$\delta(\mathbf{x} - \mathbf{x}')$ is the Dirac-delta function. Following the procedure in Kuo (2010), it can be shown that Eq. (2.22) yields

$$\Phi_m(z) = \Phi_{\text{ext}}(z) + \sum_{l=1}^N \sum_{n=1}^{\infty} B_m^{\Phi(l)} e^{-n\omega_l}. \quad (2.23)$$

This is the consistency equation which relates the external applied fields to the local potential expansions. Note that the field identity (2.23) is written based on different coordinates.

To proceed, we shift the origin of the expansions (2.23) to a fixed point, say Z_p , the centroid of the p th ellipse, by expanding the term $e^{-m\omega_l}$ as (Kushch *et al.*, 2005)

$$e^{-n\omega_l} = \sum_{m=-\infty}^{\infty} \eta_{nm}^{lp} e^{-m\omega_p}, \quad (2.24)$$

with

$$\eta_{nm}^{lp} = (-1)^m n \left(\frac{d_l}{d_{lp}} \right)^n \sum_{s=0}^{\infty} v_{lp}^{-(n+m+2s)} \sum_{t=0}^s \frac{(-1)^{s-t}}{(s-t)!} \left(\frac{d_p}{d_{lp}} \right)^{m+2t} M_{nmt}(d_l, d_p) \frac{(n+m+t+s-1)!}{(s-t)!}, \quad (2.25)$$

where $d_{lp} \equiv d_l + d_p$, $v_{lp} \equiv Z_{lp}/d_{lp} + \sqrt{(Z_{lp}/d_{lp})^2 - 1}$ and

$$M_{nmt}(d_l, d_p) = \sum_{k=0}^t \frac{(d_l/d_p)^{2k}}{k!(t-k)!(k+n)!(m+t-k)!}. \quad (2.26)$$

Introducing (2.24) into (2.23), we have the expansion

$$\Phi_{m, \text{near}}^{(p)}(z) = \Gamma^{\Phi} Z_p + \sum_{n=-\infty}^{\infty} (B_n^{\Phi(p)} + b_n^{\Phi(p)}) e^{-n\omega_p}, \quad (2.27)$$

where

$$b_n^{\Phi(p)} = \Gamma^{\Phi} \frac{d_p}{2} \delta_{n,\pm 1} + \sum_{l \neq p}^N \sum_{m=1}^{\infty} B_m^{\Phi(l)} \eta_{mn}^{lp} \quad (2.28)$$

valid for the domain within an ellipse centered in Z_p with inter-foci distance $2d_{lp}$ and passing the pole of l th elliptic coordinate systems closest to Z_p (Kushch *et al.*, 2005). Further, since z lies in the matrix domain, Equations (2.27) and (2.16) should be identical. This provides the condition

$$\sum_{n=-\infty}^{\infty} A_n^{\Phi(p)} e^{-n\omega_p} = \Gamma^{\Phi} Z_p + \sum_{n=-\infty}^{\infty} b_n^{\Phi(p)} e^{-n\omega_p}. \quad (2.29)$$

Taking the real part and the imaginary part of (2.29), we find the two conditions

$$A_{nR}^{\Phi(p)} = (\Gamma_R^{\Phi} \operatorname{Re} Z_p - \Gamma_I^{\Phi} \operatorname{Im} Z_p) \delta_{n,0} + b_{nR}^{\Phi(p)}, \quad (2.30)$$

and

$$A_{nI}^{\Phi(p)} = (\Gamma_I^{\Phi} \operatorname{Re} Z_p + \Gamma_R^{\Phi} \operatorname{Im} Z_p) \delta_{n,0} + b_{nI}^{\Phi(p)}. \quad (2.31)$$

Equations (2.30), (2.31) and (2.19)₁ constitute an infinite set of linear algebraic equations. Upon appropriate truncations of the expansions terms, we can determine the expansion coefficients $A_n^{\Phi(p)}$, $B_n^{\Phi(p)}$, $C_n^{\Phi(p)}$. Here we make one further remark.

