

Physically-Based Patination for Underground Objects

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Abstract

Although current photorealistic rendering techniques can produce very impressive images, the rendered objects are often too clean and shiny. Thus, the resulting images look unnatural. This paper proposes a physically-based model to simulate the appearance of patinas on ancient Chinese bronzes. Buried in the soil for thousands of years, many patinas are found on the surface of ancient bronzes as a result of the aging process and the physical and chemical conditions of the soil environment. The development of patinas is modulated herein by L-systems according to tendencies based on the environmental factors and object geometry. The tendencies are employed to represent the accumulative effect of all factors on patination. The proposed model can be extended to simulate a variety of metallic patinas including the ancient Chinese bronzes discovered at San-hsing-tui, Sichuan, China.

1. Introduction

Computer graphics research has increasingly focused on enhancing the realism of computer generated images to produce more accurate illumination models and rendering schemes^{3, 14, 16, 21, 38}. However, the surfaces of most real-world objects are imperfect, whereas the rendered objects are often too clean to be realistic. The appearance of most real materials is changed because of the influence of nature, such as rain washing, dirt accumulation, sedimentation of deposits, and metal erosion. On the other hand, real-world objects are often covered with various kinds of imperfections that challenge the illusion of the real world. Therefore, environmental and aging effects of objects must be taken into account to enhance the realism of rendered images.

Many different kinds of blemishes occur on object surfaces over time. The metallic patina of underground objects is one of the most diverse among these imperfections. The constituents and patina formation processes vary according to the metal type. Moreover, metals might combine with mineral content in the soil to produce special patinas. Thus, metallic patinas of the underground objects often exhibit a multiplicity of color and appearance. This work focuses on synthesizing patinas on bronze, particularly the ancient Chinese bronze. Bronze objects were uncovered at San-hsing-tui, in the west of the Sichuan Basin, 40 km north of Chengtu, in the summer of 1986 as illustrated in Figure 1.

Many patinas appear on the surface of these ancient bronzes as a result of the aging process and physical and chemical conditions of soil environments since they were buried in soil for thousands of years. A procedural approach for modeling and rendering patinas on the ancient Chinese bronze is proposed herein.



Figure 1: Sample ancient bronzes discovered at the San-hsing-tui.

Among the various approaches for generating imperfections on metallic objects in a rendered image, texture mapping¹⁷ is easier and less costly than the others. Texture mapping, although providing some degree of reality, generally suffers from distortions and provides no automation for generating blemishes. Procedural texture synthesis^{1, 8, 18, 25, 39} is perhaps a more appropriate approach for generating specific kinds of defects. Becket and Badler¹ developed fractal

subdivision techniques and applied them to simple distribution models to generate a wide class of surface blemishes. They adopted a rule-based system to position the surface imperfections and they specified the kind of defects by a natural language interface. Miller²⁵ discussed two distinct algorithmic approaches for determining surface accessibility, which is closely related to the distribution of patinas on tarnished surfaces. Hsu and Wong¹⁸ modeled the amount of dust accumulation, taking into account the surface inclination and exposure. They first determined a normal amount of accumulated dust based on surface properties, and then modified the value by some external factors. Based on this technique, Wong et al.³⁹ also developed a geometry dependent, texture generation framework for simulating the surface blemishes. This procedural framework allows the user to intuitively control the distribution of blemishes and automatically generate blemish details by utilizing geometric information. Dorsey and Hanrahan⁸ examined a metallic surface as a series of layers and then simulated the patina with a collection of procedural operators. They proposed a technique based on the Kubelka-Munk model²¹ to model the reflection and transmission of light through the layered patinas.

In light of above developments, the tendencies of distinct layers to model the patination potential according to an object's geometric and environmental factors are examined herein. The diffusion development of patinas is simulated by a set of production rules via the L-system. The resulting strings of the L-systems are graphically interpreted by a ray tracer. The rest of this paper is organized as follows. Section 2 describes the characteristics of underground bronze patinas. Section 3 presents the distribution of tendencies that reflect the influence of environmental factors on the development of patinas. The geometric properties of the objects are also discussed. The L-system interacts with the environment to impact the growing situations is depicted in Section 4. A simple illumination model to visualize the blemished model is developed in Section 5. Experimental results are discussed in Section 6, while a conclusion and future research considerations are provided in Section 7.

