



International Journal of Systems Science

Publication details, including instructions for authors and subscription information:

<http://www.tandfonline.com/loi/tsys20>

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Published online: 24 Jan 2011.

To cite this article: Van-Tsai Liu, Chun-Liang Lin & Home-Young Wing (2012) New realisation of Preisach model using adaptive polynomial approximation, International Journal of Systems Science, 43:9, 1642-1649, DOI:

[10.1080/00207721.2010.549585](https://doi.org/10.1080/00207721.2010.549585)

To link to this article: <http://dx.doi.org/10.1080/00207721.2010.549585>

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New realisation of Preisach model using adaptive polynomial approximation

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(Received 12 February 2010; final version received 2 December 2010)

Modelling system with hysteresis has received considerable attention recently due to the increasing accurate requirement in engineering applications. The classical Preisach model (CPM) is the most popular model to demonstrate hysteresis which can be represented by infinite but countable first-order reversal curves (FORCs). The usage of look-up tables is one way to approach the CPM in actual practice. The data in those tables correspond with the samples of a finite number of FORCs. This approach, however, faces two major problems: firstly, it requires a large amount of memory space to obtain an accurate prediction of hysteresis; secondly, it is difficult to derive efficient ways to modify the data table to reflect the timing effect of elements with hysteresis. To overcome, this article proposes the idea of using a set of polynomials to emulate the CPM instead of table look-up. The polynomial approximation requires less memory space for data storage. Furthermore, the polynomial coefficients can be obtained accurately by using the least-square approximation or adaptive identification algorithm, such as the possibility of accurate tracking of hysteresis model parameters.

Keywords: hysteresis; Preisach model; polynomial approximation

Notation

$u(t)$	signal input	T	transpose operation
$f(t)$	Preisach model output	$x(t)$	function of the input
$\mu(\alpha, \beta)$	Preisach density function	η	constant step size
$\gamma_{\alpha\beta}[u(\cdot)](t)$	hysteresis operator	N	normalisation factor
$\alpha(\beta)$	hysteresis operator corresponds to 'up' ('down') switching values of input		
$\pm u_{\text{sat}}$	divides the range of input data		
$u(\tau)$	input signal		
$\{M_1, M_2, \dots, M_{n-1}\}$	peaks of the previous input signal		
$\{m_1, m_2, \dots, m_{n-1}\}$	valleys of the previous input signal		
$f_{\alpha\beta}(M, m)$	polynomial function		
$c_i(M)$	indicates the coefficient corresponding with M		
$c_0(M), c_1(M), c_2(M)$	polynomial coefficients		
f_n	represents the effect of previous input		
$f_p(t)$	polynomial approximation		
$e(t)$	output error		
$f(t)$	plant output		
$f_p(t)$	adaptive model output		
θ	update algorithm for the vector		

1. Introduction

Hysteresis is a nonlinear phenomenon occurring in many engineering devices, such as the micro-piezoelectric actuators (PZA), the shape memory alloys (SMA) and the ferromagnetic elements (Hughes and Wen 1995; Agrawal, Elshafei, and Song 1997; Bertotti 1988; Mayhan, Srinivasan, Watechagit, and Washington 2000; Torre 2000; Dlala, Saitz, and Arkkio 2005). A system with hysteresis is usually difficult to describe accurately and may result in unstable behaviours in closed loop without appropriate feedback control. Therefore, an accurate model of hysteresis is the key of developing suitable control algorithms or applications with these systems (Croft, Shed, and Devasia 2001; Jia, Luo, and Chen 2005). Several models have been developed to demonstrate systems with hysteresis, and the classical Preisach model (CPM) has been known as the most familiar one to demonstrate hysteresis behaviours (Mayergoyz

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1991); this model can be further represented by an infinite but countable first-order reversal curves (FORCs), which can be measured in many experimental applications. The conventional approach to realise the CPM with an hysteresis system approximates the representation by sampling a finite number of FORCs and preserves the sample data in tables (Ge and Jouaneh 1996, 2005; Song, Zhao, Zhou, and De Abreu-Garcia 2005). The model output is then calculated from the tabled data and the given input by performing simple linear interpolation. This approach, however, face two drawbacks. One is that it requires a large amount of memory space to obtain an accurate prediction of hysteresis because the number of FORCs and the sampling resolution should be increased. Another drawback is that it is difficult or even impossible to adjust the tabled data to immediately reflect the parameter variations with ageing or timing effect. Hence, the resulting accuracy may degrade with time and eventually needs to rebuild the table.