Remark The essential step of the framework is to establish the generalized Rayleigh's identities, (2.30) and (2.31). We observe, however, that the derivation of the identities is independent of inclusions' expansions. In other words, these identities can be applicable to inclusions with inhomogeneous constituents provided that the admissible fields in the inclusions and the transition relations, similar to (2.19) can be constructed.

2.3 Periodic arrays

The analysis carried out in the previous section for the arbitrary system with a finite number of cylinders may also be adapted for the case of a periodic array of cylinders. Here we concentrate on the rectangular lattice, and we sketch the outline of the derivation focussing on the differences from the previous situation.

Let us first introduce a Cartesian coordinate system (x, y) positioned at the centroid O of one of the ellipses in a rectangular array, as shown in Figure 2. The sides of the rectangular cell parallel to the x and y coordinates are, respectively, denoted by α and β . The elliptic cylinders are of the same orientation, elliptic radius $\mu = a$ and inter-foci distance $2d$. Uniform intensities $\overline{E}_x$ and $\overline{H}_x$ are applied along the positive x axis, and an anti-plane shear deformation $\overline{\varepsilon}_{zx}$ is applied out of the xy plane. In terms of elliptic coordinates, the general solution has the admissible form

$$\Phi_i = \sum_{n=-\infty}^{\infty} C_n^{\Phi} e^{-n\omega}, \quad C_n^{\Phi} = C_{-n}^{\Phi} \quad (2.32)$$

for $\mu < a$, and

$$\Phi_m = \sum_{n=-\infty}^{\infty} A_n^{\Phi} e^{-n\omega} + \sum_{n=1}^{\infty} B_n^{\Phi} e^{-n\omega}, \quad A_n^{\Phi} = A_{-n}^{\Phi}, \quad (2.33)$$

for $\mu > a$. The coefficients A_n^{Φ} , B_n^{Φ} , and C_n^{Φ} are unknown complex constants to be determined from the interface and boundary conditions.

Analogous to (2.19), the continuity conditions at the interface will give constraints (2.19) between the coefficients. Next, imposing the periodicity conditions analogous

to the boundary condition we did to derive (2.30) and (2.31), lead to generalized Rayleigh's identities

$$A_{n\Lambda}^\Phi = b_{n\Lambda}^\Phi, \quad \Lambda = R, I. \quad (2.34)$$

Here the quantities

$$b_n^\Phi = -\Gamma^\Phi \frac{d}{2} \delta_{n,\pm 1} + \sum_{m=1}^{\infty} B_m^\Phi \sum_{l \neq o}^{\infty} \eta_{mn}^{lo}, \quad (2.35)$$

$$\sum_{l \neq o}^{\infty} \eta_{mn}^{lo} = (-1)^m n \sum_{t=0}^{\infty} \left(\frac{d}{2} \right)^{n+m+2t} M_{nmt}(d, d) (n+m+2t-1)! S_{n+m+2t}, \quad (2.36)$$

where $M_{nmt}(\cdot)$ is defined in (2.26), and

$$S_m = \sum_{l \neq o} Z_l^{-m}, \quad (2.37)$$

are the lattice sums characterizing the geometry of the periodic structure, and Z_l is the centroid of the l th cylinder when measured in the complex plane centered at the central point O . The index l runs over all cylinders' centers underlying the periodic array except the central one. Previous studies (Rayleigh, 1892) have reported that the sum S_2 is conditionally convergent and its value depends upon the shape of the exterior boundary of the array. A list of S_2 for different values of α/β can be found in Nicrovici and McPhedran (1996).

Equations (2.34) and (2.19)₁ constitute an infinite set of linear algebraic equations. Upon appropriate truncations of the expansion terms at a finite order M , we can determine the expansion coefficients A_n^Φ , B_n^Φ , and C_n^Φ .