2. The Patination of Bronze

Ancient Chinese bronze, or tin bronze, is a copper-tin alloy with a small amount of lead. Although ancient bronzes have various proportions of alloy constituents for different purposes, copper is the primary component. Hence, the patinations of bronze and copper have essentially similar behavior. A thin tarnish layer that consists of cuprous oxides and cuprites is generated on the copper surface after exposure to the atmosphere for a period of time^{2, 8, 12, 27}. This firmly adherent reddish oxide layer provides the basis for which a green or blue-green patina will develop¹⁹. The primary chemical constituents of green patinas include copper carbonates, copper sulfates, as well as inorganic and organic

species. These chemical compounds gradually develop to form a well patinated and coherent surface with shining coloring under some particular conditions. Bronze surfaces are inevitably attacked by chlorine and mineralize a compact epidermis of atacamite in a burial environment²⁷. Coarse epidermis generally contains many rugged bubbles and tends to break down unevenly, resulting in variegated appearances. Copper chlorate produces a green surface with a powdery appearance if it forms quickly. The mineral salts of the soil periodically adhere to object surfaces to produce a specific kind of patina, named adherent patina.

Copper, soil ingredients, and the burial time period affect the characteristics and appearances of the patinas. The patinas typically discovered in a burial environment are as follows:

- *Tarnish* : A dull brown layer that formed quickly on objects changes gradually to a reddish color as they are exposed to the environment. This reddish layer is a thin appressed film of cuprite mineral and copper oxide on the object surfaces. The tarnish layer is normally covered with other patinas.
- *Cavitation* : A particular form of patinas caused by the formation of bubbles on bronze surfaces that has an uneven and grained appearance, perhaps with some cracks. The bubbles change the shape of objects and destroy the pattern on surfaces as depicted in Figure 2a.
- *Monomer patina* : Copper oxide generally gently interacts with carbohydrate and sulfide in the soil to generate a firm, smooth enamel of copper carbonate and sulfate. This layer is the characteristic green and blue colors of aged bronze as illustrated in Figure 2b.
- *Powdery patina* : A green patina with a powdery appearance is formed when bronze hastily reacts with chloride. This layer is a fragile patina that generally mars object surfaces whose basic components are copper chloride and copper chlorate as depicted in Figure 2c.
- *Adherent patina* : Clay or silt may combine with some patinas to form an adherent patina on the top layer that typically is grayish green or yellowish brown as illustrated in Figure 2d.

Natural patinas are produced by metallic corrosion. It is an electrochemical process, actually an oxidation-reduction reaction, in which some grains of metal act as anodes and corrode while others act as cathodes and are protected. Metal yields electrons and becomes positive ions at the anode. The positively charged ions either combine with some species in the electrolyte to produce metallic salts or diffuse toward the cathode to gain electrons and are reduced. The soil acts as the electrolyte to provide the conducting path between the anode and the cathode during underground corrosion. Hence, the soil corrosivity is the primary factor of corrosion in soil, excepting the metal itself. Corrosion in soil is a very complex phenomenon and can be only qualitatively described. Aeration, acidity values, salt content, electrical conductivity, and moisture content are among the corrosive characteristics

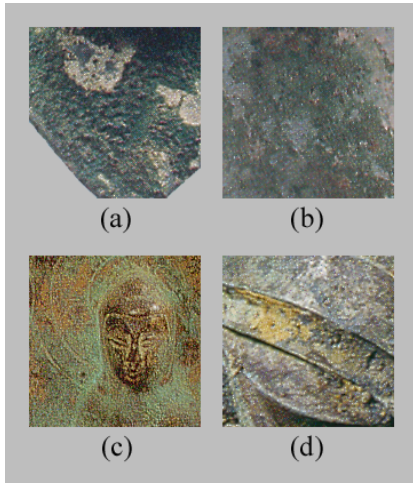


Figure 2: A collection of representative patinas.

of a soil environment^{2,9}. To date, poor aeration, high electrical conductivity, and high salt and water contents confessedly are typical of corrosive soils. Soil corrosion is primarily dependent on the moisture content of the soil since patinas pullulate more easily when the object surfaces contact very moist soil. The alteration of moisture influences soil aeration, value of acidity, and conductivity of electrolyte because it changes the content of oxygen, carbon dioxide, salt, and organic and inorganic substances dissolved in water.

