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Chaos control and anticontrol of a tachometer system by Ge–Yao–Chen partial region stability theory

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Abstract: In this paper, chaos control and anticontrol of a tachometer system by Ge–Yao–Chen (GYC) partial region stability are proposed. The Lyapunov function becomes a simple linear homogeneous function and the controllers become simpler by using the GYC partial region stability theory. The simulation results are more precise because the controllers are in lower degree than that of traditional controllers. Finally, chaos control and anticontrol of the tachometer system by GYC partial region stability are obtained and verified by numerical simulations.

Keywords: tachometer system, Lyapunov exponent, hyperchaos, Ge–Yao–Chen partial region stability theorem, chaos control, anticontrol

1 INTRODUCTION

Since the chaos control problem was first considered by Ott *et al.* [1, 2], it has been studied extensively. Chaos control is used to suppress or eliminate the chaotic dynamical behaviour in non-linear systems. There are many control methods for chaos control, such as feedback and non-feedback control [3–6], adaptive control [7, 8], observer-based control [9], inverse optimal control [10], and active control [11]. However, when Lyapunov asymptotical stability of zero solution of states is used, the asymptotical stability of solutions on the whole neighbourhood region of the origin is demanded.

Anticontrol [12–18] is an interesting, new, and challenging phenomenon. As a reverse process of suppressing or eliminating chaotic behaviours in order to reduce the complexity of an individual system or a coupled system, anticontrol of chaos aims at creating or enhancing the system complexity for some special applications. More precisely, anticontrolling chaos is to generate some chaotic behaviours from a given system, which is non-chaotic or even stable originally. By fully exploiting the intrinsic non-linearity,

this control technique provides another dimension for feedback systems design. Its potential applications can be easily found in many fields, typically including physics, biology, engineering, and medical as well as social sciences.

In this paper, a new method to achieve chaos control and anticontrol by the Ge–Yao–Chen (GYC), the partial region stability theory [19–21], is proposed. In this theory, when the asymptotical stability of zero solution of states is studied, the asymptotical stability of solution only on the partial neighbourhood region of the origin is demanded. Using this stability theory, the Lyapunov function becomes a simple linear homogeneous function of error states, and every term of the controllers are of lower degree than that of the controllers when the traditional Lyapunov asymptotical stability theory is used. The simulation results are more precise since the controllers are of a lower degree than that of traditional controllers.

This paper is organized as follows. In section 2, a chaos control and anticontrol scheme by the GYC partial region stability theory is proposed. In section 3, a tachometer system and a new hyperchaotic Mathieu–Duffing is introduced. In section 4, chaos control and anticontrol of the tachometer system by the GYC partial region stability theory are presented in three examples by simulations. Finally, conclusions are drawn in section 5. The partial region stability theory is introduced in the Appendix.

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2 CHAOS CONTROL AND ANTICONTROL SCHEME

Consider the following chaotic systems

$$\dot{\mathbf{x}}' = \mathbf{f}(t, \mathbf{x}') \quad (1)$$

where $\mathbf{x}' = [x'_1, x'_2, \dots, x'_n]^T \in R^n$ is a state vector and $\mathbf{f}: R_+ \times R^n \rightarrow R^n$ is a vector function. Translate the origin of the $\mathbf{x}'$ coordinate system to a new $\mathbf{x}$ coordinate system, where $\mathbf{x}' = \mathbf{x} - \mathbf{g}$ and $\mathbf{g}$ is a constant vector with suitable positive entries. Equation (1) becomes

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x} - \mathbf{g}) \quad (2)$$

The target system, which can be either chaotic or regular, is

$$\dot{\mathbf{y}} = \mathbf{h}(t, \mathbf{y}) \quad (3)$$

where $\mathbf{y} = [y_1, y_2, \dots, y_n]^T \in R^n$ is a state vector, and $\mathbf{h}: R_+ \times R^n \rightarrow R^n$ is a vector function. In order to make the chaotic state $\mathbf{x}$ approaching the target $\mathbf{y}$, define $\mathbf{e} = \mathbf{x} - \mathbf{y}$ as the error vector. The aim of adding $\mathbf{g}$ is to make the error dynamics always happen in the first quadrant of the error space where the GYC partial region stability theory can be used. When $\mathbf{y}$ is zero or a regular motion, chaos control is achieved; when $\mathbf{y}$ is a hyperchaotic motion, chaos anticontrol is achieved. The chaos control or anticontrol is accomplished in the sense that [22–24]

$$\lim_{t \rightarrow \infty} \mathbf{e} = \lim_{t \rightarrow \infty} (\mathbf{x} - \mathbf{y}) = 0 \quad (4)$$

The Lyapunov function becomes a simple linear homogeneous function of error states, and the controllers become simpler by using the GYC partial region stability theory. The simulation results are more precise because the controllers are in lower degree than that of traditional controllers.

3 CHAOTIC TACHOMETER SYSTEM AND NEW HYPERCHAOTIC MATHIEU–DUFFING SYSTEM

The tachometer system is shown in Fig. 1 [25]. In Fig. 1, the masses of the rods and vertical axis O_1O_2 are neglected, and balls A and B are assumed as particles with equal mass m_1 . The vertical axis rotates with a constant speed η and is subjected to a vertical vibration $A \sin x_3$, where x_3 is a state variable and A is the amplitude of the vibration. m_2 is the mass of the sleeve C, l is the length of rod BC, $2l$ is the length of AB, ϕ is the angle between rod AB and the vertical axis O_1O_2 , k_1 is the spring constant of a restoring spiral spring

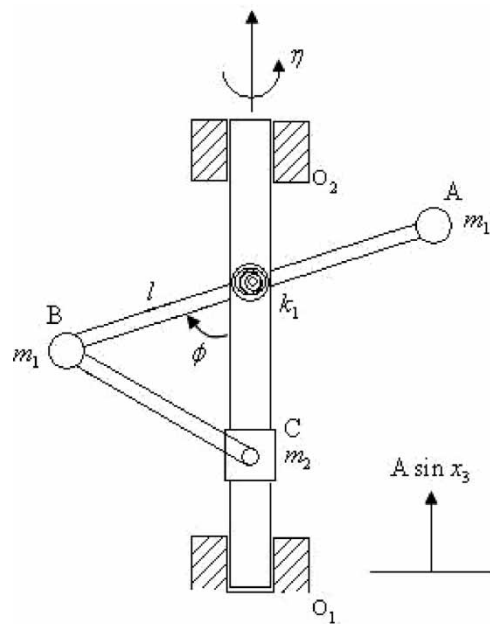


Fig. 1 Sketch of a tachometer with vibrating base

that is used to restrain the angle ϕ caused by centrifugal forces of A and B, and k_2 is the viscous damping coefficient caused by friction in the bearings. When $A = 0$, the vertical axis becomes a fixed axis, and the whole system rotates about this fixed axis. By dynamic equation, when η is constant, ϕ is a corresponding constant. η and ϕ are one to one corresponding to each other. Angle ϕ can be used to indicate the speed η . The system can be used as a tachometer. Let $x_1 = \phi$, $x_2 = \dot{\phi}$, and $x_4 = \dot{x}_3$, then by the Lagrange equation, the state equations for the autonomous tachometer system are

$$\begin{cases} \frac{d}{dt}x_1 = x_2 \\ \frac{d}{dt}x_2 = \frac{1}{2m_1 + 4m_2 \sin^2 x_1} \left(\frac{-2m_2 g \sin x_1}{l} + \frac{2m_2 A \sin x_3 \sin x_1}{l} - 4m_2 x_2^2 \sin x_1 \cos x_1 + 2m_1 \sin x_1 \cos x_1 \eta^2 - \frac{k_1 x_1}{l^2} - \frac{k_2 x_2}{l^2} \right) \\ \frac{d}{dt}x_3 = x_4 \\ \frac{d}{dt}x_4 = -A \sin x_3 \end{cases} \quad (5)$$

where x_1, x_2, x_3 , and x_4 are state variables and k_1, k_2, A, l, g, m_1 , and m_2 are constants, $k_1 = 4, k_2 = 1, m_2 = 3, m_1 = 3, A = 5, \eta = 4.5, g = 9.8$, and $l = 1.5$. The system exhibits chaotic behaviour with one positive Lyapunov