The purpose of this article is to include an instant adjustment mechanism which had not been adequately concluded in the past study. The coefficients of the proposed approximating polynomial can be obtained by using the least-square optimisation (Recktenwald 2001; Villien and Ostertag 2009) or the adaptive identification algorithm (Haykin 2002), so the tracking of hysteresis model parameters is possible. For a review of alternate identification approaches, one may consult Hong et al. (2008) and Taarit, Belkoura, Ksouri, and Richard (2011). The proposed approach has been applied in computer simulations of the prediction of CPM output. We apply the least mean square (LMS) adaptive algorithm to identify the polynomial coefficients of the FORCs. The demonstrating results compared with deduced results from the table look-up method yields similar performance in model accuracy while using less memory size. Furthermore, the proposed approach combined with the adaptation feature can reach the goal of modifying the parameters of CPM instantaneously rather than rebuilding the tables.

2. Traditional realisations of CPM

This section introduces the mathematical form and the numerical implement of the CPM – the model of hysteresis nonlinearity.

2.1. Classical Preisach model

The CPM (Mayergoyz 1991; Jia et al. 2005) represents an hysteresis system with input $u(t)$ and output $f(t)$ in

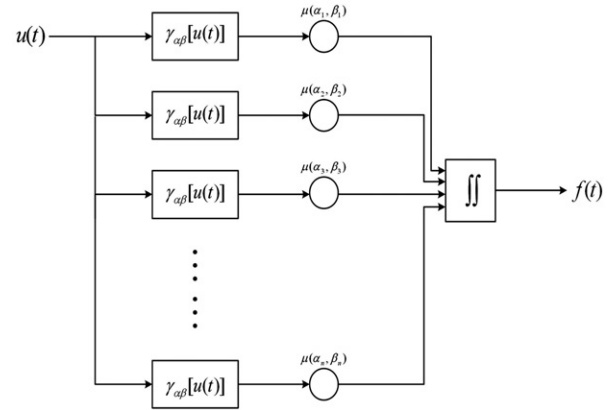


Figure 1. Interpretation of the CPM.

a double integral equation below:

$$f(t) = \int \int_{\alpha \geq \beta} \mu(\alpha, \beta) \gamma_{\alpha\beta}[u(\cdot)](t) d\alpha d\beta, \quad (1)$$

where $\gamma_{\alpha\beta}[u(\cdot)](t)$ is an hysteresis operator with its value being 1 when $u(t) \geq \alpha$, or -1 when $u(t) \leq \beta$, or retaining the value of previous moment when $\alpha > u(t) > \beta$ and the function $\mu(\alpha, \beta)$, often referred to as the Preisach density function, serves as the weighting on the corresponding hysteresis operator. This realisation, however, is considered to be time-consuming because numerical implementation of double integrals is needed. The idea of the CPM, therefore, is to regard an hysteresis transducer as the superposition of an infinite set of weighted hysteresis operators shown in Figure 1 (Mayergoyz 1985; Iyer and Shirley 2004; Tan and Baras 2005).

The values α and β of an hysteresis operator correspond to ‘up’ and ‘down’ switching values of input, respectively. Here, we assume that $\alpha \geq \beta$. The operator is characterised by its switching values α and β , and is depicted by a rectangular loop on its input–output phase diagram in Figure 2. As shown in the figure, the operator output of $\gamma_{\alpha\beta}[u(\cdot)](t)$ remains to be -1 until the input is above the value α and the output remains to be $+1$ until the input is below the value β . A practical anticipation of actual applications is the limits of integration which require us to set saturation values of input, that is $u_{\text{sat}} \geq \alpha \geq \beta \geq -u_{\text{sat}}$.