3 Confocally multicoated elliptic cylinders

From the previous remark, we now consider that the inclusions are confocally multicoated elliptic cylinders with the outer elliptic radius $a_p^{(1)}$, $p = 1, 2, \dots, N$, where N is the number of inclusions. We denote the matrix as phase 0, with transversely isotropic material parameters $C_{44}^{(0)}$, $e_{15}^{(0)}$, $q_{15}^{(0)}$, $\kappa_{11}^{(0)}$, $\mu_{11}^{(0)}$ and $\lambda_{11}^{(0)}$. The multicoated cylinder consists of a core, with radius $\mu_p = a_p^{(\mathcal{M})}$, surrounded by $(\mathcal{M} - 1)$ layers of coating. The j th layer of the coatings occupies the annulus $V_p^{(j)} : a_p^{(j+1)} \leq \mu_p \leq a_p^{(j)}$, $j = 1, 2, \dots, \mathcal{M}$, in which $V_p = V_p^{(1)} \cup V_p^{(2)} \cup \dots \cup V_p^{(\mathcal{M})}$. Here the innermost core is solid so that we have $a_p^{(\mathcal{M}+1)} = 0$. We assume that the material properties of j th constituent layer of the p th multicoated cylinder are $C_{44}^{(p,j)}$, $e_{15}^{(p,j)}$, $q_{15}^{(p,j)}$, $\kappa_{11}^{(p,j)}$, $\mu_{11}^{(p,j)}$ and $\lambda_{11}^{(p,j)}$.

The admissible potentials in each constituent layer of the multicoated inclusion

can be expressed as

$$\Phi^{(p,j)} = \sum_{n=-\infty}^{\infty} A_n^{\Phi(p,j)} e^{-nw_p} + \sum_{n=1}^{\infty} B_n^{\Phi(p,j)} e^{-nw_p}, \quad A_n^{\Phi(p,j)} = A_{-n}^{\Phi(p,j)}, \quad (3.1)$$

where $A_n^{\Phi(p,j)} = A_{nR}^{\Phi(p,j)} + i A_{nI}^{\Phi(p,j)}$ and $B_n^{\Phi(p,j)} = B_{nR}^{\Phi(p,j)} + i B_{nI}^{\Phi(p,j)}$ are unknown complex constants to be determined. Note that the potential at $\mu \rightarrow 0$ should be finite and thus we can set

$$B_n^{\Phi(p,\mathcal{M})} = 0. \quad (3.2)$$

We consider that the interfaces are perfectly bonded, the potential and the normal component of flux are continuous across the interfaces,

$$\begin{aligned} \text{Re } \Phi^{(p,j-1)} \Big|_{\mu_p=a_p^{(j)}} &= \text{Re } \Phi^{(p,j)} \Big|_{\mu_p=a_p^{(j)}}, \\ (\boldsymbol{\Sigma}^{\Phi})^{(p,j-1)} \cdot \mathbf{n}_p^{(j)} \Big|_{\mu_p=a_p^{(j)}} &= (\boldsymbol{\Sigma}^{\Phi})^{(p,j)} \cdot \mathbf{n}_p^{(j)} \Big|_{\mu_p=a_p^{(j)}}. \end{aligned} \quad (3.3)$$

These continuity conditions lead to

$$\begin{pmatrix} \mathbf{a}_{n\Lambda}^{(p,j-1)} \\ \mathbf{b}_{n\Lambda}^{(p,j-1)} \end{pmatrix} = \mathbf{k}_{n\Lambda}^{(p,j)} \begin{pmatrix} \mathbf{a}_{n\Lambda}^{(p,j)} \\ \mathbf{b}_{n\Lambda}^{(p,j)} \end{pmatrix}, \quad \Lambda = R, I, \quad j = 1, 2, \dots, \mathcal{M}, \quad (3.4)$$