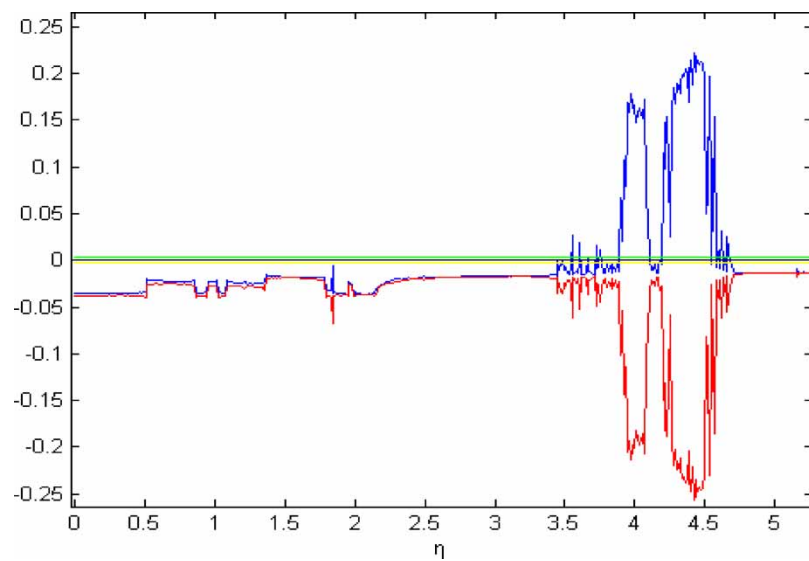


Fig. 2 Chaotic Lyapunov exponents for the tachometer system

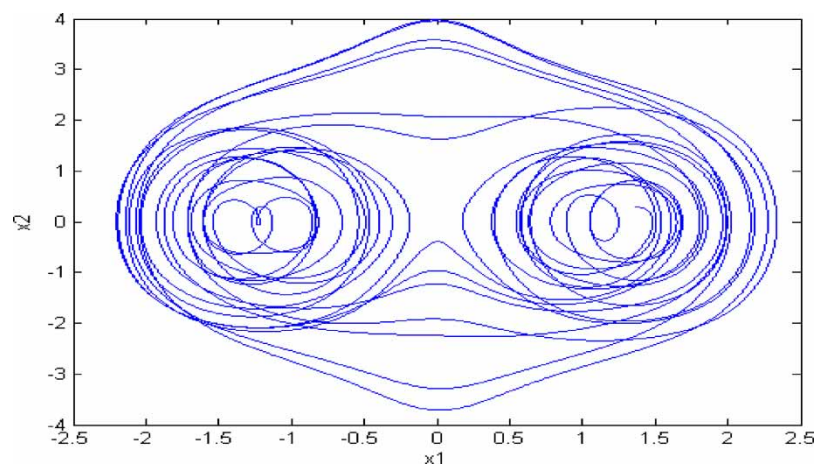


Fig. 3 Phase portrait for the tachometer system

exponent, as shown in Fig. 2. Its phase portraits are shown in Fig. 3.

A new hyperchaotic Mathieu–Duffing system is given by

$$\begin{cases} \frac{d}{dt}z_1 = z_2 \\ \frac{d}{dt}z_2 = -az_1 - bz_3z_1 - az_1^3 - bz_3z_1^3 - cz_2 + dz_3 \\ \frac{d}{dt}z_3 = z_4 \\ \frac{d}{dt}z_4 = -z_3 - z_3^3 - ez_4 + fz_1 \end{cases} \quad (6)$$

where its initial states are given by $(2, 3, 4, 2.5)$ and system parameters are $a = 20.30$, $b = 0.5970$, $c = 0.005$, $d = -24.441$, $e = 0.002$, and $f = 14.63$. The system exhibits hyperchaotic behaviour with two positive Lyapunov exponents as shown in Fig. 4. Its phase portraits are shown in Fig. 5.

4 CHAOS CONTROL AND ANTICONTROL OF TACHOMETER SYSTEM BY GYC PARTIAL REGION STABILITY THEORY

Case 1. Control the chaotic motion to zero.

The old origin of the coordinate system of the tachometer system equations is translated to $(x_1, x_2, x_3, x_4) = (g_1, g_2, g_3, g_4)$, where $g_1 = 5, g_2 = 5, g_3 = 5, g_4 = 5$.

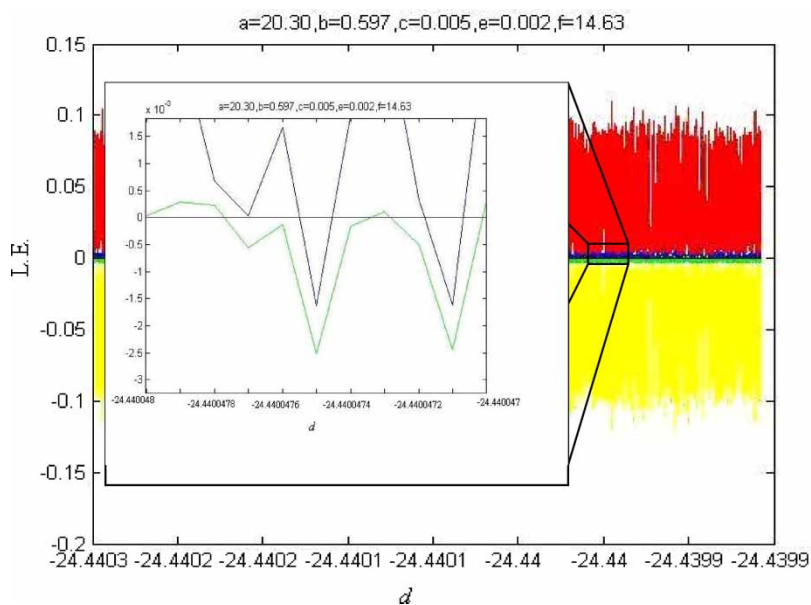


Fig. 4 The new Mathieu–Duffing system with hyperchaotic Lyapunov exponents

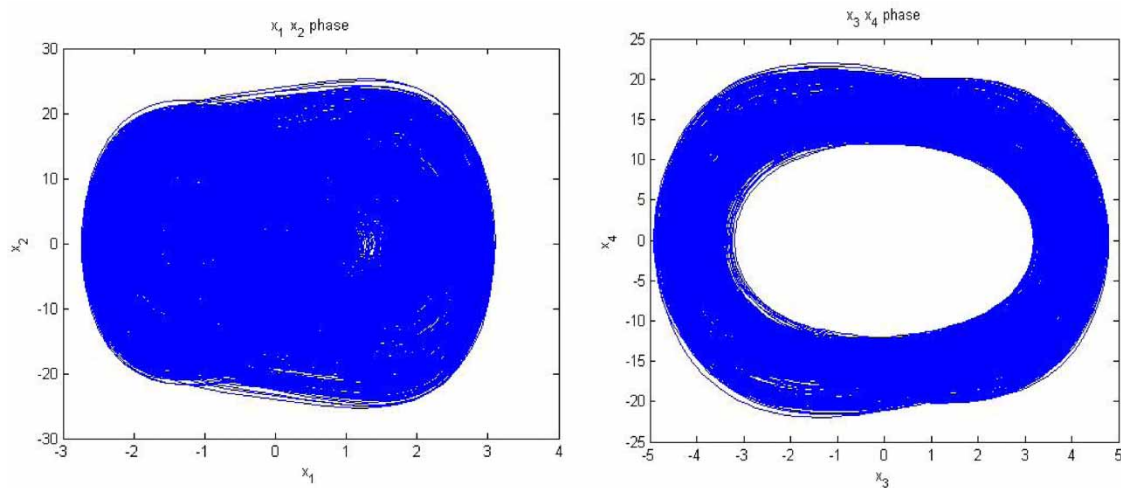


Fig. 5 Chaotic phase portraits for the Mathieu–Duffing system in the first quadrant