Another common and numerically more efficient realisation exploits the property of hysteresis operator so that the output value $f(t)$ can be realised by a set of the so-called the FORC, which can be obtained directly from the measured experimental data and need not identify Preisach density function. We briefly discuss this realisation below and the employed notations which follow closely. To assume that the interested range of input signals is limited in $-u_{\text{sat}} \leq u(t) \leq u_{\text{sat}}$,

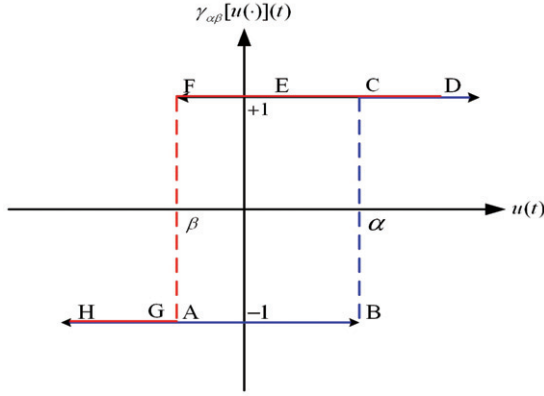


Figure 2. The hysteresis operator: the output follows the trajectory $ABCD$ when the input is increasing, and follows $EFGH$ when the input is decreasing.

an FORC denoted by $f_{\alpha\beta}(M, m)$ is obtained as the output of the hysteresis system when its input $u(t)$ starts at $-u_{\text{sat}}$, then increases to value of M , and finally decreases to value of m . Note that for a fixed M value we can obtain a curve $f_{\alpha\beta}(M, m)$ along with varying m . In terms of FORCs, this approach realises $f(t)$ from $u(t)$ ascending at the moment t below

$$f(t) = -f^+ + \sum_{k=1}^{n-1} [f_{\alpha\beta}(M_k, m_k) - f_{\alpha\beta}(M_k, m_{k-1})] + [f_{\alpha\beta}(u(t), u(t)) - f_{\alpha\beta}(u(t), m_{n-1})], \quad (2)$$

where f^+ is the value of $f(t)$ for $u(t) \geq u_{\text{sat}}$ and the series $\{M_1, M_2, \dots, M_{n-1}\}$ and $\{m_1, m_2, \dots, m_{n-1}\}$ are peaks and valleys of the previous input signal $u(\tau)$, $\tau < t$ conforming to the wiping-out property (Mayergoyz 1991). Similarly, the output value $f(t)$ for $u(t)$ descending at the moment t is given by

$$f(t) = -f^+ + \sum_{k=1}^{n-2} [f_{\alpha\beta}(M_k, m_k) - f_{\alpha\beta}(M_k, m_{k-1})] + [f_{\alpha\beta}(M_{n-1}, u(t)) - f_{\alpha\beta}(M_{n-1}, m_{n-2})]. \quad (3)$$

It is important to notice that in demonstration, the FORCs are experimentally obtained and stored in memory and retrieved when evaluating $f(t)$ for a given input $u(t)$. To evaluate $f(t)$ exactly via this approach, therefore, requires to store countable infinite FORCs; in practice, approximation is necessary to yield a viable result of the realisation.

2.2. Table look-up method

The table look-up method for CPM realisation creates a table for storing samples of several FORCs, it then employs the linear interpolation technique for evaluating the model output $f(t)$ from the input $u(t)$.

Divide the range of input data from $-u_{\text{sat}}$ to u_{sat} uniformly into N levels with $u_i = -u_{\text{sat}} + (i-1)\Delta$, $\Delta = 2u_{\text{sat}}/(N-1)$ for $i = 1, \dots, N$. The table stores the samples of N FORCs with the i -th FORC having data $f_{\alpha\beta}(u_i, u_j)$ with $i = 1, \dots, N$ and $j = 1, \dots, i$. Note that these data can be experimentally obtained; for example, the value $f_{\alpha\beta}(u_i, u_j)$ is the plant output with the input data $u(t)$ increasing from $-u_{\text{sat}}$ to u_i and then decreasing from u_i to u_j . It is clear that the number of data needed for constructing the table is $N(N+1)/2$.

In evaluating the model output $f(t)$ using (2) and (3), we have to compute the sample of FORC, for example $f_{\alpha\beta}(x, y)$. If (x, y) perfectly matches (u_i, u_j) , then an exact FORC sample can be obtained. In most cases, (x, y) will lie on an area surrounded by sample nodes (u_i, u_j) . Hence, most table look-up search approaches the approximate sampled value $f_{\alpha\beta}(x, y)$ via the neighbouring nodes by linear interpolation (Ge and Jouaneh 1996). The performance of linear interpolation technique, obviously, will be improved if the number N is increased. The cost, however, is the increase in the size of required memory for storing the data. The advantage of table look-up method is its simplicity. In the following section, we present a method which requires less memory space and possesses the capability to solve for the phenomenon of parameter drift in the hysteresis systems.