where

$$\mathbf{a}_{n\Lambda}^{(p,j)} = \begin{pmatrix} A_{n\Lambda}^{w(p,j)} \\ A_{n\Lambda}^{\varphi(p,j)} \\ A_{n\Lambda}^{\psi(p,j)} \end{pmatrix}, \quad \mathbf{b}_{n\Lambda}^{(p,j)} = \begin{pmatrix} B_{n\Lambda}^{w(p,j)} \\ B_{n\Lambda}^{\varphi(p,j)} \\ B_{n\Lambda}^{\psi(p,j)} \end{pmatrix}, \quad (3.5)$$

$$\begin{aligned} \mathbf{k}_{nR}^{(p,j)} &\equiv \begin{pmatrix} 2 \cosh na_p^{(j)} \mathbf{I} & e^{-na_p^{(j)}} \mathbf{I} \\ -2 \sinh na_p^{(j)} \mathbf{L}^{(j-1)} & e^{-na_p^{(j)}} \mathbf{L}^{(j-1)} \end{pmatrix}^{-1} \begin{pmatrix} 2 \cosh na_p^{(j)} \mathbf{I} & e^{-na_p^{(j)}} \mathbf{I} \\ -2 \sinh na_p^{(j)} \mathbf{L}^{(j)} & e^{-na_p^{(j)}} \mathbf{L}^{(j)} \end{pmatrix}, \\ \mathbf{k}_{nI}^{(p,j)} &\equiv \begin{pmatrix} -2 \sinh na_p^{(j)} \mathbf{I} & e^{-na_p^{(j)}} \mathbf{I} \\ 2 \cosh na_p^{(j)} \mathbf{L}^{(j-1)} & e^{-na_p^{(j)}} \mathbf{L}^{(j-1)} \end{pmatrix}^{-1} \begin{pmatrix} -2 \sinh na_p^{(j)} \mathbf{I} & e^{-na_p^{(j)}} \mathbf{I} \\ 2 \cosh na_p^{(j)} \mathbf{L}^{(j)} & e^{-na_p^{(j)}} \mathbf{L}^{(j)} \end{pmatrix} \end{aligned} \quad (3.6)$$

and $\mathbf{I}$ is the 3×3 identity matrix. Now, repeated use of (3.4) gives

$$\begin{pmatrix} \mathbf{a}_{n\Lambda}^{(p,0)} \\ \mathbf{b}_{n\Lambda}^{(p,0)} \end{pmatrix} = \mathbf{K}_{p\Lambda}^{(j,n)} \begin{pmatrix} \mathbf{a}_{n\Lambda}^{(p,j)} \\ \mathbf{b}_{n\Lambda}^{(p,j)} \end{pmatrix}, \quad j = 1, 2, \dots, \mathcal{M}, \quad (3.7)$$

where

$$\mathbf{K}_{p\Lambda}^{(j,n)} \equiv \mathbf{k}_{n\Lambda}^{(p,1)} \mathbf{k}_{n\Lambda}^{(p,2)} \dots \mathbf{k}_{n\Lambda}^{(p,j)}. \quad (3.8)$$

For $j = \mathcal{M}$, we have

$$\begin{pmatrix} \mathbf{a}_{n\Lambda}^{(p,0)} \\ \mathbf{b}_{n\Lambda}^{(p,0)} \end{pmatrix} = \mathbf{K}_{p\Lambda}^{(\mathcal{M},n)} \begin{pmatrix} \mathbf{a}_{n\Lambda}^{(p,\mathcal{M})} \\ \mathbf{b}_{n\Lambda}^{(p,\mathcal{M})} \end{pmatrix}. \quad (3.9)$$

Further, according to (3.2), (3.9) implies that

$$\mathbf{a}_{n\Lambda}^{(p,0)} = \left[\mathbf{K}_{p\Lambda}^{(\mathcal{M},n)} \right]_{11} \left[\mathbf{K}_{p\Lambda}^{(\mathcal{M},n)} \right]_{21}^{-1} \mathbf{b}_{n\Lambda}^{(p,0)}. \quad (3.10)$$