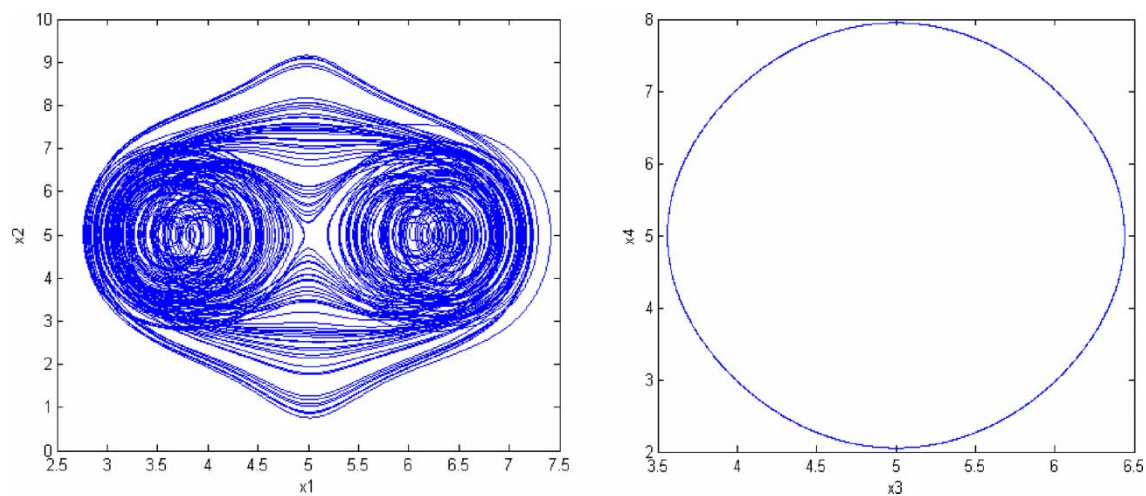


Fig. 6 Chaotic phase portraits for the tachometer system in the first quadrant

The tachometer system in the new coordinate system becomes

$$\left\{ \begin{array}{l} \frac{d}{dt}x_1 = x_2 - g_1 \\ \frac{d}{dt}x_2 = \frac{1}{2m_1 + 4m_2 \sin^2(x_1 - g_2)} \\ \quad \times \left[\frac{-2m_2 g \sin(x_1 - g_2)}{l} \right. \\ \quad + \frac{2m_2 A \sin(x_3 - g_2) \sin(x_1 - g_2)}{l} \\ \quad - 4m_2 (x_2 - g_2)^2 \sin(x_1 - g_2) \cos(x_1 - g_2) \\ \quad + 2m_1 \sin(x_1 - g_2) \cos(x_1 - g_2) \eta^2 \\ \quad \left. - \frac{k_1(x_1 - g_2)}{l^2} - \frac{k_2(x_2 - g_2)}{l^2} \right] \\ \frac{d}{dt}x_3 = x_4 - g_3 \\ \frac{d}{dt}x_4 = -A \sin(x_3 - g_4) \end{array} \right. \quad (7)$$

and the chaotic motion always happens in the first quadrant of the coordinate system (x_1, x_2, x_3, x_4) . This tachometer system is presented as simulated examples where the initial conditions are $x_{10} = 6.53$, $x_{20} = 5.53$, $x_{30} = 6.415$, and $x_{40} = 5.5$. The chaotic motion in the first quadrant is shown in Fig. 6.

In order to lead (x_1, x_2, x_3, x_4) to the target, controllers u_1, u_2, u_3, u_4 are added to each equation of equation (7), respectively

$$\left\{ \begin{array}{l} \frac{d}{dt}x_1 = x_2 - g_1 + u_1 \\ \frac{d}{dt}x_2 = \frac{1}{2m_1 + 4m_2 \sin^2(x_1 - g_2)} \\ \quad \times \left[\frac{-2m_2 g \sin(x_1 - g_2)}{l} \right. \\ \quad + \frac{2m_2 A \sin(x_3 - g_2) \sin(x_1 - g_2)}{l} \\ \quad - 4m_2 (x_2 - g_2)^2 \sin(x_1 - g_2) \cos(x_1 - g_2) \\ \quad + 2m_1 \sin(x_1 - g_2) \cos(x_1 - g_2) \eta^2 \\ \quad \left. - \frac{k_1(x_1 - g_2)}{l^2} - \frac{k_2(x_2 - g_2)}{l^2} \right] + u_2 \\ \frac{d}{dt}x_3 = x_4 - g_3 + u_3 \\ \frac{d}{dt}x_4 = -A \sin(x_3 - g_4) + u_4 \end{array} \right. \quad (8)$$

In this case, the chaotic motion of the tachometer system (7) will be controlled to zero. The target is $y = 0$. The state error is $e_i = x_i - y_i = x_i$, $i = 1, 2, 3, 4$. The purpose here is $\lim_{t \rightarrow \infty} e = 0$. The error dynamics becomes

$$\left\{ \begin{array}{l} \dot{e}_1 = \dot{x}_1 = x_2 - g_1 + u_1 \\ \dot{e}_2 = \dot{x}_2 = \frac{1}{2m_1 + 4m_2 \sin^2(x_1 - g_2)} \\ \quad \times \left[\frac{-2m_2 g \sin(x_1 - g_2)}{l} \right. \\ \quad + \frac{2m_2 A \sin(x_3 - g_2) \sin(x_1 - g_2)}{l} \\ \quad - 4m_2 (x_2 - g_2)^2 \sin(x_1 - g_2) \cos(x_1 - g_2) \\ \quad + 2m_1 \sin(x_1 - g_2) \cos(x_1 - g_2) \eta^2 \\ \quad \left. - \frac{k_1(x_1 - g_2)}{l^2} - \frac{k_2(x_2 - g_2)}{l^2} \right] + u_2 \\ \dot{e}_3 = \dot{x}_3 = x_4 - g_3 + u_3 \\ \dot{e}_4 = \dot{x}_4 = -A \sin(x_3 - g_4) + u_4 \end{array} \right. \quad (9)$$

In Fig. 7, it is noted that the error dynamics always exists in the first quadrant of the coordinate system.

By the GYC partial region stability theory, one can easily choose a linear homogeneous Lyapunov function in the form of a positive definite function in the first quadrant as

$$V = e_1 + e_2 + e_3 + e_4 \quad (10)$$

Its time derivative through the error dynamics (9) is

$$\begin{aligned} \dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\ &= (e_2 - g_1 + u_1) + \left\{ \frac{1}{2m_1 + 4m_2 \sin^2(e_1 - g_2)} \right. \\ &\quad \times \left[\frac{-2m_2 g \sin(e_1 - g_2)}{l} \right. \\ &\quad + \frac{2m_2 A \sin(e_3 - g_2) \sin(e_1 - g_2)}{l} \\ &\quad - 4m_2 (e_2 - g_2)^2 \sin(e_1 - g_2) \cos(e_1 - g_2) \\ &\quad + 2m_1 \sin(e_1 - g_2) \cos(e_1 - g_2) \eta^2 \\ &\quad \left. - \frac{k_1(e_1 - g_2)}{l^2} - \frac{k_2(e_2 - g_2)}{l^2} \right] + u_2 \Big\} \\ &\quad + (e_4 - g_3 + u_3) + [-A \sin(e_3 - g_4) + u_4] \end{aligned} \quad (11)$$