3. New realisations of CPM

This section develops a polynomial approximation (PA) for CPM realisation and derive the adaptive algorithm for parameter tracking of the hysteresis systems.

3.1. Polynomial approximation

It is common to investigate the measured data using an analysable function called curve fitting and the PA. Our goal is to fit the samples of a finite number of FORCs by a polynomial. Realisation of the PA, similar to the table look-up method, also exploits the FORC approach to realise an hysteresis system. The difference between the table look-up method and the PA is that the former stores sample data of each FORC directly in a table while the latter transforms sample data of each FORC into a set of parameters by polynomial curve-fitting techniques and stores these parameters in the table. In practice, the model's output can be obtained by evaluating the corresponding polynomial and introducing simple linear interpolation techniques. Detailed procedure and mathematical expression of the PA are discussed as follows.

For simplicity, we consider a second-order polynomial function. Hence, for an FORC $f_{\alpha\beta}(M, m)$ with a specific M , we find a polynomial for approximation, denoted as

$$f_{\alpha\beta}(M, m) \approx c_0(M) + c_1(M)m + c_2(M)m^2. \quad (4)$$

Note that approximation with the least-square error is adopted here (Recktenwald 2001). Specifically, the FORC $f_{\alpha\beta}(M, m)$ for a specific M can be experimentally measured for various m values; these data are used to compute, via the least-square error technique, the coefficients $c_0(M)$, $c_1(M)$, $c_2(M)$. Hence, instead of storing the sample data of an FORC in the conventional table look-up method, the PA converts the sampled data by a set of coefficients. Each FORC corresponds to three coefficients through the second-order polynomial. If one considers N FORCs with the value of N which has been uniformly divided among the input range with $u_i = -u_{\text{sat}} + (i-1)\Delta$, $\Delta = 2u_{\text{sat}}/(N-1)$ for $i = 1, \dots, N$, then only $3N$ coefficients denoted as $c_j(u_i)$, $j = 0, 1, 2$, and $i = 1, \dots, N$ are required for characterising the N FORCs. For an FORC $f_{\alpha\beta}(M, m)$ with $u_i < M \leq u_{i+1}$, the corresponding polynomial coefficients are obtained by the simple linear interpolation as follows:

$$\begin{aligned} c_j(M) &= \frac{(u_{i+1} - M)c_j(u_i) + (M - u_i)c_j(u_{i+1})}{u_{i+1} - u_i} \\ &= \frac{(u_{i+1} - M)c_j(u_i) + (M - u_i)c_j(u_{i+1})}{\Delta}, \quad j = 0, 1, 2. \end{aligned} \quad (5)$$

In realisation of the PA method, we can derive information from below that a simple linear form can be obtained. Consider first the ascending input and get a further explanation of (2) from the following equation

$$f(t) = f_n + [f_{\alpha\beta}(u(t), u(t)) - f_{\alpha\beta}(u(t), m_{n-1})], \quad (6)$$

where f_n represents the effect of previous input and is

$$f_n = -f^+ + \sum_{k=1}^{n-1} [f_{\alpha\beta}(M_k, m_k) - f_{\alpha\beta}(M_k, m_{k-1})]. \quad (7)$$

Using the PA (4) to replace $f_{\alpha\beta}(u(t), u(t))$ and $f_{\alpha\beta}(u(t), m_{n-1})$ in (6) yields

$$\begin{aligned} f(t) &\approx f_P(t) \\ &= f_n + [c_2(u(t))u(t)^2 + c_1(u(t))u(t) + c_0(u(t))] \\ &\quad - [c_2(u(t))m_{n-1}^2 + c_1(u(t))m_{n-1} + c_0(u(t))] \\ &= f_n + c_2(u(t))(u(t)^2 - m_{n-1}^2) + c_1(u(t))(u(t) - m_{n-1}), \end{aligned} \quad (8)$$

where we use $f_P(t)$ to denote the PA. Note that the coefficient $c_0(u(t))$ is not needed in realisation; this

property further reduces the required memory for storing the coefficients of each polynomial corresponding to an FORC.