Here $\left[\mathbf{K}_{p\Lambda}^{(\mathcal{M},n)} \right]_{11}$ represents the upper-left (3×3) submatrix of $\mathbf{K}_{p\Lambda}^{(\mathcal{M},n)}$ and $\left[\mathbf{K}_{p\Lambda}^{(\mathcal{M},n)} \right]_{21}$ is the lower-left (3×3) submatrix of $\mathbf{K}_{p\Lambda}^{(\mathcal{M},n)}$. The formulation implies that the effect of the multiple coatings can be incorporated through a recurrence procedure and is solely represented by a (6×6) array alone. We mention that once we construct the admissible field (3.1) and the transition relation (3.4) in the inhomogeneous inclusions, we can follow the remaining generalized Rayleigh's framework proposed in previous section to determine the potential distribution.

4 Effective moduli

We are interested in the effective behavior for a situation where we have a large number of cylinders. The effective material properties are defined in terms averaged fields,

$$\langle \boldsymbol{\Sigma}_j \rangle \equiv \mathbf{L}_j^* \langle \mathbf{Z}_j \rangle, \text{ no summation}, \quad (4.1)$$

where the angular brackets denote the area averages over the representative volume element (unit cell in the case of periodic composites)

$$\langle \boldsymbol{\Sigma}_j \rangle = \frac{1}{V} \int_V \boldsymbol{\Sigma}_j d\mathbf{x}, \quad \langle \mathbf{Z}_j \rangle = \frac{1}{V} \int_V \mathbf{Z}_j d\mathbf{x}, \quad (4.2)$$

and $\mathbf{L}_j^*$ denotes the effective magneto-electroelastic parameters of the composite. Note that since the microstructure is nonsymmetric, the responses of the composite along x and y -axes are different:

$$\mathbf{L}_x^* = \begin{pmatrix} C_{55}^* & e_{15}^* & q_{15}^* \\ e_{15}^* & -\kappa_{11}^* & -\lambda_{11}^* \\ q_{15}^* & -\lambda_{11}^* & -\mu_{11}^* \end{pmatrix}, \quad \mathbf{L}_y^* = \begin{pmatrix} C_{44}^* & e_{24}^* & q_{24}^* \\ e_{24}^* & -\kappa_{22}^* & -\lambda_{22}^* \\ q_{24}^* & -\lambda_{22}^* & -\mu_{22}^* \end{pmatrix}. \quad (4.3)$$

Let the composite is subjected to uniform intensities $\bar{\varepsilon}_{zx}$, $-\bar{E}_x$, and $-\bar{H}_x$ along the positive x -axis. We can compute the average $\mathbf{Z}_x$ by noting that each component

is a gradient and applying the divergence theorem. We obtain:

$$\langle Z_x^\Phi \rangle = \Gamma_R^\Phi. \quad (4.4)$$

Next, to find $\langle \Sigma_x^\Phi \rangle$, we again use the divergence theorem and the equilibrium condition (including the interface conditions) to obtain

$$\langle \Sigma_x^\Phi \rangle = \frac{1}{V} \int_V \Sigma_x^\Phi d\mathbf{x} = \frac{1}{V} \int_V \nabla \cdot (x \Sigma^\Phi) d\mathbf{x} = \frac{1}{V} \int_{\partial V} x (\Sigma^\Phi)_m \cdot \mathbf{n} ds, \quad (4.5)$$

where Σ^Φ is defined in (2.18). We then use the expansions (2.16) for the fields to obtain

$$\frac{1}{V} \int_{\partial V} x (\mathbf{Z}^\Phi)_m \cdot \mathbf{n} ds = \Gamma_R^\Phi - \frac{\pi d B_{1R}^\Phi}{\alpha \beta} \quad (4.6)$$

where

$$\mathbf{Z}^w = (\varepsilon_{zx}, \varepsilon_{zy}), \quad \mathbf{Z}^\varphi = -(E_x, E_y), \quad \mathbf{Z}^\psi = -(H_x, H_y). \quad (4.7)$$