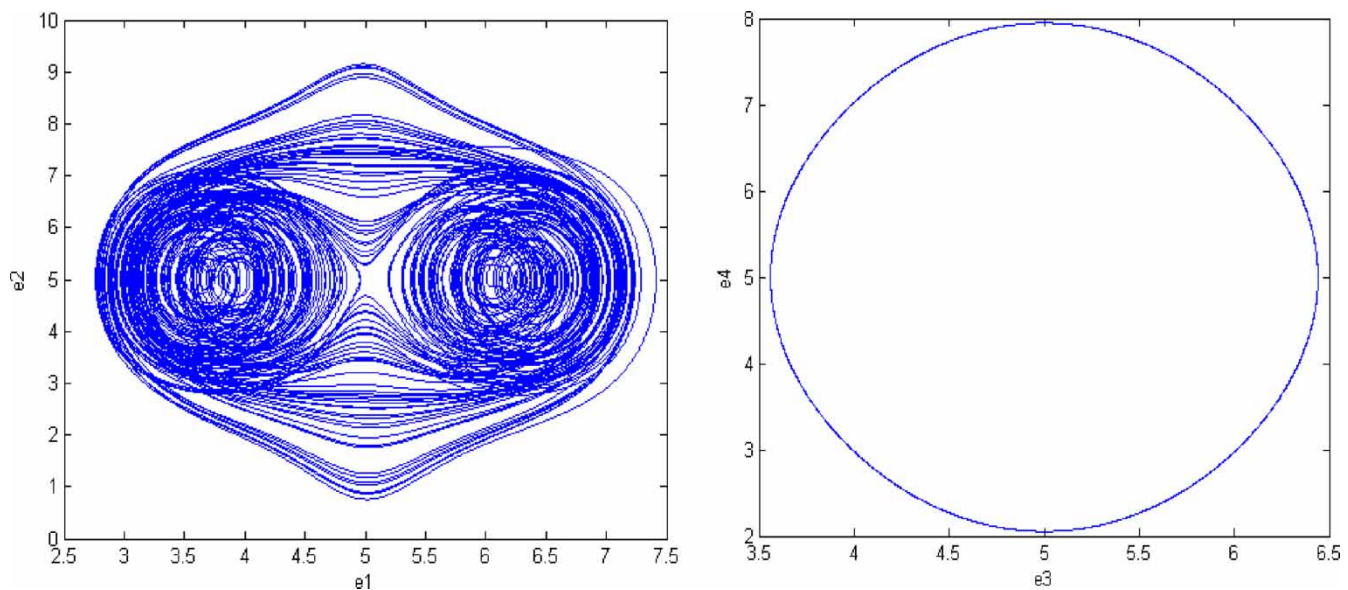


Fig. 7 Phase portraits of error dynamics for Case 1

Choose

$$\begin{aligned}
 u_1 &= -e_2 + g_1 - e_1 \\
 u_2 &= -\frac{1}{2m_1 + 4m_2 \sin^2(e_1 - g_2)} \left[\frac{-2m_2 g \sin(e_1 - g_2)}{l} \right. \\
 &\quad + \frac{2m_2 A \sin(e_3 - g_2) \sin(e_1 - g_2)}{l} \\
 &\quad - 4m_2 (e_2 - g_2)^2 \sin(e_1 - g_2) \cos(e_1 - g_2) \\
 &\quad + 2m_1 \sin(e_1 - g_2) \cos(e_1 - g_2) \eta^2 \\
 &\quad \left. - \frac{k_1 (e_1 - g_2)}{l^2} - \frac{k_2 (e_2 - g_2)}{l^2} \right] - e_2 \\
 u_3 &= -e_4 + g_3 - e_3 \\
 u_4 &= +A \sin(e_3 - g_4) - e_4
 \end{aligned} \quad (12)$$

One can obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \quad (13)$$

which is a negative definite function in the first quadrant. By the asymptotical stability theorem for the partial region, $\lim_{t \rightarrow \infty} \mathbf{e} = 0$. The numerical results are shown in Fig. 8. When the controllers act, the motion trajectories approach the origin within 5 s.

Case 2. Control the chaotic motion to a sine function.

The tachometer system of the old origin is translated to $(x_1, x_2, x_3, x_4) = (g_1, g_2, g_3, g_4)$, where $g_1 = 30, g_2 = 30, g_3 = 30$, and $g_4 = 30$, and the chaotic motion always happens in the first quadrant of the new coordinate system. This tachometer system is presented

as simulated examples where the initial conditions is $x_{10} = 31.53, x_{20} = 29.53, x_{30} = 31.415$, and $x_{40} = 30.5$.

In this case, the chaotic motion of the tachometer system (7) is controlled to a sine function of time. The target is $y = F \sin \omega t$. The error dynamics are

$$\begin{aligned}
 e_i &= x_i - y_i = x_i - F \sin \omega_i t \\
 \dot{e}_i &= \dot{x}_i - \dot{y}_i = \dot{x}_i - \omega_i F \cos \omega_i t
 \end{aligned} \quad (14)$$

where $F = 0.5, \omega_1 = 0.1, \omega_2 = 0.2, \omega_3 = 0.15$, and $\omega_4 = 0.3$. The goal is

$$\lim_{t \rightarrow \infty} e_i = \lim_{t \rightarrow \infty} (x_i - F_i \sin \omega_i t) = 0, \quad i = 1, 2, 3, 4$$

The error dynamics are

$$\begin{aligned}
 \dot{e}_1 &= \dot{x}_1 - \omega_1 F \cos \omega_1 t = x_2 - g_1 - \omega_1 F \cos \omega_1 t + u_1 \\
 \dot{e}_2 &= \dot{x}_2 - \omega_2 F \cos \omega_2 t \\
 &= \frac{1}{2m_1 + 4m_2 \sin^2(x_1 - g_2)} \left[\frac{-2m_2 g \sin(x_1 - g_2)}{l} \right. \\
 &\quad + \frac{2m_2 A \sin(x_3 - g_2) \sin(x_1 - g_2)}{l} \\
 &\quad - 4m_2 (x_2 - g_2)^2 \sin(x_1 - g_2) \cos(x_1 - g_2) \\
 &\quad + 2m_1 \sin(x_1 - g_2) \cos(x_1 - g_2) \eta^2 \\
 &\quad \left. - \frac{k_1 (x_1 - g_2)}{l^2} - \frac{k_2 (x_2 - g_2)}{l^2} \right] - \omega_2 F \cos \omega_2 t + u_2 \\
 \dot{e}_3 &= \dot{x}_3 - \omega_3 F \cos \omega_3 t = x_4 - g_3 - \omega_3 F \cos \omega_3 t + u_3 \\
 \dot{e}_4 &= \dot{x}_4 - \omega_4 F \cos \omega_4 t = -A \sin(x_3 - g_4) \\
 &\quad - \omega_4 F \cos \omega_4 t + u_4
 \end{aligned} \quad (15)$$

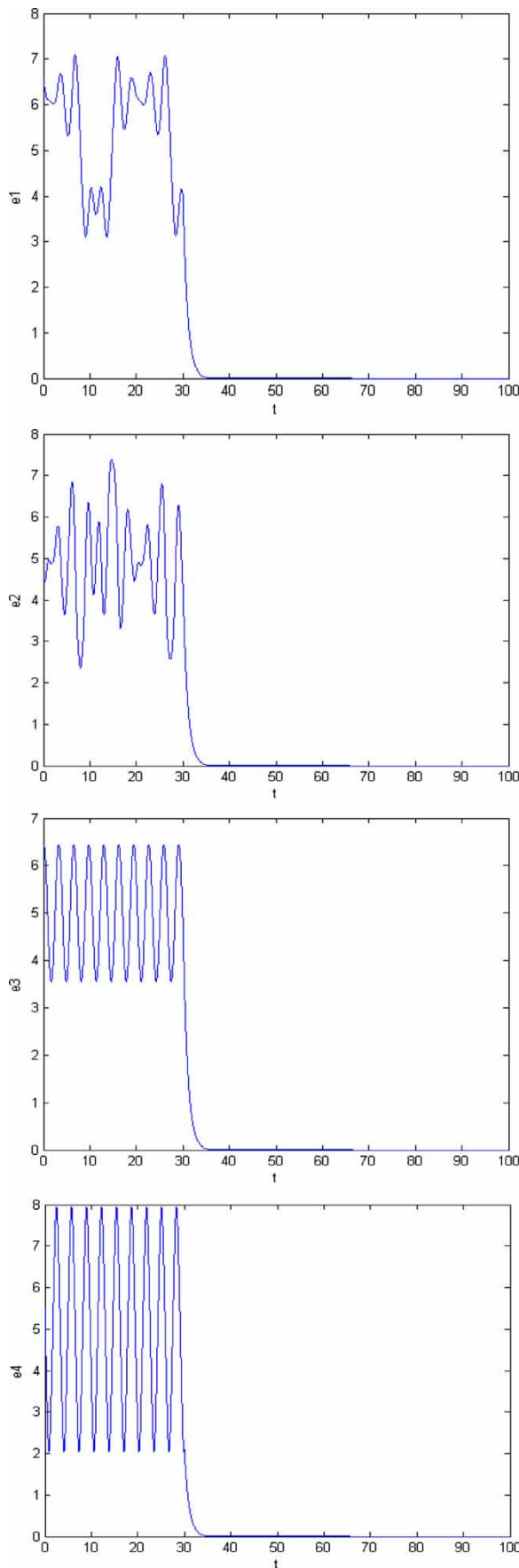


Fig. 8 Time histories of errors for Case 2

In Fig. 9, the error dynamics always exists in the first quadrant.