3. Tendency Distribution

Corrosion in soil deteriorates metals by the chemical, mechanical, and biological action of the soil environment. Although the formation of patinas is complex and somewhat unknown, one could still more or less observe the underlying systematic distribution. The distributions of tendency³⁹, which represent the potential occurrence of patinas, must first be defined to model the influences of environmental factors on the development of patinas. In addition, the orientation and local geometric properties of objects are also affiliated with these distributions because the local properties can reflect the details of bronze disease. Thus, tendency T is defined at a surface point of interest as follows:

$$T = \alpha * C + \beta * A + \gamma * G + \delta * S$$

where C is the surface curvature
 A is the surface accessibility
 G is the tropism
 S is the characteristic of the soil
 α, β, γ , and δ are weighting constants

The surface curvature describes several significant surface properties such as bumps, hollows, and saddle points. The surface curvature is estimated by the approach³⁶ proposed by Turk since the polygonal representation is employed to

model the smooth object. Let $PQ_1, PQ_2, \dots$, and PQ_n represent the edges of an object. Let N be an approximate normal vector at vertex P . Then, θ_i is the angle between PQ_i and N . The radius estimate of the curvature for edge PQ_i is $r_i = \frac{|PQ_i|}{2} \cdot \tan(\theta_i)$. Hence the estimate of the minimum radius of the curvature at vertex P can be obtained as the minimum of all r_i 's. The estimate of the maximum principle curvature can be deduced from the curvature definition. Similarly, the minimum principle curvature can be estimated by deriving the maximum of all the r_i 's.

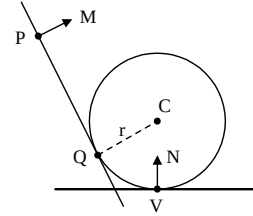


Figure 3: Tangent-sphere accessibility.

The surface accessibility is a measure of how easily a surface would be touched by a spherical probe. It describes the possibility of contact between an object's surface and the soil. Miller²⁵ presented a tangent-sphere method that measures the surface accessibility as displayed in Figure 3. The center C of the sphere, which is tangential to surface point V , is represented by $C = V + rN$, where N is the surface normal. The radius r of the sphere should be equal to the distance between the infinite plane containing a polygon and the center of the sphere, as defined by $r = (C - P) \cdot M$, where M is the polygon normal and P is the center of the polygon. Then, the value of radius can be derived as $r = \frac{(V-P) \cdot M}{1 - (N \cdot M)}$. The point $Q = V + r(N - M)$, which is the tangent point on the infinite plane, is checked whether it lies within the polygon or not. If it does, then the value of r is the accessibility of surface point V . Otherwise, the vertices of the polygon are adopted to compute the tangential radius, which is defined as $r = \frac{-(V-P)(V-P)}{2(N \cdot (V-P))}$.

The soil is a sophisticated, heterogeneous environment, and it is difficult to observe directly without destructive sampling methods²². Although many factors determine the corrosive properties of the soil, the moisture content of the soil is the most significant factor. Therefore, a solid texture is employed to describe the moisture content of the soil. The density of chlorine in the soil should also be depicted since the generation of copper chlorite is an unavoidable process in a burial environment. The same solid texture is utilized to represent the chloric density because the chloride always dissolves in groundwater. Moreover, the patina formation is also influenced by the groundwater current. Since water flows downward under the influence of gravity, upward surfaces may retain more water than downward ones. In addi-

tion, horizontal or inclined surfaces may retain more water than vertical surfaces⁸. Accordingly, *tropism* $G = N \cdot g$, is defined where g is the gravity and N is the surface normal, to indicate the water retention of a surface. Figure 4 depicts an experimental example of tendency distributions where high color values depict high tendency values, while the weighting parameters are described in Table 1. These two tendencies represent preferable conditions for two distinct patinas. Tendency1 describes the distribution of moisture on a Chinese bronze mask, and tendency2 denotes the distribution of chlorine.

	Accessi- bility	Gravity	Curvature			Soil
			Max	Min	Mean	
Tendency1	-0.2	0.15	0.65	-0.2	0.1	0.3
Tendency2	0.1	0.0	0.3	0.1	0.0	0.65

Table 1: The Weighted Parameters for Tendencies.

4. The Patination Model

The transport of metallic ions and electrons plays an essential role in the formation of patinas since patination is an oxidation-reduction reaction^{9, 27}. Metals become positive ions and move to cathodes to gain electrons through the oxidation. Ions may combine with chemical constituents of soil to form patinas in some preferable conditions during transport. The opportunities to form patinas increase with the number of metallic ions absorbed. Hence, L-systems can accurately simulate the transport of ions and the diffusion of patina formation on object surfaces.