Putting (4.5) and (4.6) together, and recalling the constitutive relation (2.2) for the matrix, we obtain

$$\begin{pmatrix} \langle \sigma_{zx} \rangle \\ \langle D_x \rangle \\ \langle B_x \rangle \end{pmatrix} = \begin{pmatrix} C_{44} & e_{15} & q_{15} \\ e_{15} & -\kappa_{11} & -\lambda_{11} \\ q_{15} & -\lambda_{11} & -\mu_{11} \end{pmatrix}^{(m)} \begin{pmatrix} \bar{\varepsilon}_{zx} - \frac{\pi d B_{1R}^w}{\alpha \beta} \\ -\bar{E}_x - \frac{\pi d B_{1R}^\varphi}{\alpha \beta} \\ -\bar{H}_x - \frac{\pi d B_{1R}^\psi}{\alpha \beta} \end{pmatrix}. \quad (4.8)$$

Putting together (4.1) and (4.8) and noting that the coefficients B_1^Φ depend linearly on the applied field Γ_R^Φ , we obtain set of equations for the effective property $\mathbf{L}_x^*$. We can determine this by applying different loading combinations between $\bar{\varepsilon}_{zx}$, $\bar{E}_x$ and $\bar{H}_x$. Similarly we can determine $\mathbf{L}_y^*$ by applying different loading combinations between $\bar{\varepsilon}_{zy}$, $\bar{E}_y$ and $\bar{H}_y$.

5 Results and discussion

In order to have a better understanding for the theoretical results above, we first perform a numerical computation for a two-phase transversely isotropic piezoelectric-piezomagnetic composite with $6mm$ material symmetry about the fiber axis. Specifically we consider a composite of BaTiO₃ and CoFe₂O₄ which has been studied by other researchers. We consider the square arrays, and two cases: BTO fibers in a CFO matrix and BTO coated Terfenol-D fibers in a CFO matrix. The independent material constants of these constituents are given in Table 1, where the xy plane is isotropic and the fiber axis is along the z -direction. Note that in all materials magnetoelectric coefficients are zero, i.e. $\lambda_{11} = 0$.

We begin with a composite of BTO fibers in a CFO matrix. Figure 3 (a-c) show

the contours of displacement, electric potential and magnetic potential for a square array with an applied magnetic field. The ratio of the semi-axes l_x and l_y is 1.2. The magnetic field induces a mechanical stress in the CFO which in turn results in an electric displacement in the BTO fiber. The magnetic field is attracted by the BTO since it has a smaller magnetic permeability. Further, Figure 3 (d-f) show the potential contours for BTO fibers coated Terfenol-D in a CFO matrix for a square array with an applied magnetic field. The ratio of semi-axis l_x between BTO and Terfenol-D is 0.7. Since the magnetic permeability of Terfenol-D is almost the same as that of BTO, the magnetic potential is similar to its homogeneous counterpart. However, in this case the contours of vertical displacement and the electric potential have dramatically difference from those with the homogeneous fiber. The fields in the fiber are now not uniform, and there are field concentrations at the BTO and Terfenol-D interfaces.

We now turn to effective moduli of the composite. Figure 4 shows the effective elastic, dielectric, magnetic, piezoelectric, piezomagnetic and magnetoelectric moduli for this composite. We assume that the inclusions are circular cylinders, i.e., $l_x \rightarrow l_y$, and the inclusions are in a square array. Therefore there is no distinction between $\mathbf{L}_x^*$ and $\mathbf{L}_y^*$. The effective moduli vary nonlinearly with volume fraction, and the curve stops at $f = \frac{\pi}{4}$ when the inclusions touch. The magnetoelectric coefficient is non-zero for every non-zero volume fraction, then reaches a maximum before dropping just as the fibers are close to touching. Further, Figure 4 also compares the effective moduli with those predicted by Benveniste (1995) who used the composite cylinder assemblage (CCA) model. In CCA, there is no upper limit on the volume fraction since one can have fibers with various sizes. Still, the overall magnitudes and trends agree well between our periodic and his CCA. In addition, our numerical results fulfil the compatibility conditions given in Eq. (21) of the work by Benveniste (1995). Figure 5 shows the effective moduli for the BTO fibers coated Terfenol-D in a CFO matrix. The ratio of the radius of BTO and Terfenol-D is 0.8. We find that the magnetoelectric coefficient λ_{11}^* has dramatically enhanced, and the enhancement is increased as the particles touch.