By GYC partial region stability, one can easily choose a linear homogeneous Lyapunov function in the form of a positive definite function in the first quadrant as

$$V = e_1 + e_2 + e_3 + e_4 \quad (16)$$

Its time derivative is

$$\begin{aligned} \dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 \\ &= (x_2 - g_1 - \omega_1 F \cos \omega_1 t + u_1) \\ &\quad + \left\{ \frac{1}{2m_1 + 4m_2 \sin^2(x_1 - g_2)} \left[\frac{-2m_2 g \sin(x_1 - g_2)}{l} \right. \right. \\ &\quad + \frac{2m_2 A \sin(x_3 - g_2) \sin(x_1 - g_2)}{l} \\ &\quad - 4m_2 (x_2 - g_2)^2 \sin(x_1 - g_2) \cos(x_1 - g_2) \\ &\quad + 2m_1 \sin(x_1 - g_2) \cos(x_1 - g_2) \eta^2 \\ &\quad \left. - \frac{k_1 (x_1 - g_2)}{l^2} - \frac{k_2 (x_2 - g_2)}{l^2} \right] - \omega_2 F \cos \omega_2 t + u_2 \Big\} \\ &\quad + (x_4 - g_3 - \omega_3 F \cos \omega_3 t + u_3) \\ &\quad + [-A \sin(x_3 - g_4) - \omega_4 F \cos \omega_4 t + u_4] \quad (17) \end{aligned}$$

Choose

$$\begin{aligned} u_1 &= -x_2 + g_1 + \omega_1 F \cos \omega_1 t - e_1 \\ u_2 &= -\frac{1}{2m_1 + 4m_2 \sin^2(x_1 - g_2)} \left[\frac{-2m_2 g \sin(x_1 - g_2)}{l} \right. \\ &\quad + \frac{2m_2 A \sin(x_3 - g_2) \sin(x_1 - g_2)}{l} \\ &\quad - 4m_2 (x_2 - g_2)^2 \sin(x_1 - g_2) \cos(x_1 - g_2) \\ &\quad + 2m_1 \sin(x_1 - g_2) \cos(x_1 - g_2) \eta^2 - \frac{k_1 (x_1 - g_2)}{l^2} \\ &\quad \left. - \frac{k_2 (x_2 - g_2)}{l^2} \right] + \omega_2 F \cos \omega_2 t - e_2 \\ u_3 &= -x_4 + g_3 + \omega_3 F \cos \omega_3 t - e_3 \\ u_4 &= A \sin(x_3 - g_4) + \omega_4 F \cos \omega_4 t - e_4 \quad (18) \end{aligned}$$

One can obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \quad (19)$$

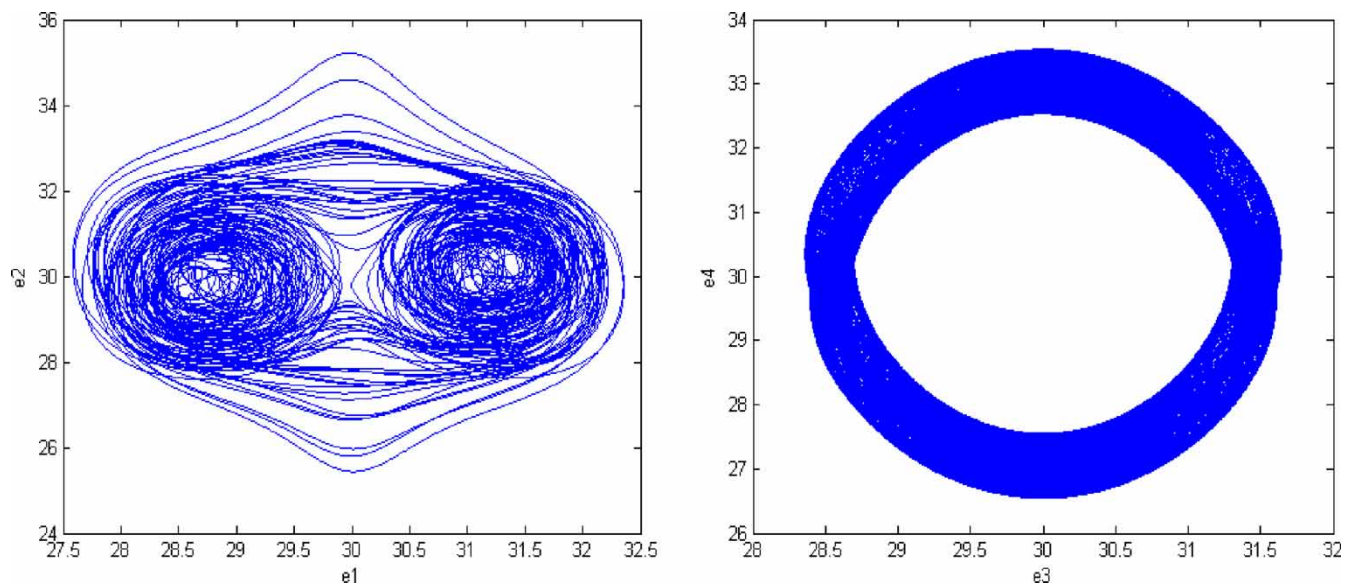


Fig. 9 Phase portraits of error dynamics for Case 2

which is a negative definite function in the first quadrant. By the asymptotical stability theorem for the partial region, $\lim_{t \rightarrow \infty} \mathbf{e} = 0$. The numerical results are shown in Figs 10 and 11. When the controllers act, the errors approach zero and the motion trajectories approach to sine functions within 5 s.

Case 3. Anticontrol of the chaotic motion of the tachometer system to the hyperchaotic motion of a new Mathieu–Duffing system.

The tachometer system of the old origin is translated to $(x_1, x_2, x_3, x_4) = (g_1, g_2, g_3, g_4)$ where $g_1 = 30, g_2 = 30, g_3 = 30$, and $g_4 = 30$, and the chaotic motion always happens in the first quadrant of new coordinate system. This chaotic tachometer system is presented as simulated examples where the initial conditions are $x_{10} = 51.53, x_{20} = 49.53, x_{30} = 51.415$, and $x_{40} = 50.5$.

In this case, the chaotic motion to the hyperchaotic motion of a new Mathieu–Duffing system is controlled. The target system is a new Mathieu–Duffing system

$$\begin{cases} \frac{d}{dt} z_1 = z_2 \\ \frac{d}{dt} z_2 = -az_1 - bz_3 z_1 - az_1^3 - bz_3 z_1^3 - cz_2 + dz_3 \\ \frac{d}{dt} z_3 = z_4 \\ \frac{d}{dt} z_4 = -z_3 - z_3^3 - tz_4 + fz_1 \end{cases} \quad (20)$$

The error state is $e_i = x_i - z_i$ ($i = 1, 2, 3, 4$). Our goal is $\lim_{t \rightarrow \infty} \mathbf{e} = 0$.