Since the input $u(t)$ may not be equal to the value of any u_i , $i = 1, \dots, N$, the linear interpolation (5), therefore, is employed for realisation. If $u_i < u(t) \leq u_{i+1}$, then we use the simple linear interpolation (5) to substitute for $c_1(u(t))$ and $c_2(u(t))$ in (8), after rearranging, yielding a simple linear form as shown below

$$f_P(t) = \theta^T x(t), \quad (9)$$

where the superscript T denotes transpose and

$$\theta = \begin{bmatrix} f_n \\ c_2(u_i) \\ c_2(u_{i+1}) \\ c_1(u_i) \\ c_1(u_{i+1}) \end{bmatrix}, \quad x(t) = \begin{bmatrix} 1 \\ \frac{[u_{i+1} - u(t)][u^2(t) - m_{n-1}^2]}{\Delta} \\ \frac{[u(t) - u_i][u^2(t) - m_{n-1}^2]}{\Delta} \\ \frac{[u_{i+1} - u(t)][u(t) - m_{n-1}]}{\Delta} \\ \frac{[u(t) - u_i][u(t) - m_{n-1}]}{\Delta} \end{bmatrix}.$$

Note that the linear expression of the model output also makes the derivation of adaptive algorithm easier for tracking time-varying parameters of an hysteresis system. This part will be discussed later. For the input $u(t)$ descending at the moment t with $u_i < M_n \leq u_{i+1}$, via the interpolation technique and the procedure above, the model output can be similarly derived, and have the same result as (9). Thus, the difference between the ascending input and descending input lies on the determination of FORCs for interpolation, the former uses the input $u(t)$ while the latter uses the previous peak M_n . The model realisation for either ascending or descending input results in a simple linear equation in (9). It is worth pointing out that the polynomial coefficients indicate the parameters of the CPM and are suitable for model identification because they are adjustable and hence exhibiting flexibility in identification.

In summary, the proposed PA method requires to store $N(P+1)$ coefficients of P -th order polynomials to characterise N FORCs. If the second-order polynomial is used, then the required table size is $3N$. Compared with the table look-up method requiring $N(N+1)/2$ memory, the proposed method is more efficient. The number of sampled data in the look-up table realisation, $N(N+1)/2$, will be larger than $N(P+1)$ when $N > 2P+1$, i.e. $P < \frac{N}{2} - 1$.

We propose a simple criterion for selecting the polynomial order P in the following.

Step 1: Set $P=1$. Set N_ε as a threshold of order for the candidate polynomials. Choose the number N and construct the table as a

baseline for determining the candidate polynomials.

Step 2: Determine the corresponding error cost $\varepsilon_N = \|f(t) - f_N(t)\|$ where $f_N(t)$ is the approximation function based on the table look-up method and $\|\cdot\|$ is the L_2 -norm.

Step 3: Determine the error cost $\varepsilon_P = \|f(t) - f_P(t)\|$. If $\varepsilon_P < \varepsilon_N$ then go to Step 7, otherwise proceed to the next step.

Step 4: $P \leftarrow P + 1$.

Step 5: If $P < \frac{N}{2} + N_\varepsilon$ then go to Step 3, otherwise proceed to the next step.

Step 6: If $N_\varepsilon \geq 1$ then $N_\varepsilon \leftarrow N_\varepsilon - 1$, otherwise stop and report 'no solution'.

Step 7: Output the polynomial order P and stop.

For example, the polynomial order $P = 1$, and $N = 100$ and $N_\varepsilon = 1$ were set in our simulation and experimental study.

It is important to notice that the proposed method may not obtain more accurate approximation than the table look-up method because the low-order PA may yield a large error in some regions of an FORC. The most important advantage of the PA method, in our view, is that the polynomial coefficients can be adaptively modified to track a time-varying hysteresis system. This is developed in the following sections.

3.2. Adaptive PA

Based on PA, we obtain a set of coefficients that can be used to describe FORC; however, these coefficients may be inaccurate. A system model with inaccurate parameters can be regulated by updating its parameters via adaptive algorithms: the adaptive model output is compared with the plant output, and the error is used to update the adaptive system parameters. The steepest descent algorithm is an illustration of the most common and easiest one of adaptive algorithms. Assume the adaptive model output $f_p(t) = (\theta, x(t)) = \theta^T x(t)$ is a function of parameters θ and the input x , and the plant output $f(t)$ is a function of the input t .