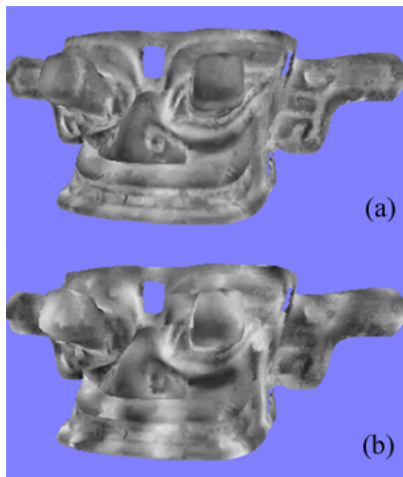


Figure 4: Tendency distributions. (a):Tendency1. (b):Tendency2.

A Lindenmayer system (L-system)²³ is a parallel rewriting system that generates a new string from an initial axiom according to the successor of matching production rules.

The resulting string can be graphically interpreted to visualize models in a post-processing step. The interpretation proceeds sequentially, with reserved modules acting as commands to a LOGO-style turtle³¹. L-systems are commonly adopted as a general framework for plant modeling. Although several models^{5, 22, 24, 30, 31, 32} were proposed for synthesizing images of plants based on the L-system, few considered the interaction between plants and the environment. Prusinkiewicz et al.³⁰ introduced an environmentally-sensitive L-system to simulate the influence of environment on plants by employing a simple function of the turtle's position to limit the growth of plants. Měch and Prusinkiewicz²⁴ extended the concept with a bi-directional communication module (termed an "open L-system") between two concurrent processes, plants and the environment. The growth of plants can be considered as a chain of causally linked events based on alteration of environments according to the definition of open L-systems. Accordingly, an L-system is proposed to simulate the development of patinas guided by the environmental tendencies.

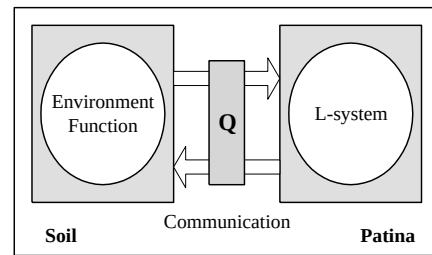


Figure 5: Conceptual framework of patination.

The patination process framework consists of two modules: the patina module and the soil module, as illustrated in Figure 5. A communication interface acts as a bridge between these two modules. A reserved symbol, Q , denotes the communications. The patina module sends a message, which includes the position of turtle, the type of patinas, and the lifetime of turtle, to the soil module before the L-system is derived. The soil module will then activate a query process that makes the soil module tackle the corresponding turtle information sent by the patina module. The soil module gathers corresponding tendency values from the turtle's neighbors according to what type of patinas the turtle belongs to and its surrounding environment. The next state of the turtle is evaluated and a record is kept in the module. Similarly, the results of query are sent back to the patina module. The next derivation step starts according to the production rules of L-system after all feedback generated by the soil environment has been received.

The metallic patina is a layered structure and each layer is composed of several chemical constituents¹⁰. First, a simple homogeneous patination model is considered. For ease of explanation, each layer of patinas is assumed to be homogeneous so that there are only two layers: the green patina

and the adherent patina. A set of vertices can be selected randomly at the start of patinas development. The patina development from these vertices simultaneously proceeds and follows the edges that are connected to these vertices according to the L-system. In addition to determining the growing direction of patinas at the current position requested by the patina module, the soil module determines whether another patina is triggered or not. The soil module begins to gather corresponding tendencies from neighboring vertices of the queried vertex, except those already have patinas, after receiving a message from patina module. Among the neighbors, vertices are produced if its tendency value is larger than both a threshold value and the tendency of the queried vertex. The soil module decides whether to stop growing or transform to another type of patina if no proper vertex is picked up to grow. The production rules of the patina module are depicted as follows:

```
#define L1 80 /* initial activity of patina type1 */
#define L2 2 /* initial activity of patina type2 */
p1: Q(M,n) < A(p,l,t) : l == 0 → R(p,t)
p2: Q(M,n) < A(p,l,t) : n == 0 & t == 1
      → R(p,t)Q(M,0)A(p,L2,t+1)
p3: Q(M,n) < A(p,l,t) : n == -1 → R(p,t)
p4: Q(M,n) < A(p,l,t) : * → R(p,t)Q(M,0)A(M[0],l-1,t)
      Q(M,0)A(M[1],l-2,t) ... Q(M,0)A(M[n-1],l-n,t)

axiom : Q(M,0)A(V1,L1,1)Q(M,0)A(V2,L1,1) ...
      Q(M,0)A(Vk,L1,1)
```