The error dynamics become

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{z}_1 = x_2 - g_1 - z_2 + u_1 \\ \dot{e}_2 &= \dot{x}_2 - \dot{z}_2 \\ &= \frac{1}{2m_1 + 4m_2 \sin^2(x_1 - g_2)} \left[\frac{-2m_2 g \sin(x_1 - g_2)}{l} \right. \\ &\quad + \frac{2m_2 A \sin(x_3 - g_2) \sin(x_1 - g_2)}{l} \\ &\quad - 4m_2 (x_2 - g_2)^2 \sin(x_1 - g_2) \cos(x_1 - g_2) \\ &\quad + 2m_1 \sin(x_1 - g_2) \cos(x_1 - g_2) \eta^2 \\ &\quad \left. - \frac{k_1 (x_1 - g_2)}{l^2} - \frac{k_2 (x_2 - g_2)}{l^2} \right] \\ &\quad + az_1 + bz_3 z_1 + az_1^3 + bz_3 z_1^3 + cz_2 - dz_3 + u_2 \\ \dot{e}_3 &= \dot{x}_3 - \dot{z}_3 = x_4 - g_3 - z_4 + u_3 \\ \dot{e}_4 &= \dot{x}_4 - \dot{z}_4 = -A \sin(x_3 - g_4) + z_3 + z_3^3 + tz_4 \\ &\quad - fz_1 + u_4 \end{aligned} \quad (21)$$

Figure 12 shows that the error dynamics always exists in the first quadrant.

By GYC partial region stability, one can easily choose a Lyapunov function in the form of a positive definite function in the first quadrant as

$$V = e_1 + e_2 + e_3 + e_4 \quad (22)$$

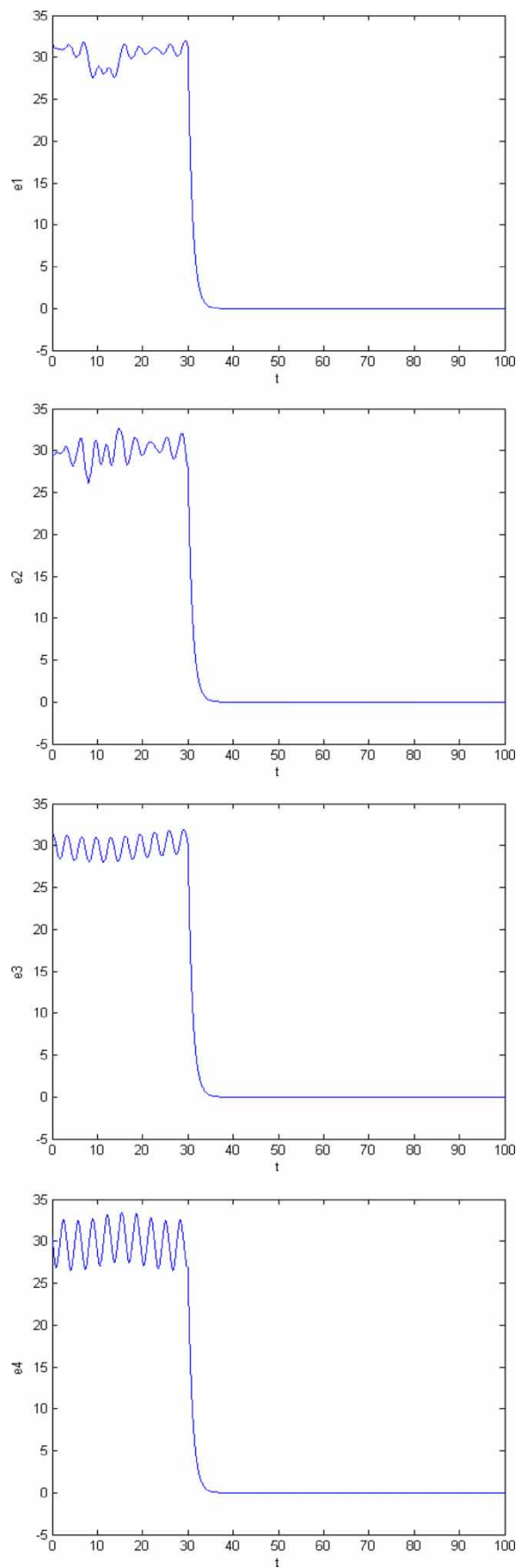
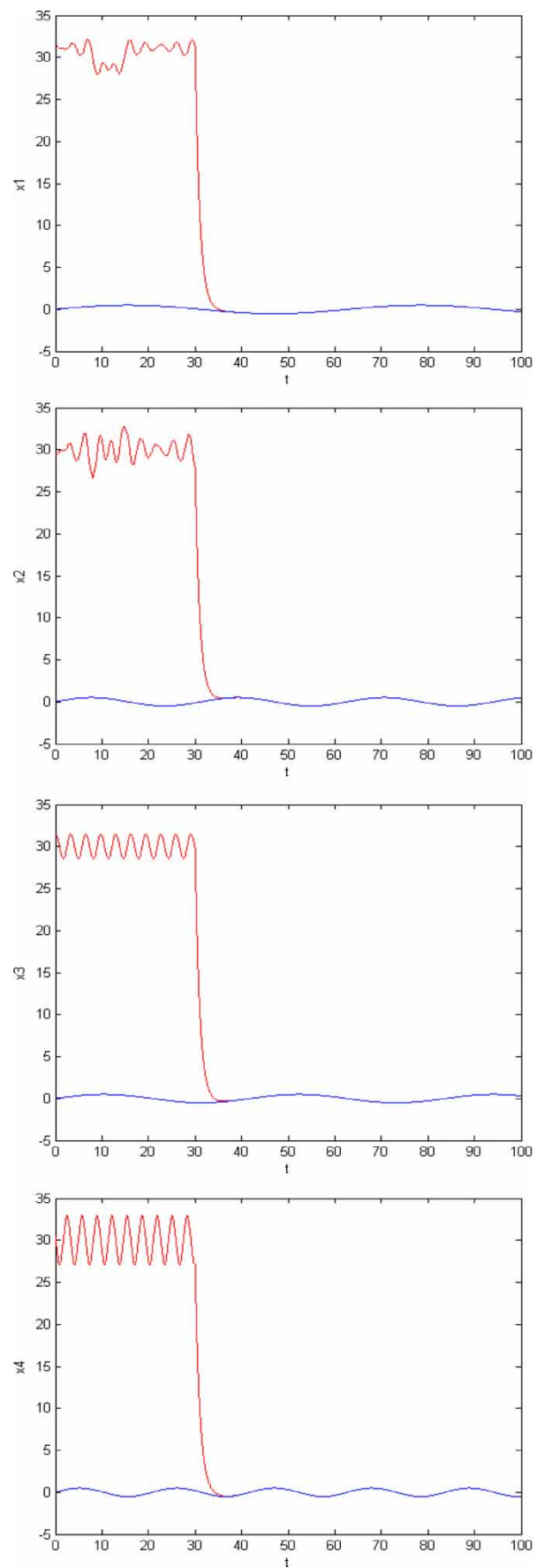