Since we have obtained that via the PA, the model output can be expressed in a linear form (9), the adaptive techniques such as the LMS or recursive least-squares (RLS) method (Haykin 2002) can be directly employed to derive the adaptive algorithm for adjusting the vector θ in (9). For simplicity, we elaborate the LMS algorithm below; the RLS algorithm can be similarly derived.

The LMS uses the idea of the steepest descending method. It first computes the error $e(t)$ between the

system output $f(t)$ and the model output $f_p(t)$ as given below

$$e(t) = f(t) - f_p(t) \quad (10)$$

$$= f(t) - \theta^T x(t). \quad (11)$$

The LMS update algorithm for determining θ becomes

$$\theta(t) = \eta e(t)x(t), \quad (12)$$

where η is a constant step size. The effectiveness of the adaptive algorithm will be demonstrated in the later simulation study.

4. Simulation results and discussion

Here, we give the computer simulations to demonstrate our proposed method. The following simulations use a Gaussian-Gaussian Preisach density function (Henze and Rucker 2002) which is defined as follows:

$$\mu(\alpha, \beta) = N_0 \exp \left\{ -\frac{\left(\frac{\alpha-\beta}{2} - H_0\right)^2}{2\sigma^2 H_0^2} - \frac{\left(\frac{\alpha+\beta}{2}\right)^2}{2\sigma^2 H_0^2} \right\}. \quad (13)$$

The parameters N_0 is the normalised factor, the maximum H_0 and the standard deviation σ are unknown, note that all the output regions below are normalised. The reference outputs of CPM are calculated by numerical integral using Matlab. The problem is that a double integral must be solved numerically and hence extra amount of computational time is needed.

4.1. Polynomial approximation

The first experiment considers the performance of the PA method when the parameters in (13) are determined. The parameter settings for this experiment are: $1 \geq \alpha \geq \beta \geq -1$, $N_0 = 1$, $\sigma = 0.5829$ and $H_0 = 0.4251$. To implement the PA method, first we need to identify the polynomial coefficients. A total number of eight FORCs are used and are shown in Figure 3. The fourth-order polynomials ($P = 4$) are used here and each polynomial uses 100 samples ($N = 100$), which are obtained by uniformly dividing the input range to the corresponding FORC during the procedure of polynomial curve-fitting technique. Table 1 lists the polynomial coefficients and its first column denotes the values of M of $f_{\alpha\beta}(M, m)$. Note that the required number of memory to store the parameters is 40. Simulation results of the experiment are shown in Figures 4–6.

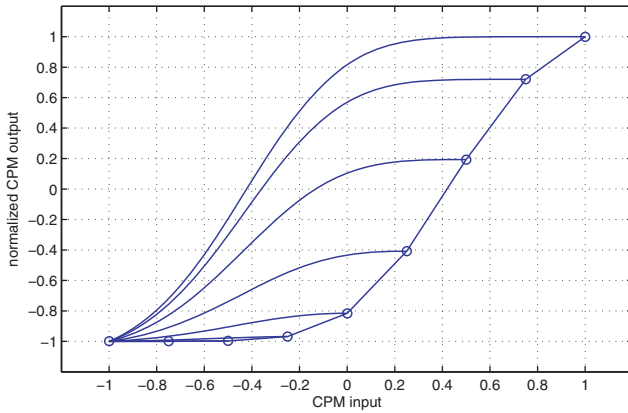


Figure 3. FORCs with the value of eight uniformly divided among the input range of the CPM in the first experiment.

Table 1. Polynomial coefficients.

M	$C_4(M)$	$C_3(M)$	$C_2(M)$	$C_1(M)$	$C_0(M)$
-0.75	-0.0094	-0.0359	-0.0508	-0.0316	-0.6697
-0.5	-0.0431	-0.1453	-0.178	-0.0902	-0.6769
-0.25	-0.108	-0.3463	-0.3755	-0.1307	-0.6559
0	-0.1078	-0.4629	-0.514	-0.0389	-0.5417
0.25	0.1624	-0.2359	-0.5749	0.1936	-0.2858
0.5	0.5861	0.0416	-0.8154	0.4383	0.0655
0.75	0.7743	0.0146	-1.1124	0.6578	0.3716
1	0.6624	-0.1393	-1.1648	0.821	0.528