where M represents an array which stores candidates for growing patina returned by the soil, and n is the number of candidates. The soil module is fixed at $n = -1$ if it decides to stop growing. The parameter l denotes the lifetime of apex A , where an apex denotes a cell with metallic ions that could continuously move on surfaces, and t is the type of patinas. The lifetime of an apex decreases gradually until it reaches zero, meaning that the apex dies naturally. The communication symbol $Q(M,n)$ delineates a request to the soil environment as it receives the result of query. The symbol $R(p,t)$ indicates that there is a type t of patina at position p . The simulated development begins with an initial string $V_1, V_2, \dots, V_k$, that is chosen randomly, with an initial lifetime L_1 . The initial string is denoted as an *axiom* in the L-system. The derivation begins after all query processes have proceeded. First, the patina module checks whether the apex has run out of its lifetime or not according to the context-sensitive production p_1 . The module stops growing and replaces the pair of the apex $A(p,l,t)$ and the left context symbol $Q(M,n)$ by a patina node $R(p,t)$ if the lifetime equals zero. The soil module determines the next state according to the states of the vertex and its neighbors when no proper vertex is available to grow the same patina. A patina is induced at the second layer reposed on production p_2 if the vertex is not covered with a patina at the second layer. Otherwise, production p_3 generates a patina node and terminates the growing process. The growth stops (production p_3) when all neighbors are covered with patinas. The primary growth of patinas

is described by production p_4 . An apex A with a query Q creates a patina node R and several new apices A 's led by query nodes Q 's in each iteration after receiving the query result from the soil module. The new apex with higher tendency has a longer lifetime than those with a lower tendency. Figure 6a illustrates the patination result of this homogeneous model.

The above model demonstrates that the L-system of a patination process consists of the following components: terminating productions, patina type changing productions, a patina growing production, and an axiom. The axiom is a patination starting string that is composed of pairs of communication symbol Q and apex symbol A . The patination process starts from the apices. The production p_1 and p_3 are the termination criteria of the patination. The patinas stop developing when the lifetime runs out or no other apex is alive. The production p_2 is activated and starts growing another type of patina when the apex of a certain type of patina is dead and it is not covered with other patinas.

The proposed model can be extended to allow for two or more patinas on each layer for more complicated patinas. Although still having the same functionality, the soil module uses a probability function to determine which type of patina is triggered on a layer. The essential features of this non-homogeneous model are described by the following production rules.

```
#define L1 80 /* initial activity of patina type 1 */
#define L2 5 /* initial activity of patina type 2 */
#define L3 4 /* initial activity of patina type 3 */
#define L4 2 /* initial activity of patina type 4 */
p1: Q(M,n) < A(p,l,t) : l == 0 → R(p,t)
p2: Q(M,n) < A(p,l,t) : n == 0 & t == 1
      → R(p,t)Q(M,0)A(p,L2,t+1) : with prob = 0.5
p3: Q(M,n) < A(p,l,t) : n == 0 & t == 1
      → R(p,t)Q(M,0)A(p,L3,t+2) : with prob = 0.5
p4: Q(M,n) < A(p,l,t) : n == 0 & t == 2
      → R(p,t)Q(M,0)A(p,L4,t+2)
p5: Q(M,n) < A(p,l,t) : n == 0 & t == 3
      → R(p,t)Q(M,0)A(p,L4,t+1)
p6: Q(M,n) < A(p,l,t) : n == -1 → R(p,t)
p7: Q(M,n) < A(p,l,t) : * → R(p,t)Q(M,0)A(M[0],l-1,t)
      Q(M,0)A(M[1],l-2,t) ... Q(M,0)A(M[n-1],l-n,t)

axiom : Q(M,0)A(V1,L1,1)Q(M,0)A(V2,L1,1) ...
      Q(M,0)A(Vk,L1,1)
```

Similarly, the generation of patinas begins with k random apices. Each apex has a query node for sending a message to the soil module. Production p_1 describes the termination of growth when the lifetime equals zero. Two types of patinas are stimulated at the second layer according to production p_2 and p_3 with equal probability. The fourth type of patina is created from previous patinas when the triggering condition is satisfied at the top layer, whereas a patina node is produced at the current position and spreads out to the neighbors. The former patina is the functionality of production p_4 and p_5 . Production p_6 terminates the growth the same as the pro-

duction p_3 in the previous homogeneous model. The diffusion development of all patinas is depicted in production p_7 . The latter result generated by the non-homogeneous model is depicted in Figure 6b.