Fig. 10 Time histories of errors for Case 2

Fig. 11 Time histories of x_1 , x_2 , x_3 , and x_4 for Case 2

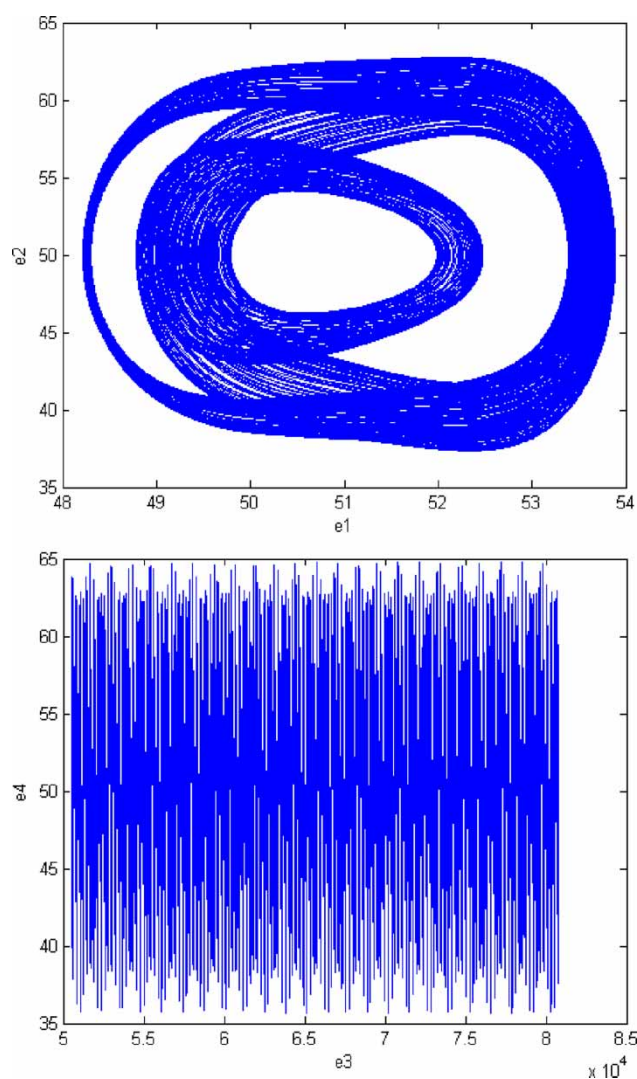


Fig. 12 Phase portraits of error dynamics for Case 3

Its time derivative is

$$\begin{aligned}
 \dot{V} &= \dot{e}_1 + \dot{e}_2 + \dot{e}_3 + \dot{e}_4 \\
 &= (x_2 - g_1 - z_2 + u_1) \\
 &\quad + \left\{ \frac{1}{2m_1 + 4m_2 \sin^2(x_1 - g_2)} \left[\frac{-2m_2 g \sin(x_1 - g_2)}{l} \right. \right. \\
 &\quad + \frac{2m_2 A \sin(x_3 - g_2) \sin(x_1 - g_2)}{l} \\
 &\quad - 4m_2 (x_2 - g_2)^2 \sin(x_1 - g_2) \cos(x_1 - g_2) \\
 &\quad + 2m_1 \sin(x_1 - g_2) \cos(x_1 - g_2) \eta^2 \\
 &\quad \left. - \frac{k_1 (x_1 - g_2)}{l^2} - \frac{k_2 (x_2 - g_2)}{l^2} \right] \\
 &\quad + az_1 + bz_3 z_1 + az_1^3 + bz_3 z_1^3 + cz_2 - dz_3 + u_2 \Big\} \\
 &\quad + (x_4 - g_3 - z_4 + u_3) \\
 &\quad + [-A \sin(x_3 - g_4) + z_3 + z_3^3 + tz_4 - fz_1 + u_4]
 \end{aligned} \tag{23}$$