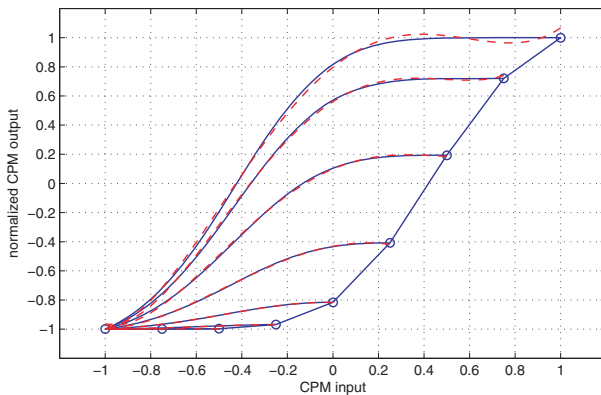


Figure 4. FORCs (solid line) compared with their PAs (dashed line).

In Figure 4, we compare the FORCs in Figure 3 with their PA values. We can observe that when the input range of an FORC is large, the differentials between the FORC and its PA values are apparent. This is because when the input is approximate to the saturation values $\pm u_{\text{sat}}$, it has very slight variation of

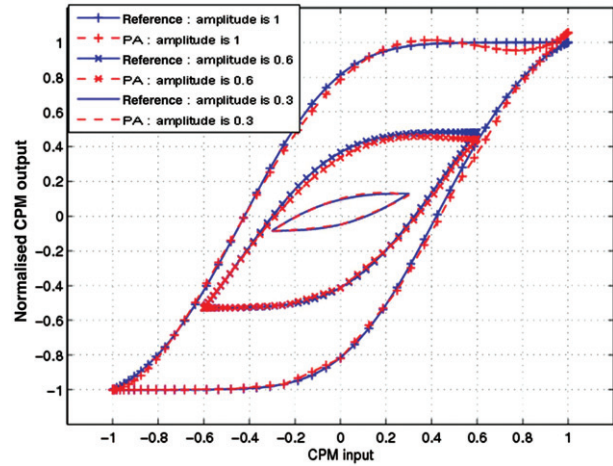


Figure 5. Simulation results for the CPM with the sinusoidal inputs and amplitudes of 1, 0.6 and 0.3. The CPM outputs (solid line) vs. those of PA (dashed line).

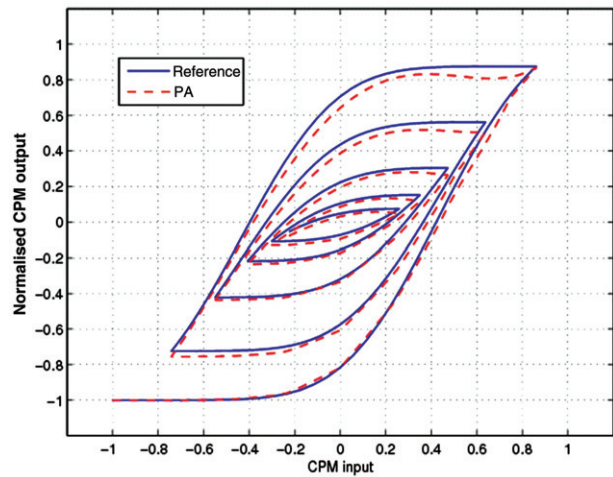


Figure 6. Simulation results in the first experiment when the CPM input is a sinusoidal signal with decaying amplitude. The CPM outputs (solid line) vs. PA (dashed line).

CPM output and the polynomial is not suitable for the curve-fitting technique. To improve the accuracy for realisation of the CPM, we can use the saturation operation or linear interpolation in the regions that the PA does not work well. It is easy but tedious, and we do not include these concepts in this article; even so, the PA method is useful when the input region of CPM is not large. Figures 5 and 6 illustrate this fact. Different sinusoidal signals with various amplitudes 1, 0.6 and 0.3, frequency of 0.5 Hz and sampling frequency of 1 kHz are applied to the CPM, see Figure 5 for the results. Excellent coincidences between the

predictions obtained by the PA method and the CPM reference outputs are observed, except the amplitude of input is 1. This result is due to the bad approximation for the FORC and is expectable.

Consider next the CPM input with decaying amplitude as described below is applied:

$$u(t) = -e^{-0.15t} \cos(\pi t). \quad (14)$$

The total amount of time is set to be 8 s, and the sampling rate is 1 kHz. The input–output phase plot of CPM and its approximation are illustrated together in Figure 6. The PA method causes a bad prediction in the wide input range, and the prediction for small input region will result in a bias. However, the values of variation of the realised output and reference output are very close.