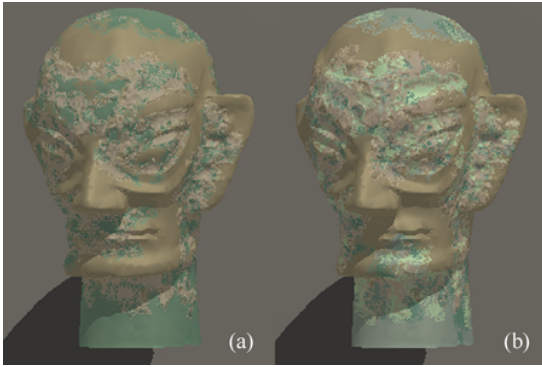


Figure 6: Bronze object with patinas. (a):Homogeneous patina model. (b):Non-homogeneous patina model.

5. Rendering

The Whitted illumination model³⁸ with a simple ray tracer simulates the reflection and transmission of light through patina layers to render the bronze model. The patina constituent of P is determined according to the vertices of the triangle containing it when a ray hits the object at a point P . A Perlin's noise function²⁹ determines the patina of P if the triangle vertices have distinct types of patina. Otherwise, P exhibits the patinas the same as the triangle. The boundary of patinas presents an irregular appearance according to this noise function. A simple sine function is applied to bump up adherent patinas at boundaries at the top layer of patinas, generally the adherent patina. In addition, the normal vectors of interior vertices of adherent patinas are also agitated to generate a rugged appearance. Patinas with high density would allow less light to be reflected from the subsurface because the high density implies a compact structure. The coefficient of patinas transmittance is adjusted, according to corresponding tendency values, to simulate the transmission of light through layers of varying densities.

6. Results

The proposed patination model is applied to various object models in this section. Different patina constructions can be realistically simulated according to distinct geometric properties and environmental factors. The non-homogeneous patina model simulates three layers of patinas as in Figure 7. The first layer is a thin film of copper chloride. A monomer patina composed of copper sulfate and chlorate develops in the second layer, and the top layer is an adherent patina. The Bronze Head model was created from a 3D digital scan and consisted of approximately 16,000 small evenly

sized triangles. The weighting coefficients of environmental factors and geometric factors are described in Table 2 and the distributions are depicted in Figure 8. The areas with high tendency values might cumulate more patinas than the ones with low tendencies because a high tendency value indicates a suitable environment for developing patinas. Figure 9 demonstrates distinct views of the same result.



Figure 7: Patinas on the ancient Bronze Head.



Figure 8: Tendency distributions of different patinas. (from left to right : tarnish, monomer1, monomer2, adherent)

Patina	Accessi- bility	Gravity	Curvature			Soil
			Max	Min	Mean	
Tarnish	-0.1	-0.1	0.2	0.2	0.1	0.28
Monomer1	-0.2	0.15	0.65	-0.2	0.1	0.4
Monomer2	0.1	0.0	0.3	0.1	0.0	0.68
Adherent	0.35	0.0	0.25	0.25	0.0	0.25

Table 2: The Weighted Parameters for Tendencies of Bronze Head.

Distinct environmental factors may result in various patina developments. The influence of gravity direction indicates the orientation of bronze objects buried in the soil. Different gravity directions are applied as depicted in Figure 10. However, the same L-system (non-homogeneous)



Figure 9: Different viewing directions of Bronze Head.

model simulates the influence of burial direction on a Bronze Mask. The burial direction determines the water retention of object surfaces. Upward surfaces might get wetter than downward ones. The values of gravity tropism are displayed in the left side of Figure 10. White colors indicate high values of tropism represent upward surfaces. Notices that the nose and the left prominent eye are covered with more patinas when they face upward. The Bronze Mask consists of 21,000 triangles and was discovered at San-hsing-tui like the Bronze Head.