We finally turn to the magnetoelectric voltage coefficient, which is the important figure of merit for magnetic field sensors. It relates the overall electric field that is generated in the composite when it is subjected to a magnetic field. It combines the coupling and dielectric coefficients, and is defined by

$$\alpha_{11}^* = \frac{\lambda_{11}^*}{\kappa_{11}^*}. \quad (5.1)$$

Figure 6 shows how this coefficient depends on the fiber volume fraction for the above two cases. Note that there is a qualitative difference between the case of BTO fibers in CFO and BTO fibers coated Terfenol-D in the CFO matrix. In the former, the maximum coefficient is for intermediate volume fraction of $f = 0.35$ where $\alpha_{11}^* = 0.0306\text{V/cmOe}$. In contrast, in the case of BTO fibers coated Terfenol-D in the

CFO matrix, the maximum is attained as the fibers near close touching at $f = 0.725$ where $\alpha_{11}^* = 0.5732\text{V/cmOe}$, which is one order of magnitude enhancement of the coupling coefficient.

6 Concluding remarks

In summary, we have extended Rayleigh's formalism on periodic conductive composites to a magneto-electroelastic composite consisting of arbitrarily distributed or periodic arrays of elliptic cylinders under anti-plane shear deformation, in-plane electric field and in-plane magnetic intensities. The cylinders can be homogeneous or confocally multicoated. Expressions for the elastic, electric and magnetic potentials for the cylinders and the matrix are derived, and used to compute the effective moduli. This extension is a hybrid technique: the admissible potentials for the matrix and inclusions are expanded in complex planes, while the interface conditions are directly satisfied by using elliptic coordinates. It is shown that the effective properties solely depend on one particular constant B_{1R}^Φ among the infinite number of expansion coefficients. Finally, as a practical example, explicit numerical calculations for field distributions and the magnetoelectric effects in $\text{BaTiO}_3\text{-CoFe}_2\text{O}_4$ and $\text{BaTiO}_3\text{-Terfenol-D-CoFe}_2\text{O}_4$ composites are presented and discussed. This example shows the important difference between the case of BTO fibers in a CFO matrix from the case of BTO fibers coated Terfenol-D in a CFO matrix. We expect that these results will be beneficial as design tools for functionally graded tunable composites.

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Table 1: Material parameters of BaTiO_3 , CoFe_2O_4 (Li and Dunn, 1998a), and Terfenol-D (Liu *et al.*, 2003, Liu *et al.*, 2004)

Figure 1: The cross-section of the fiber composite.

Figure 2: A schematic representation of a unit cell.

Figure 3: Potential contours for a square array composite ($f = 0.2$, $\bar{\varepsilon}_{zx} = 0$, $\bar{E}_x = 0$, $\bar{H}_x = 1\text{C/ms}$) (a)-(c) BTO fibers embedded in a CFO matrix, (d)-(f) BTO fibers coated Terfenol-D embedded in a CFO matrix, (a), (d) Vertical displacement (m), (b), (e) Electric potential (V), (c), (f) Magnetic potential (C/s).

Figure 4: Effective moduli of a composite of BTO fibers in a CFO matrix versus fiber volume fractions (a) Effective elastic modulus, (b) Effective dielectric permittivity, (c) Effective magnetic permeability, (d) Effective piezoelectric modulus, (e) Effective piezomagnetic modulus, (f) Effective magnetoelectric coefficient.

Figure 5: Effective moduli of a composite of BTO fibers coated Terfenol-D in a CFO matrix versus fiber volume fractions (a) Effective elastic modulus, (b) Effective dielectric permittivity, (c) Effective magnetic permeability, (d) Effective piezoelectric modulus, (e) Effective piezomagnetic modulus, (f) Effective magnetoelectric coefficient.