4.2. Adaptive polynomial approximation

In the second experiment, the adaptive polynomial approximation (APA) method possessing the capability of tracking time-varying hysteresis system is demonstrated. We consider that a CPM is a system which consists of drifted parameters: $\sigma = 0.47$ and $H_0 = 0.53$. A sinusoidal CPM input with an amplitude of 0.8, a frequency of 0.5 Hz and a sampling frequency of 1 kHz is used to adjust the polynomial coefficients by the LMS update algorithm (12) with $\eta = 0.05$. Here we use the coefficients obtained in the first experiment as the initial values of the updated parameters.

New polynomial coefficients are obtained via the updating procedure and they are used to realise the CPM output. We apply the same inputs in the first experiment to the altered CPM and simulated results of realisation for the CPM output are shown in Figures 7 and 8. The input–output phase diagram of CPM with drifted parameters and realisation from the original polynomial coefficients are plotted together in Figures 7 and 8. It can be seen that the differences in the realisation results are improved because of the parameter updating feature fulfilled by (12) of the method.

5. Conclusions

A system with hysteresis nonlinearity is often difficult to precisely describe and the nonlinearity may result in unstable behaviours in a feedback loop without appropriate control. In this article, we adopt the Preisach model to characterise the hysteresis behaviour and propose to use the PA for model realisation. The approach is applied to model the motion of a real micro-PZA. This outperforms the conventional table

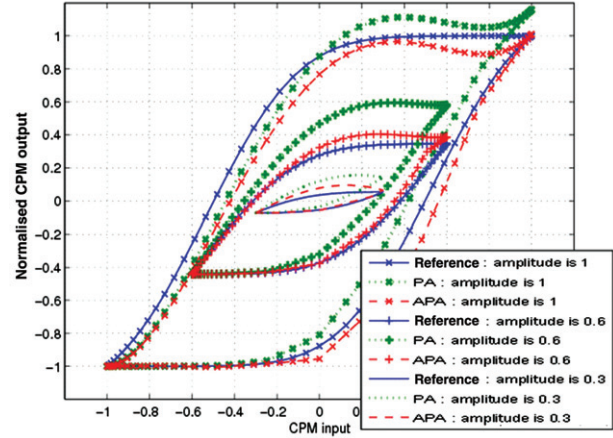


Figure 7. Simulation results in the second experiment when the CPM input are sinusoidal signals with amplitudes 1, 0.6 and 0.3. The CPM outputs (solid line) vs. adaptive PA (APA) (dashed line) and PA (dotted line).

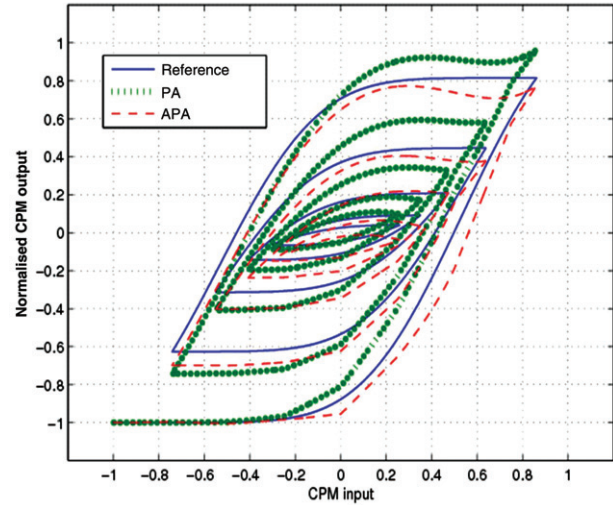


Figure 8. Simulation results in the second experiment when the CPM input is a sinusoidal signal with decaying amplitude. The CPM outputs (solid line) vs. APA (dashed line) and vs. PA (dotted line).

look-up method in the memory space utilised and capability of the parameter tracking of hysteresis elements.

Acknowledgements

This article is in memory of Dr Mu-Huo Cheng who was a professor in the Department of Electrical and Control Engineering, National Chiao Tung University, Hsinchu, Taiwan, ROC. This research was sponsored by National Science Council of Taiwan under the Grant NSC96-2221-E-150-016.

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