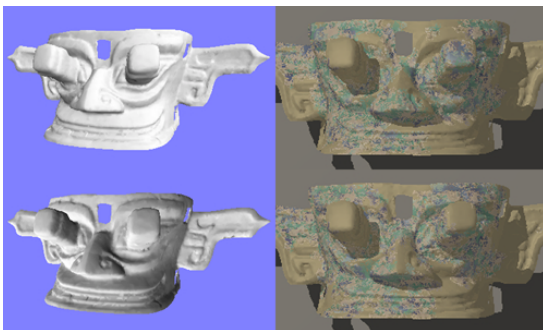


Figure 10: The influence of burial direction.

Bronze objects buried in the soil are always attacked by chlorine and produce a green patina. The influence of various chlorine distributions is displayed in Figure 11. The white color in the left part of Figure 11 depicts high density of chlorine. Thus, there are various sizes and distributions of the verdant patina. The face of a Buddha consists of approximately 12,000 triangles. Figure 12 illustrates the aging effect is a sequence of images with varying patination steps that correspond to different burial time periods.

7. Conclusions

This work has presented a physically-based model to simulate the development of patinas on underground objects. The

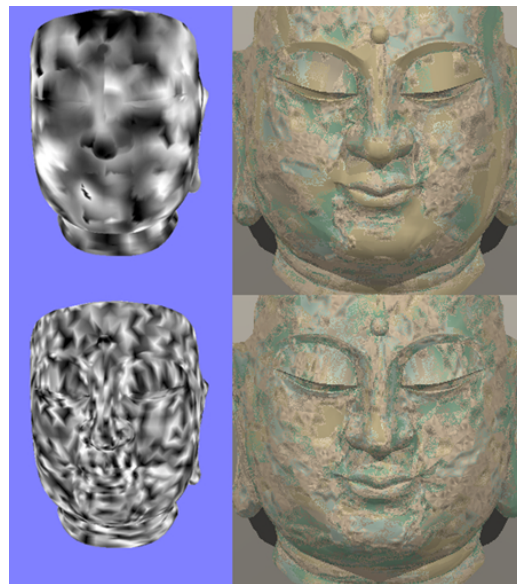


Figure 11: Two different distributions of chlorine in the soil.

model can automatically generate the patina patterns of distinct time periods while simultaneously considering the influence of environmental factors. The framework of open L-systems was employed to describe the relationship between the growth of patinas and the surrounding environment. A distribution of tendency is introduced to analyze the effect of environmental factors in patination. The L-systems applications were successfully extended to a process distinct from the growth of plants. Furthermore, the proposed approach successfully simulated the development of patinas on San-hsing-tui bronze objects. We shall focus on the diffusive development of patinas in the near future.

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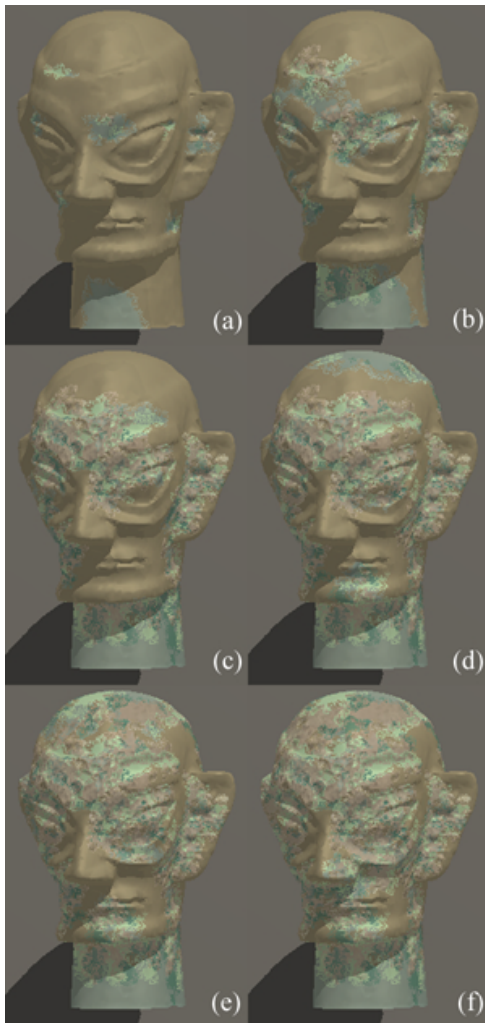


Figure 12: A sequence of images corresponds to different patination steps.

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