Figure 6: Effective magnetoelectric voltage coefficient of the composite versus the fiber volume fraction.

*2nd International Conference on Material Modelling
incorporating the 12th European Mechanics of
Materials Conference*

第二屆國際材料模擬會議
暨
第十二屆歐洲材料力學會議

服務機關：國立交通大學土木工程學系
姓名職稱：郭心怡 助理教授
前往國家：法國Ecole des Mines de Paris
出國期間：100/08/31~09/02
報告日期：100/09/13

國科會補助專題研究計畫項下出席國際學術會議心得 報告

日期：99 年 9 月 13 日

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計畫名稱	含橢圓纖維壓電壓磁複合材料之電磁耦合現象		
出國人員姓名	郭心怡	服務機構及職稱	國立交通大學土木工程學系助理教授
會議時間	100 年 8 月 31 日至 100 年 9 月 2 日	會議地點	法國 Ecole des Mines de Paris
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發表論文題目	(中文)磁電鍍膜纖維複合材料之等效行為 (英文)Effective behavior of coated fibrous magnetoelectric composites		

一、參加會議經過

國際材料模擬會議（**International Conference on Material Modelling**）為每兩年舉辦之會議，今年為第二屆並與歐洲材料力學會議合併舉行。今年投稿及與會者眾，約七百人次。會議自8/31 起至9/2 間於法國巴黎礦業大學Ecole des Mines de Paris舉行，本人的報告於8/31 第一場次第一位舉行，並受邀擔任該場次之Session Chair；報告進行順利，也能理解並回答提問。此次會議為歐洲大陸會議，且場次眾多並無任何參觀觀光行程，純為學術交流與思想激盪。



二、與會心得

國際材料模擬會議雖因主辦者主要為歐洲國家（第一屆於德國Dortmund舉行）以致與會者多為歐洲大陸之學者或研究生，然由於以材料模擬為主題的會議甚少，國際間材料模擬領域內的許多學者因此皆會與會以彼此交流。藉由聆聽報告、問答互動，拓展大家的研究視野，促使自己保有研究熱忱與反思能力。

此次會議於Ecole des Mines de Paris舉行，地處巴黎拉丁區，位於盧森堡公園旁，附近有多所大學，學術風氣鼎盛，且離巴黎聖母院僅約一公里。另會議所提供的餐點及設備雖然簡單樸實，但已足以使會議流暢進行。



三、建議

由於中午用餐時間需外出自行用餐，雖可便於瀏覽巴黎景緻，然卻減少與會人員間之交流機會，甚為可惜，建議午餐仍應由大會統籌辦理。

四、攜回資料名稱及內容

1. 會議行程
2. Book of abstracts 會議文章摘要

國科會補助計畫衍生研發成果推廣資料表

日期:2011/09/13

國科會補助計畫	計畫名稱：含橢圓纖維壓電壓磁複合材料之電磁耦合現象
	計畫主持人：郭心怡
	計畫編號：99-2221-E-009-053-學門領域：結構應力
無研發成果推廣資料	

99 年度專題研究計畫研究成果彙整表

計畫主持人：郭心怡			計畫編號：99-2221-E-009-053-				
計畫名稱：含橢圓纖維壓電壓磁複合材料之電磁耦合現象							
成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	1	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	4	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無
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	成果項目	量化	名稱或內容性質簡述
科 教 處 計 畫 加 填 項 目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

☒ 達成目標

☐ 未達成目標（請說明，以 100 字為限）

☐ 實驗失敗

☐ 因故實驗中斷

☐ 其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：☒ 已發表 ☐ 未發表之文稿 ☐ 撰寫中 ☐ 無

專利：☐ 已獲得 ☐ 申請中 ☒ 無

技轉：☐ 已技轉 ☐ 洽談中 ☒ 無

其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）