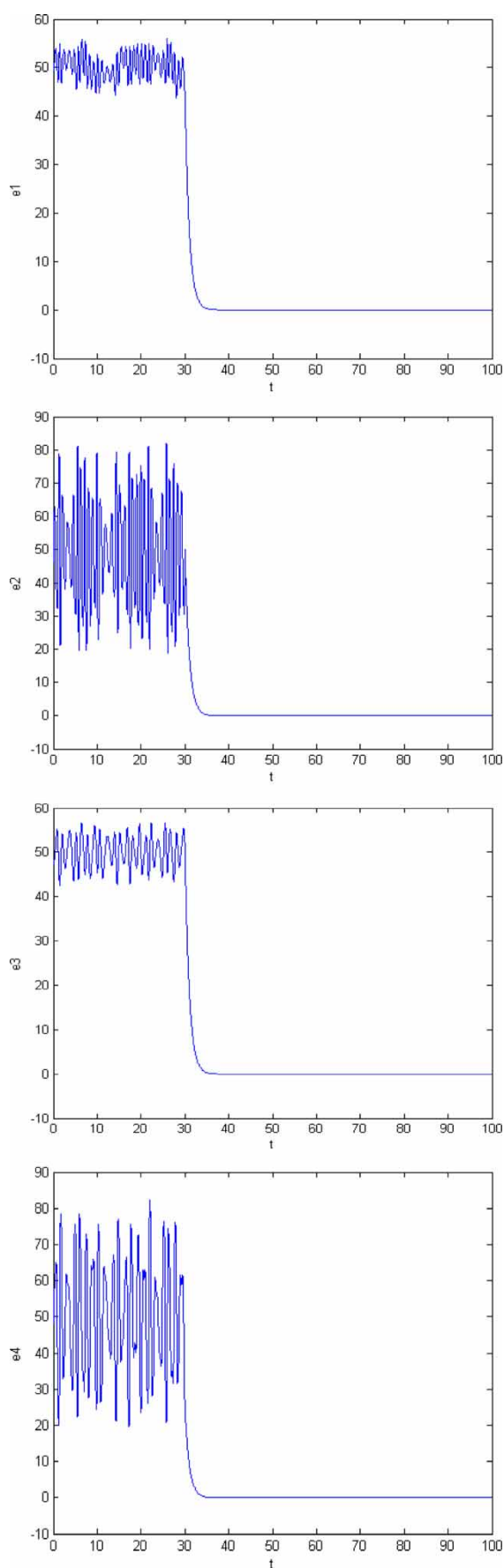


Fig. 13 Time histories of errors for Case 3

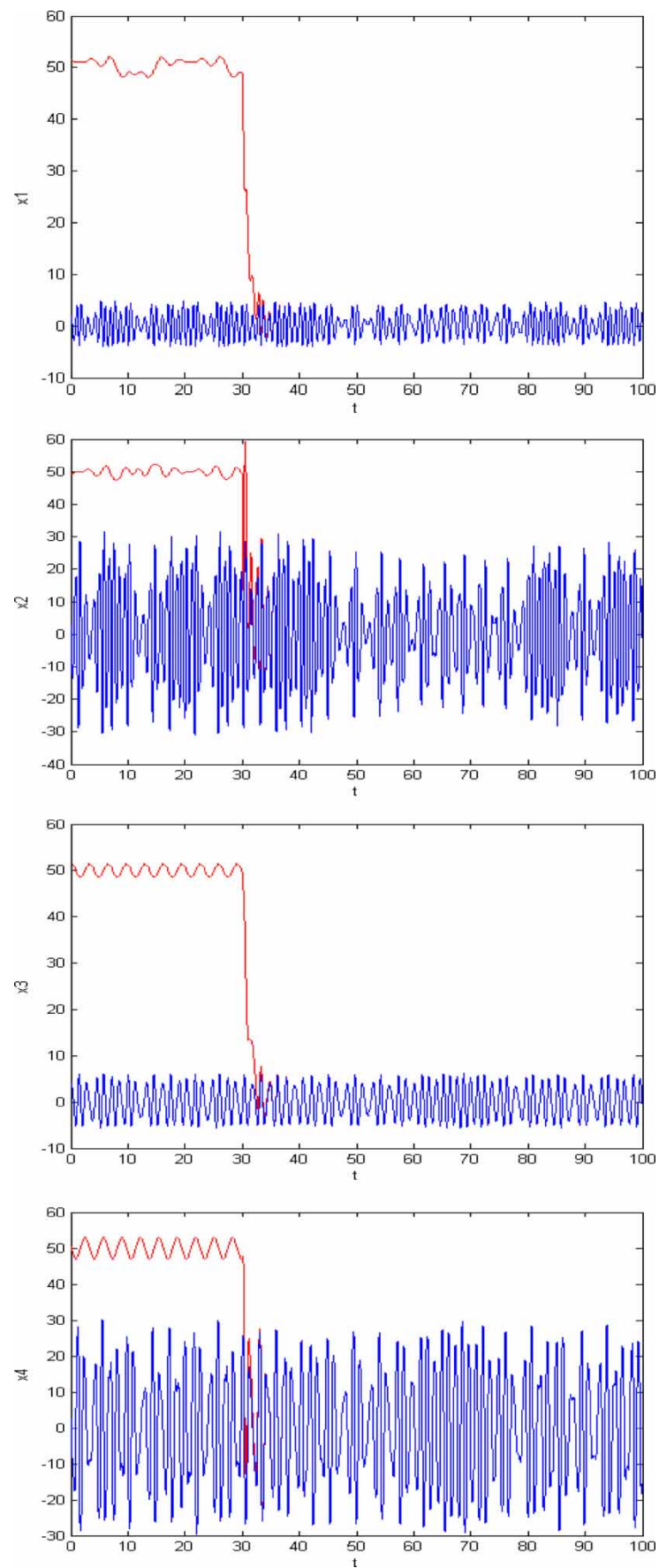


Fig. 14 Time histories of x_1 , x_2 , x_3 , and x_4 for Case 3

Choose

$$\begin{aligned}
 u_1 &= -x_2 + g_1 + z_2 - e_1 \\
 u_2 &= -\frac{1}{2m_1 + 4m_2 \sin^2(x_1 - g_2)} \left[\frac{-2m_2 g \sin(x_1 - g_2)}{l} \right. \\
 &\quad + \frac{2m_2 A \sin(x_3 - g_2) \sin(x_1 - g_2)}{l} \\
 &\quad - 4m_2 (x_2 - g_2)^2 \sin(x_1 - g_2) \cos(x_1 - g_2) \\
 &\quad + 2m_1 \sin(x_1 - g_2) \cos(x_1 - g_2) \eta^2 \\
 &\quad \left. - \frac{k_1 (x_1 - g_2)}{l^2} - \frac{k_2 (x_2 - g_2)}{l^2} \right] \\
 &\quad - az_1 - bz_3 z_1 - az_1^3 - bz_3 z_1^3 - cz_2 + dz_3 - e_2 \\
 u_3 &= -x_4 + g_3 + z_4 - e_3 \\
 u_4 &= +A \sin(x_3 - g_4) - z_3 - z_3^3 - tz_4 + fz_1 - e_4
 \end{aligned} \tag{24}$$

One can obtain

$$\dot{V} = -e_1 - e_2 - e_3 - e_4 < 0 \tag{25}$$

which is a negative definite function in the first quadrant. By the asymptotical stability theorem for the partial region, $\lim_{t \rightarrow \infty} e = 0$. The numerical results are shown in Figs. 13 and 14. When the controllers act, the errors approach zero and the chaotic motion of tachometer system approaches to the hyperchaotic motion of the new Mathieu–Duffing system within 5 s.

5 CONCLUSIONS

In this paper, chaos control and anticontrol of a chaotic tachometer system by GYC partial region stability are studied. The simulation results are more precise because the controllers are of a lower degree than that of traditional controllers. Three simulation examples are given. When controllers are in action, the error states rapidly approach zero within 5 s in all three examples, which means this chaos control and anticontrol scheme is very effective. Simulations show that for other continuous systems, this method is also effective. Besides, traditional anticontrol of a chaotic system is limited to increase the chaos or hyperchaos of the chaotic system itself. In this paper, chaos anticontrol is extended to make the chaos of a chaotic system increase to the hyperchaos of any other system.

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APPENDIX: GYC PARTIAL REGION STABILITY THEORY

Definitions of the stability on partial region

Consider the differential equations of disturbed motion of a non-autonomous system in the normal form

$$\frac{dx_s}{dt} = X_s(t, x_1, \dots, x_n), \quad s = 1, \dots, n \quad (26)$$

where the function X_s is defined on the intersection of the partial region Ω (shown in Fig. 15) and

$$\sum_s x_s^2 \leq H \quad (27)$$

and $t > t_0$, where t_0 and H are certain positive constants. X_s , which vanishes when the variables x_s are all zero, is a real valued function of $t, x_1, \dots, x_n$. It is assumed that X_s is smooth enough to ensure the existence and uniqueness of the solution of the initial value problem. When X_s does not contain t explicitly, the system is autonomous [19–21].

Obviously, $x_s = 0$ ($s = 1, \dots, n$) is a solution of equation (26). The asymptotical stability of this zero solution on the partial region Ω (including the boundary) of the neighbourhood of the origin, which in

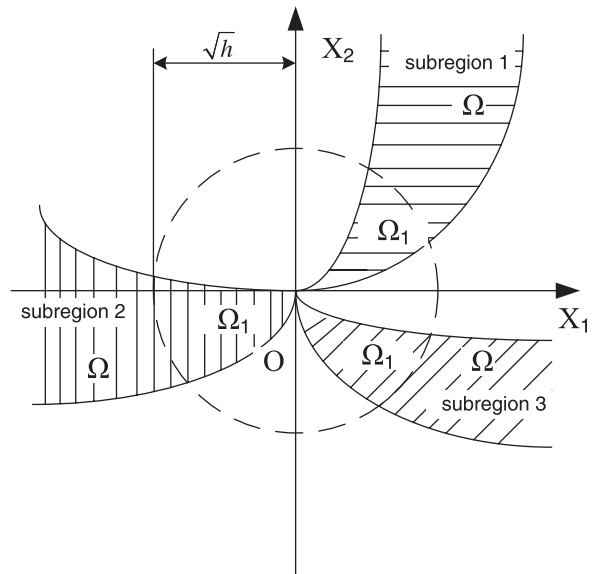


Fig. 15 Partial regions Ω and Ω_1

general may consist of several subregions (Fig. 15) is of interest.

Definition 1

If the undisturbed motion is stable on the partial region Ω , and there exists a $\delta' > 0$, so that on the given partial region Ω when

$$\sum_s x_{s0}^2 \leq \delta', \quad s = 1, \dots, n \quad (28)$$

If the equality

$$\lim_{t \rightarrow \infty} \left(\sum_s x_s^2 \right) = 0 \quad (29)$$

is satisfied for the solutions of equation (26) on Ω , then the undisturbed motion $x_s = 0$ ($s = 1, \dots, n$) is asymptotically stable on the partial region Ω .

The intersection of Ω and the region defined by equation (30) is called the region of attraction.

Definition of functions. $V(t, x_1, \dots, x_n)$.

Consider the functions $V(t, x_1, \dots, x_n)$ given on the intersection Ω_1 of the partial region Ω and the region

$$\sum_s x_s^2 \leq h, \quad (s = 1, \dots, n) \quad (30)$$

for $t \geq t_0 > 0$, where t_0 and h are positive constants. Suppose that the functions are single valued and have continuous partial derivatives and become zero when $x_1 = \dots = x_n = 0$ [26–28].

Definition 2

If there exists $t_0 > 0$ and a sufficiently small $h > 0$, so that on the partial region Ω_1 and $t \geq t_0$, $V \geq 0$ (or ≤ 0), then V is a positive (or negative) semi-definite, in general semi-definite, function on Ω_1 and $t \geq t_0$.

Definition 3

If there exists a positive (negative) definitive function $W(x_1, \dots, x_n)$ on Ω_1 , so that on the partial region Ω_1 and $t \geq t_0$

$$V - W \geq 0 \text{ (or } -V - W \geq 0) \quad (31)$$

then $V(t, x_1, \dots, x_n)$ is a positive definite function on the partial region Ω_1 and $t \geq t_0$.

Definition 4: Function with infinitesimal upper bound

If V is bounded, and for any $\lambda > 0$ there exists $\mu > 0$, so that on Ω_1 when $\sum_s x_s^2 \leq \mu$, and $t \geq t_0$, one can obtain

$$|V(t, x_1, \dots, x_n)| \leq \lambda$$

then V admits an infinitesimal upper bound on Ω_1 .

GYC theorem of stability and of asymptotical stability on partial region*Theorem 2*

If there can be found for the differential equations of the disturbed motion (equation (26)) a definite function $V(t, x_1, \dots, x_n)$ on the partial region, and for which the derivative with respect to time based on these equations as given by the following

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + \sum_{s=1}^n \frac{\partial V}{\partial x_s} X_s \quad (32)$$

is a semidefinite function on the partial region whose sense is opposite to that of V , or if it becomes zero identically, then the undisturbed motion is stable on the partial region.

Theorem 3

If in satisfying the conditions of Theorem 1, the derivative dV/dt is a definite function on the partial region with opposite sign to that of V and the function V itself permits an infinitesimal upper limit, then the undisturbed motion is asymptotically stable on the partial region.